Answer Key

Chapter 1: Critical Thinking and Problem Solving

1.1 Exercises

1. Inductive **3.** False

5. Deductive reasoning

7. Deductive reasoning

9. Deductive reasoning

11. Inductive reasoning

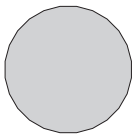
13. 9, 11, 13; arithmetic; common difference = 2

15. −5, −10, −15; arithmetic; common difference = −5

17. $\frac{1}{16}, \frac{1}{32}, \frac{1}{64}$; geometric; common ratio = $\frac{1}{2}$

19. 100,000, 1,000,000, 10,000,000; geometric; common ratio = 10

21.



Geometric; common ratio = 2. (A figure with 24 sides)

23. Answers will vary. There are seven days in a week, so if the first Tuesday of the month was the 2nd (an even day), then seven days later that Tuesday's date would be the 9th (an odd day). Therefore, not every Tuesday in the month is an even day.

25. Answers will vary.

$5 - 3 = 2$. The difference between any two odd numbers is always an even number. Therefore, if the difference between two numbers is even, the numbers do not have to both be even.

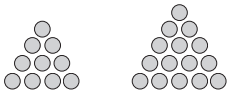
27. Answers will vary.

If $a = 5$, $b = 1$, and $c = 2$ then $a > b$ and $a > c$, but $b < c$.

29. b **31.** b

33. 10 and 15

35.



9	3	4	5	6	8	1	2	7
8	2	6	7	1	4	5	9	3
1	5	7	9	2	3	4	6	8
2	7	8	1	5	9	3	4	6
6	4	1	3	8	7	2	5	9
3	9	5	6	4	2	7	8	1
5	6	3	4	9	1	8	7	2
7	8	9	2	3	5	6	1	4
4	1	2	8	7	6	9	3	5

37. **a.** 1,333,332 **b.** 1,333,332

c. Inductive reasoning

1.2 Exercises

1. order of magnitude

3. True

5. Answers may vary. Ten thousands

7. Answers may vary. Tens

9. **b.** Knowing the average weight of a newborn Asian elephant can help determine how large it must grow during the gestation period.

11. **a.** Knowing the number of employees who work in the office can be helpful if you can also estimate the amount of time a single person spends in a meeting.

13. $540,000 **15.** $7875

17. Answers will vary. Need to provide food, drinks, cake or cupcakes, decorations, and games.

19. Answers will vary. Need to lease or buy a building, to purchase all kitchen equipment, to purchase dining tables and chairs, and to set aside money to cover employee paychecks for 6 months.

21. Answers may vary.

a. $1120; **b.** $7840

23. Answers may vary.

a. 5400 pounds

b. 64,800 pounds

25. Approximately 29,450 ears of corn per acre and 2,945,000 ears of corn for the season.

27. Approximately 7254 sprinkles

29. 810 tacos

31. 319 jelly beans

33. Answers will vary. For example,

$\$120/\text{hr} \cdot 100\ \text{hr} + 2(\$90/\text{hr} \cdot 50\ \text{hr}) + 3(\$120/\text{hr} \cdot 100\ \text{hr}) = \$57{,}000$

35. Answers will vary.
Bills come to about $\$180 + \$140 + \$180 = \$500$ per month. So leftover income will be about $\$1700 - \$500 = \$1200$.

37. Answers will vary.
$(\$25{,}000 - \$7500) \div 4 = \$4375$

39. Answers will vary.
$9 \cdot \$5 + 6 \cdot \$30 = \$225$

41. $60 \cdot 160$ will be the most precise. Answers will vary. For example, these numbers are rounded to the nearest 10 instead of 100, so there will be less rounding error.

1.3 Exercises

1. True **3.** False

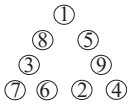
5. 20 pieces of candy

7.

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

9.

11.



13.

4	3	8
9	5	1
2	7	6

15. a. 72 · 1; 36 · 2; 24 · 3; 18 · 4; 9 · 8; 8 · 9; 4 · 18; 3 · 24; 2 · 36; 1 · 72

b. 9 · 8 or 8 · 9

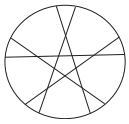
17. Marcus should purchase the miniature racetrack with cars, the fire station playset, the dinosaur expedition playset, and either the toy soldiers set or the construction truck set for a total of either $84.66 or $88.66.

19. Yes, Sofía can do the aquarium, art museum, and wine tasting for $148.

21. 405,450 **23.** 122,500

25. 140 total games

27.



29. Three 41¢ stamps and six 8¢ stamps

31. 6 packs of fudge swirls and 7 packs of extra chocolate chunk

33. 41 and 22 **35.** 4

Chapter 1 Exercises

1. Inductive reasoning

3. Answers will vary. Most Super Bowls have been played in January. For example, Super Bowl XXXVII was played on January 26, 2003.

5. Answers will vary. The sum of two odd numbers is always even, so if the sum is odd, then the numbers cannot both be odd. For example, $4 + 5 = 9$, and 4 is even, not odd.

7. 108, 216, 648; neither

9. $12{,}345{,}678 \cdot 8 + 8 = 98{,}765{,}432$

11. Answers may vary. Tens

13. Approximately 4544 people

15. 45 at $35 and 50 at $55

17.

4
2 3
9 7
5 1 8 6

19. 4; 7; 10; 13; 49

Chapter 2: Set Theory

2.1 Exercises

1. False; the set may be an infinite set.

3. True

5. True

7. True **9.** True

11. $B = \{$Nebraska, Nevada, New Hampshire, New Jersey, New Mexico, New York, North Dakota, North Carolina$\}$

13. Answers will vary.

15. $F = \{$Monday, Tuesday, Wednesday, Thursday, Friday$\}$

17. $B = \{3, 6, 9, 12, 15\}$

19. $D = \{$Wyoming, Nebraska, Kansas, New Mexico, Oklahoma, Utah, Arizona$\}$

21. $|X| = 10$

23. $|Y| = 43$, as of printing

25. Neither **27.** Neither

29. $H = \{x \mid x \in \mathbb{N}, x \leq 50\}$

31. $K = \{x \mid x \in U, x \text{ is an athlete}\}$

33. $M = \{3x \mid x \text{ is a positive integer}\}$

35. $P = \{x^2 \mid x \text{ is a whole number}\}$

37. Not the empty set

39. Not the empty set

41. $A' = \{$c, d, f, g, h, i, j, l, m, n, o, p, q, r, u, v, w, x, y, z$\}$

43. $A' = \{$c, d, f, g, h, i, j, l, m, n, o, p, q, r, u, v, w, x, y, z, A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U ,V, W, X, Y, Z$\}$

45. 4 **47.** 6

49. 15

51. Yes; they have the same elements, just in a different order.

53. Yes; P, R, and S are all equivalent. They all contain 5 elements. Q is not equivalent to any of them.

55. No; they have different elements.

57. B is equivalent to C since they each have 5 elements. A and D are not equivalent to the other sets.

2.2 Exercises

1. Venn diagram **3.** element

5. True

7. $A' = \{$Alex, Georgina, Charles, Olivia, Richard, Karl, Rhonda, Matthew$\}$

9. $M = \{5,10,15,20,25\}$, $N = \{12,24\}$, $P = \{2,4,9,16,25\}$

11. $M' = \{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 24\}$

13. $Q = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$
$R = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$
$S = \varnothing$

15. $R' = \{1,3,5,7,9,11,13,15,17,19\}$

17. Yes

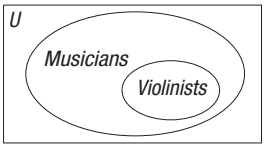
19. $\varnothing$, {*The Card Players* by Cézanne}, {*No. 5 1948* by Pollock}, {*Woman III* by de Kooning}, {*The Card Players* by Cézanne, *No. 5 1948* by Pollock}, {*The Card Players* by Cézanne, *Woman III* by de Kooning}, {*No. 5 1948* by Pollock, *Woman III* by de Kooning}, {*The Card*

Players by Cézanne, *No. 5 1948* by Pollock, *Woman III* by de Kooning}.

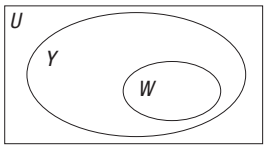
21. ∅, {lemon}, {lime}, {lemon, lime}

23. ∅, {four}, {vier}, {cuatro}, {quatre}, {four, vier}, {four, cuatro}, {four, quatre}, {vier, cuatro}, {vier, quatre}, {cuatro, quatre}, {four, vier, cuatro}, {four, vier, quatre}, {four, cuatro, quatre}, {vier, cuatro, quatre}, {four, vier, cuatro, quatre}

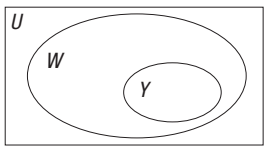
25.



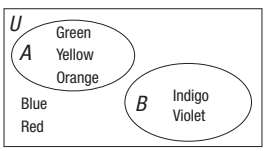
27. a.



b.



29.



31. Proper subsets are B and C.

33. ∅, {2}, {4}, {6}, {2, 4}, {2, 6}, {4, 6}

35. ∅, {cloudy}, {rainy}, {sunny}, {cloudy, rainy}, {cloudy, sunny}, {rainy, sunny}

37. ∅

39. $2^8 = 256$ subsets and $2^8 - 1 = 255$ proper subsets

41. 5 **43.** 8

45. California **47.** 22,825

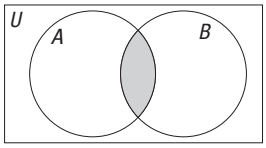
49. Because you have to eliminate the set itself (where you choose all 6 keys) and you have to eliminate the empty set (where you don't choose any keys), all proper non-empty subsets: $2^6 - 2 = 62$.

2.3 Exercises

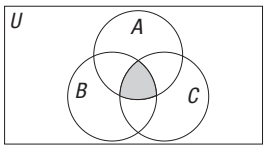
1. intersection **3.** complement

5. $A \cap B'$

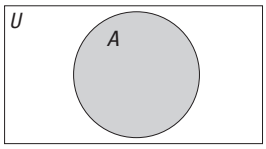
7.



9.



11.



13. $A \cap B = \{4\}$ **15.** $|A \cap B| = 1$

17. $X \cup Y = \{1, 3, 5, 13, 21\}$

19. $|X \cup Y| = 5$

21. We have $(X \cup Y)' = \{2, 8, 34\}$. Since $X' = \{2, 8, 34\}$ and $Y' = \{1, 2, 5, 8, 13, 34\}$, we have $X' \cap Y' = \{2, 8, 34\}$. Therefore, $(X \cup Y)' = X' \cap Y'$.

23. $H \cap R = \{r, i, h\}$

25. $|H \cap R| = 3$

27. $D \cup N = \{d, a, n, c, e, o, t\}$

29. $|D \cup N| = 7$

31. We have $(D \cap N)' = \{d, u, c, a, t, i, o\}$. Since $D' = \{u, t, i, o\}$ and $N' = \{d, u, c, a, i\}$, $D' \cup N' = \{d, u, c, a, t, i, o\}$. Therefore, $(D \cap N)' = D' \cup N'$.

33. $B \cup (C \cap P)$ $= \{b, i, o, g, r, a, p, h, y, c\}$

35. We have $C \cup P = \{c, h, i, p, o, t, l, e, r, a, y\}$, so $(C \cup P)' = \{g, b\}$. Similarly, $C' = \{y, r, g, a, b\}$ and $P' = \{o, g, h, t, b, l, e\}$, so $C' \cap P' = \{g, b\}$. Therefore, $(C \cup P)' = C' \cap P'$.

37. $S \cup (M \cap T) = \{s, n, o, w, m, e, l, t, d\}$

39. We have $M \cap T = \{o, d, e\}$, so $(M \cap T)' = \{m, l, t, w, n, s\}$. Similarly, $M' = \{t, w, n, s\}$ and $T' = \{m, l, n, s\}$ so $M' \cup T' = \{m, l, t, w, n, s\}$. Therefore, $(M \cap T)' = M' \cup T'$.

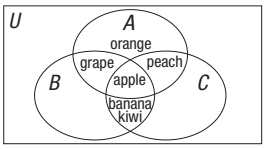
41. Will, David, Kim, Barbara, Alden, Morgan, Ali, Holly, Jessica, Jeff, Kent

43. 11 **45.** 32

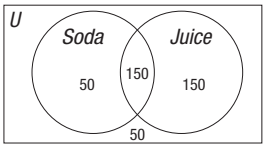
2.4 Exercises

1. four **3.** True

5.



7. 50 students

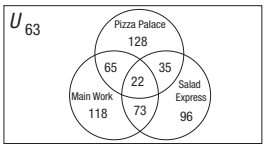


9. a. 21 **b.** 1 **c.** 20

11. a. $\left|(A \cup B \cup C)'\right| = 3$. This is the number of people who did not like any part of the meal;

b. $|B \cup C| = 69$. This is the number of people who liked the main course or dessert or both.

13. 600 students were surveyed.



15. a. 62 **b.** 17 **c.** 5

17. a. 33 **b.** 111 **c.** 24

19. 92; After accounting for each of the patients in categories A, B, and Rh, there were 92 of the 200 patients not accounted for. Therefore, those 92 must be O−.

Chapter 2 Exercises

1. False; the set containing 3 is not an element of the set.

3. True **5.** True

7. False; the cardinal number of the empty set is 0.

9. $A = \{2, 4, 6, 8, 10, 12\}$

11. $C = \{x \mid x \in \mathbb{R}, 100 < x < 1000\}$

13. $|A| = 2, |B| = 3$

15. Proper subsets of A: $\varnothing$, {Felix}, {Amber}

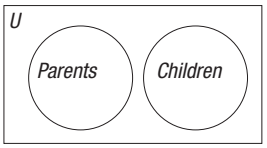
17. No, A has 2 elements and B has 3 elements.

19. Subsets of G: $\varnothing$, {I}, {II}, {III}, {I,II}, {I,III}, {II, III}, {I, II, III}; Subsets of F: $\varnothing$, {love}, {joy}, {peace}, {love, joy}, {love, peace}, {joy, peace}, {love, joy, peace}

21. No, they contain different elements.

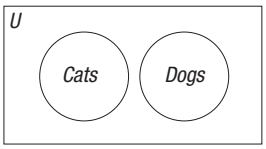
23. 255

25.



Universal set will vary.

27.



Universal set will vary.

29. {1, 3, 5}

31. {7} **33.** 3

35. {m, e, d, i, c, a, l, n, k}

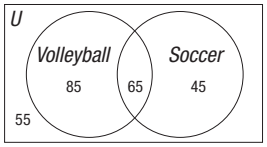
37. {n, k}

39. {b, f, h, j, o, p, q, r, s, v, w, x, y, z}

41. {a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z}

43. $|A \cap B| = 2$

45. a.



b. 85 **c.** 45 **d.** 55

47. 33%

Chapter 3: Logic

3.1 Exercises

1. paradox **3.** and, or

5. True **7.** Yes

9. No **11.** No

13. No **15.** Yes

17. Yes

19. 329 is not the number of people who applied for the same job I did.

21. Austin did not sleep until 8:00 a.m. this morning.

23. Not all students are volunteering at the food pantry this year.

25. $\sim m \vee \sim n$ **27.** $\sim b \vee a$

29. $t \Rightarrow \sim r$ **31.** $w \Rightarrow z$

33. $\sim a \Leftrightarrow b$

35. Driving makes me smile or it is sunny.

37. If it is sunny, then I grill more often than I bake.

39. If it is not sunny, then I grill no more than I bake.

41. If I do not have 1000 Facebook friends, then the home page of my website does not have a bounce rate of less than 20%.

43. The home page of my website does not have a bounce rate of less than 20% if and only if my video did not reach 1000 views on YouTube.

45. The moon is not full or I don't know if it's cloudy or bright outside.

47. The moon is full if and only if I do know if it's cloudy or bright outside.

49. If I have not lost my glasses, then I do know if it's cloudy or bright outside.

3.2 Exercises

1. truth values

3. both a and b are false

5. True

7.

w	z	$\sim w$	$\sim z$	$\sim w \vee \sim z$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

9.

m	n	$\sim n$	$m \Leftrightarrow \sim n$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	F

11.

s	t	$s \Leftrightarrow t$	$\sim(s \Leftrightarrow t)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	T	F

13.

p	r	q	$p \vee r$	$(p \vee r) \Rightarrow q$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	F	T

15.

m	n	r	$m \wedge n$	$(m \wedge n) \Leftrightarrow r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	F
T	F	F	F	T
F	T	T	F	F
F	T	F	F	T
F	F	T	F	F
F	F	F	F	T

17.

a	b	c	$\sim a$	$\sim b$	$\sim b \Leftrightarrow c$	$(\sim b \Leftrightarrow c) \Rightarrow \sim a$
T	T	T	F	F	F	T
T	T	F	F	F	T	F
T	F	T	F	T	T	F
T	F	F	F	T	F	T
F	T	T	T	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

19. a: I plan to go to the movies.; b: I plan to go out to eat this weekend.; $a \wedge b$

a	b	$a \wedge b$
T	T	T
T	F	F
F	T	F
F	F	F

21. a: My calculator is working correctly; b: I need a new calculator; c: My calculator calculates that one plus one equals three; $(\sim a \wedge b) \Leftrightarrow c)$

a	b	c	$\sim a$	$\sim a \wedge b$	$(\sim a \wedge b) \Leftrightarrow c)$
T	T	F	F	F	F
T	T	F	F	F	T
T	F	T	F	F	F
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	F
F	F	F	T	F	T

23. a: Meg gets gas.; b: Meg's car will break down.; c: Meg will miss her exam.; $\sim a \Rightarrow (b \wedge c)$

a	b	c	$\sim a$	$b \wedge c$	$\sim a \Rightarrow (b \wedge c)$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	T	F	F

25. Not a tautology

27. Tautology

29. Not a tautology; a: The bird is blue; b: The bird is green; c: The bird is from here; $(a \vee b) \Rightarrow \sim c$

a	b	c	$\sim c$	$a \vee b$	$(a \vee b) \Rightarrow \sim c$
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	T	F	T

31. Tautology; a: The animal is a penguin; b: The animal wears a bow tie; $(a \Rightarrow b) \Leftrightarrow (\sim b \Rightarrow \sim a)$

a	b	$\sim a$	$\sim b$	$a \Rightarrow b$	$\sim b \Rightarrow \sim a$	$(a \Rightarrow b) \Leftrightarrow (\sim b \Rightarrow \sim a)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

3.3 Exercises

1. Logically equivalent

3. conjunction

5. False

7. None are logically equivalent.

p	q	$p \Rightarrow q$	$\sim(p \Rightarrow q)$	$p \vee q$	$\sim(p \Rightarrow q) \Rightarrow p \vee q$
T	T	T	F	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	F	F	T

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \vee q$	$\sim(p \vee q)$	$(\sim p \wedge \sim q) \Leftrightarrow \sim(p \vee q)$
T	T	F	F	F	T	F	T
T	F	F	T	F	T	F	T
F	T	T	F	F	T	F	T
F	F	T	T	T	F	T	T

9. $p \wedge (\sim p \vee q)$ and $p \wedge q$ are logically equivalent.

p	q	$\sim p$	$\sim p \vee q$	$p \wedge (\sim p \vee q)$	$p \wedge q$	$[p \wedge (\sim p \vee q)] \Leftrightarrow (p \wedge q)$
T	T	F	T	T	T	T
T	F	F	F	F	F	T
F	T	T	T	F	F	T
F	F	T	T	F	F	T

11. Logically equivalent

m	n	$\sim n$	$n \Rightarrow m$	$m \vee \sim n$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	F
F	F	T	T	T

13. Logically equivalent

r	s	$\sim s$	$r \Rightarrow s$	$r \wedge \sim s$	$\sim(r \Rightarrow s)$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

15. Logically equivalent

p	q	$\sim p$	$q \wedge \sim p$	$p \vee q$	$p \vee (q \vee \sim p)$
T	T	F	F	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	F	T	F	F	F

17. The dog will not wear a bow tie or will wear a hat, or the dog will not have his picture taken.

19.

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

21. $p \wedge q$ **23.** $\sim(\sim p \wedge \sim q)$

25. $a \wedge (\sim(c \vee d))$

$a \wedge (\sim c \wedge \sim d)$

$a \wedge \sim c \wedge \sim d$

27. $(w \vee z) \wedge \sim(w \wedge z)$

$(w \vee z) \wedge (\sim w \vee \sim z)$

29. I leave my computer alone for 15 minutes, and it does not go to sleep.

31. I am in Charleston, South Carolina, and I am not on Eastern Standard Time.

33. Statements a and c are logically equivalent to the given conditional statement.

35. It is not true that in Biology the nucleotide bases adenine and thymine do not pair together in DNA or the bases pair together in RNA.

37. Converse: If I get an A in Biology, then I got an 89 on the final. Inverse: If I do not get an 89 on the final, then I will not get an A in Biology. Contrapositive: If I do not get an A in Biology, then I did not get an 89 on the final. Biconditional: I will get an 89 on the final if and only if I get an A in Biology.

39. Converse: If I smile, then I have seen a puppy. Inverse: If I do not see a puppy, then I will not smile. Contrapositive: If I do not smile, then I have not seen a puppy. Biconditional: I have seen a puppy if and only if I smile.

41. $p \Rightarrow \sim q$ **43.** $q \Rightarrow p$

3.4 Exercises

1. premise, conclusion

3. sound **5.** True

7. Premise: We stop burning fossil fuels today. Conclusion: There is enough carbon dioxide in the atmosphere that temperatures will continue to rise for a few hundred years.

9. Premise: Fast food is easily available in grocery shops, gas stations, and dispensers everywhere. Conclusion: Fast food obesity has strikingly increased.

11. Premise: A man is struck down by a heart attack in the street. Conclusion: Americans will care for him whether or not he has insurance.

13. Premises: Penguins are black and white.; Some old TV shows are black and white. Conclusion: Some penguins are old TV shows.

15. Premises: All potatoes have skin; I have skin. Conclusion: I must be a potato.

17. Inductive, invalid

19. Inductive, invalid

21. Deductive, valid

23. Valid **25.** Invalid

27. Valid

29. Missing piece: The hand sanitizer was bigger than 3.4 fl oz.

31. Missing piece: My guitar is tuned to an open D tuning.

33. Missing piece: Emma is over 16.

35. Missing piece: You did not buy a new car.

37. a. Premise 1: If a movie is extremely popular and financially successful, then it is considered a blockbuster, Premise 2: The 2009 movie *Avatar* was not a blockbuster, Conclusion: *Avatar* was not popular nor financially successful.

b. p: A movie is extremely popular and financially successful; q: It is considered a blockbuster; $((p \Rightarrow q) \wedge \sim q) \Rightarrow \sim p$

c. The argument is a tautology, and therefore is valid.

p	q	$\sim p$	$\sim q$	$p \Rightarrow q$	$(p \Rightarrow q) \wedge \sim q$	$((p \Rightarrow q) \wedge \sim q) \Rightarrow \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

d. The argument is not sound because premise 2 is untrue. In fact, the movie *Avatar* is one of the highest-grossing films of all time.

39. False dilemma

41. Post hoc, ergo propter hoc

43. Ad hominem

45. Dicto simpliciter

Chapter 3 Exercises

1. The puppy could keep her eyes open after 10:00.

3. Not all houses have fireplaces (or at least one house does not have a fireplace).

5.

a	b	c	$\sim b$	$a \vee \sim b$	$(a \vee \sim b) \Rightarrow c$
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	T	F

7. i: I go to the movies. y: You go to the movies. k: Kathy goes to the movies.

9. Converse: If I enroll in the next course, then my grade in this course will be an A. Inverse: If my grade in this course is not an A, then I cannot enroll in the next course. Contrapositive: If I cannot enroll in the next class, then my grade in this course was not an A. Biconditional: My grade in this course will be an A if and only if I can enroll in the next course.

11.

p	q	$p \Rightarrow q$	$p \wedge (p \Rightarrow q)$	$(p \wedge (p \Rightarrow q)) \Rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

13. a.

15. Valid argument

17. Valid argument

19. Premise: Criminals support global warming; Conclusion: If you support global warming, you are a criminal

21. False dilemma

Chapter 4: Ratios, Percentages, Rates, and Proportionality

4.1 Exercises

1. fraction

3. percentage

5. False **7.** 20%

9. 28.77%

11. one-sixth of 138

13. One-third of 93

15. 96

17. a. 10 **b.** 14

19. a. 4.5 **b.** 7.5

21. Answers will vary; possible answers must include 2 : 5 and may include 48 : 120

23. Answers will vary; possible answers must include 5 : 15 : 9 and may include 50 : 150 : 90

25. a. $\frac{\$750}{\$2800} = \frac{15}{56}$

b. $\frac{\$800}{\$2800} = \frac{2}{7}$

27. $\frac{520}{950} = \frac{52}{95}$

29. $\frac{1}{100}$ are left-handed

31. $\frac{89}{100}$ do not report having a disability

33. 16.25%

35. 30%

37. Toffee with a 76.2% delicious rating.

39.

State	Proportion of Eligible Voters who Voted	# of Eligible Voters	# of Votes Cast
Illinois	67.02%	9,027,082	6,049,950
Minnesota	79.96%	4,118,462	3,293,122
Tennessee	59.81%	5,124,867	3,065,183
Louisiana	64.61%	3,373,932	2,179,897

41. Hindi: 957,670,980 people; Christian: 2,257,367,310 people

43. 246 nonherbivores

45. City A: 2467 citizens; City B: 1898 citizens

47. Utah: 64,000 people; South Dakota: 17,700 people

49. 69 were not there to learn about online courses

51. a. $\frac{3}{2}$ **b.** 8 drops

53. a. $\frac{3}{8}$ **b.** 3 cups

55. 50,000 : 415,000; 465,000 elephants

57. 12,748 : 10,473; 23,221 students

59. $\frac{1}{2}$ or 1 : 2 or 1 to 2

61. $\frac{18}{1.3}$ or 18 : 1.3 or 18 to 1.3

63. a.

City	# of Billionaires	Population	Unit Ratio
Mumbai, India	38	20,000,000	1 : 526,316
Los Angeles, USA	44	4,000,000	1 : 90,909
Beijing, China	57	20,400,000	1 : 357,895
London, UK	66	9,300,000	1 : 140,909
Moscow, Russia	73	12,500,000	1 : 171,233

b. All five cities have a lower ratio of billionaires within the population than does New York City. Note that London has the most similar city population size while Los Angeles has the most similar ratio of billionaires within the population. For Mumbai with its population of 20,000,000 to have the same ratio of billionaires as New York City, it would need 269 billionaires.

65. 60 locals

67. 1092 uninsured cars

4.2 Exercises

1. list price **3.** 1.04

5. original amount

7. 56; 400% increase

9. 323; 68% decrease

11. 19.22; 31% decrease

13. \$16.86 **15.** \$1.23

17. \$13.27 **19.** \$1.43

21. \$2.66 **23.** \$38.49

25. \$629.85 **27.** \$243.20

29. False **31.** \$79.52

33. \$54.36

35. approximately \$34.90

37. \$189.88 **39.** \$124.20

41. \$8.81 **43.** \$523.67

45. \$416.15 **47.** \$345.60

49. \$8340 **51.** \$95,400

53. 20% decrease

55.

Change Between	Absolute Change	Percentage Change
1901 and 2001	207,518,075	267.48% increase
1901 and 2011	235,097,446	303.02% increase
2001 and 2011	27,579,371	9.67% increase

57. \$21,360 **59.** \$0.48

61. 37.04 million

63. Company Z has the better absolute change but both companies have the same percentage growth.

4.3 Exercises

1. rate

3. rate of change

5. false

7. $\frac{\$4800}{15 \text{ credit hours}} = \frac{\$320}{1 \text{ credit hour}}$

9. $\frac{15¢}{1 \text{ min}}$ **11.** $\frac{\$1473.00}{1 \text{ month}}$

13. $\frac{65 \text{ hours}}{3 \text{ weeks}}$

15. $\frac{\$3732}{\$32,750} \approx 11.4\%$

17. $\frac{3932}{29{,}070} \approx 13.5\%$

19. b. **21.** a.

23. 13.5 grams **25.** 340 milliliters

27. 56.28 inches **29.** 2484 words

31. 25.17 mpg **33.** a.

35. \$18.59/day

37. \$0.19/mile or 19.12¢/mile

39. \$0.75/pound or 74.75¢/pound

41. \$1.25/eggplant; 0.80 eggplants/\$1

43. a. US Dollars to Japanese Yen $= \frac{0.009121 \text{ USD}}{1 \text{ JPY}}$

b. Japanese Yen to US Dollars $= \frac{109.642453 \text{ JPY}}{1 \text{ USD}}$

c. 4.56 USD

45. a. Shortly after January 2020

b. Approximately –0.8% per year

c. January 2018 to January 2019 had very little change

d. Shortly after January 2020 to January 2021

4.4 Exercises

1. 1 **3.** diagonal

5. False

7. Approximately 0.28 ounces per inch

9. 120 km/hr

11. 2400 calories/lb

13. \$120/day

15. Approximately 1.89 liters per hour

17. Approximately 62 miles per hour

19. 122 centimeters/year

21. 72 ounces/day

23. 5.96 kilometer/liter

25. 0.537 kilometers/minute

27. 9 cups/day

29. 15.14 liters/minute

31. a. \$54 per hour

b. Approximately \$25.14 per hour

c. Approximately \$45.88 per hour

33. a. 50 drops/minute

b. 45 drops/minute

c. 40 drops/minute

35. Approximately 8.66×10^{-5} inches

4.5 Exercises

1. proportional

3. solving a problem

5. origin **7.** $k = 5$

9. $k = 11$

11. a. 14 N/m; **b.** 3 meters

13. a. 25 N/m; **b.** 9 meters

15. $\frac{1}{48}$ **17.** $\frac{1}{87}$

19. 26.6 inches **21.** 26.5 inches

23. 2.6 inches

25. 22 indoor swimmers

27. 2.5 boxes

29. 384 chocolate hearts

31. 120 T-shirts **33.** \$6.91

35. \$4.50

37. 250 cans

39. No, the values do not increase at the same rate each time.

41. No, the graph of the values doesn't go through the origin.

43. Yes; $k = 400$

45. No, the graph of the values doesn't go through the origin.

Chapter 4 Exercises

1. 360 people **3.** $\frac{3}{7}$

5. 68 games **7.** \$11.70

9. \$647.06 **11.** 25%

13. $\frac{42 \text{ students}}{3 \text{ advisors}} = \frac{14 \text{ students}}{1 \text{ advisors}}$

15. \$5.99 for a package of 8, since that is about \$0.75 each instead of \$1.43 each.

17. 30.72 ounces

19. a. Approximately 0.21 mm per minute;

b. Approximately 0.85 mm per minute;

c. Approximately 0.004 mm per minute

21. 2.5 cups/container

23. 5.42 feet

25. $\frac{1}{25}$

27. 184 inches

Chapter 5: Algebra: Equations, Inequalities, and Functions

5.1 Exercises

1. one

3. infinitely many

5. ordered pairs **7.** $x = \frac{8}{7} = 1\frac{1}{7}$

9. $x = 3$ **11.** No solution

13. $z = 10.9$ **15.** $y = -3$

17. Infinitely many solutions

19. $k = gt - v$ **21.** $r = \frac{I}{Pt}$

23. $h = \frac{V}{\pi r^2}$ **25.** –3

27. 856 **29.** 12

31. 1.08

33. Domain: all real numbers; Range: all real numbers greater than or equal to 5.

35. 20 attempts **37.** \$47 per ticket

39. $x = 4.9$

41. a. $x = \frac{3}{8}$ **b.** $x = \frac{3}{4}$ **c.** $x = \frac{5}{8}$

43. 1.75 hours

45. a. False. The minimum and maximum values are not the same for each graph. The range of the 2018 graph is $[82,100]$. The range of the 2019 graph is $[81,98]$. The range of the 2020 graph is $[80,99]$.

b. True. All three graphs have a domain of the months January through December.

c. True. The range of the 2019 graph is $[81,98]$, which falls within the range of the 2020 graph, $[80,99]$.

47. Domain: nonnegative integers less than or equal to 150; Range: Real numbers greater than or equal to \$7000

5.2 Exercises

1. slope **3.** slope-intercept

5. Parallel **7.** 0

9. $\frac{1}{2}$

11. $m = -4$; y-intercept: $(0,0)$

13. $m = \frac{1}{2}$; y-intercept: $\left(0,\frac{3}{2}\right)$

15. m = undefined; y-intercept: none

17. $y = -6x + 28$

19. $y = 8.4x - 1.2$

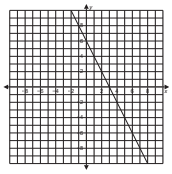
21. $m = -\frac{5}{4}$; $b = \frac{9}{2}$; $\left(\frac{13}{5},\frac{5}{4}\right)$

23. $m = -3.8$; $b = 1.4$; $(13, -48)$

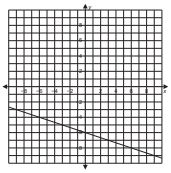
25. $y = -\frac{2}{3}x + 3$ **27.** $y = -\frac{1}{4}x + \frac{3}{2}$

29. $y = \frac{1}{8}x + \frac{17}{2}$ **31.** $y = -\frac{5}{3}x - 5$

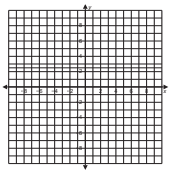
33.



35.



37.



39. a. $y = -0.051x + 17{,}545.544$

b. \$14,995.54

c. The calculated price is slightly higher. Other factors might influence the price of the car, such as body damage.

41. a. $-0.065x + 18{,}784.24$

b. \$15,534.24;

c. The value calculated with this equation is about \$500 more than the value calculated in Exercise 39. Answers will vary but might include that 50,000 is inside the range of values that should be used in the equation.

43. a. $y = 0.016x + 8.1$;

b. \$16.10

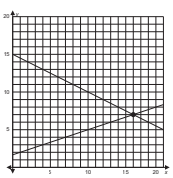
c. The actual price of \$16.50 is rather similar to the estimated one. The relationship of size to price is approximately linear.

45. a. $y = 0.022x + 6.375$

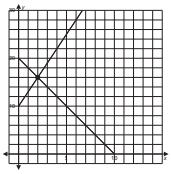
b. \$17.38;

c. The price of \$17.38 is a bit farther from the real price than \$15.95. However, it is still close enough to the real price for the same reasons.

47. The break-even point is (8,13) so the business will start to earn profit after 8 months.



49. The break-even point is (16,7) so the mass of the element should be 16 g per 100 g of the alloy.



5.3 Exercises

1. never intersect, intersect at infinitely many points, intersect at exactly one point

3. money spent, money collected

5. cost **7.** No

9. No **11.** (2,1)

13. Infinitely many solutions

15. $(-1,-3)$

17. $(0,1)$

19. Infinitely many solutions

21. $(2,1)$

23. $P(3000) = \$778.25$

25. Textbook cost, $x = 140$; Calculator cost, $y = 30$

27. Non-student tickets, $x = 542$; Student tickets, $y = 181$

29. Mutual fund, $x = \$8500$; CD, $y = \$1500$

31. $C(x) = 270 + 50x$; $R(x) = 140x$; the break-even point is (2,420) so the family consists of no more than three people.

33. a. $C(x) = 31.90x + 100{,}900$

b. $R(x) = 37.50x$

c. $P(x) = 5.60x - 100{,}900$

d. Break-even point is 18,017.8571; when 18,018 pairs of shoes are made and sold, both cost and revenue are \$675,675.

35. a. $C(x) = 18.26(0.25)x + 8.16x$
$= 12.725x$

b. $R(x) = 18.26x$

c. $P(x) = 18.26x - 12.725x$
$= 5.535x$

d. \$309.96;

e. Answers will vary. Nationwide studies show that drivers disagree strongly on whether insurance, car maintenance and depreciation are real factors in their earnings.

37. a. $C(x) = 56,000 + 12x$

b. $R(x) = 40x$

c. $P(x) = 28x - 56,000$

e. $x = 2000$

f. \$28,000

5.4 Exercises

1. equal

3. test point

5. not included

7. a. In the solution set;

b. Not in the solution set;

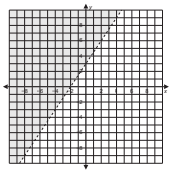
c. In the solution set

9. a. In the solution set;

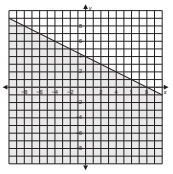
b. Not in the solution set;

c. Not in the solution set

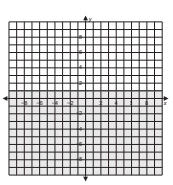
11.



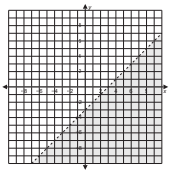
13.



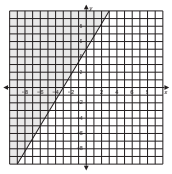
15.



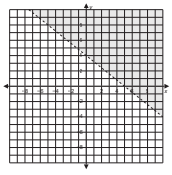
17.



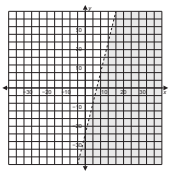
19.



21.



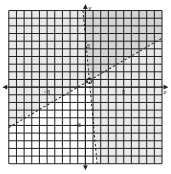
23.



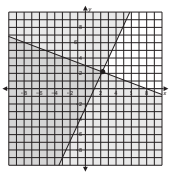
25. Open circle

27. Closed circle

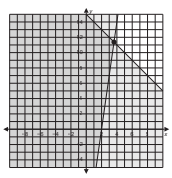
29.



31.



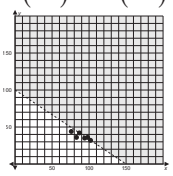
33.



35. a. $8x + 12y > 1200$

b. No, because
$8(94) + 12(26) - 1200 = -136 < 0$

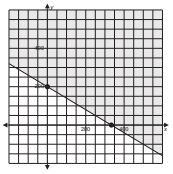
c.



d. Likely no, because most points do not lie in the region satisfying the inequality for positive profit.

37. a. $0.75x + 1.25y \geq 250$

b.

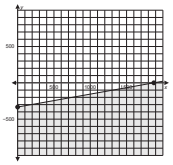


c. No **d.** 200;

e. Any whole number solution; for example, (200, 200).

39. a. $0.8x - 4.5y \geq 1500$

b.



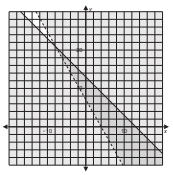
c. 1875 (the x-intercept)

d. Any whole number solution where $x > 1875$; for example, (2050, 2);

e. Although it is a solution to the inequality, it is not possible to have a negative number of broken vases.

41. a. $x + y \leq 13$; $15x + 8.5y > 60$

b.



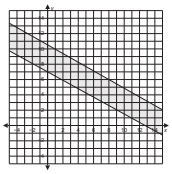
c. 4 hours of tutoring

d. 7.5 hours

e. Any point in the solution set where x is a whole number and y is a multiple of 0.5.

43. Let x be the number of short ads and y be the number of long ads.

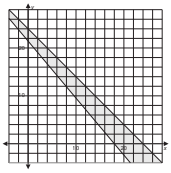
$$\begin{cases} 215x + 380y \leq 4000 \\ 110x + 200y \geq 1400 \end{cases}$$



Any point in the shaded area is a combination of how many of each type of ad the newspaper could place to meet the goal. For example, (2, 8), (5, 6), (13, 2), etc.

45. Let x be the number of hours spent producing a regular glue and y be the number of hours spent producing a glue with increased adhesiveness.

$$\begin{cases} 43x + 165y \le 2800 \\ x + y \ge 14 \end{cases}$$



Any point in the shaded area is a combination of hours the factory could spend making each type of glue to meet the goal. For example, (4, 18), (10, 12), (20, 2), etc.

5.5 Exercises

1. linear inequalities

3. limitation

5. feasible region

7. $z = 80{,}000 + 2000x - 200y$

9. $z = 10x + 6y$

11. $z = 1.5a + 1.25b + 2.1c$

13. Point c. (–5, –270) minimizes the objective function with the value $z = -48{,}850$.

15. Point b. (0, 400) maximizes the objective function with $z = 118{,}000$.

17. Point b. (0, 405) maximizes the objective function with $z = 166{,}050$.

19. The maximum value of z is 60 and occurs at the point (5, 15).

21. The minimum value of z is 93 and occurs at the point (9, 3).

23. The minimum value of z is –4 and occurs at the point (0, 4).

25. a. $z = 157x + 423y$

b. $6x \le y$

c. $2x + 3y \le 180$

d. $x \ge 0, y \ge 0$

e. 0 apple trees and 60 peach trees

f. $25,380

27. a. $z = 2.50a + 2.00b$

b. $6a + 9b \ge 132$

c. $4a + 5b \ge 82$

d. $3a + b \ge 45$

e. $a \ge 0; b \ge 0$

f. 13 packages from Company A and 6 packages from Company B

g. $44.5

29. The company should buy 8 of the 30-seat buses and 7 of the 50-seat buses to have the maximum number of seats 590. Objective function: $z = 30x + 50y$; Constraints: $45x + 60y \le 780$, $x \le 9$, $y \le 7$, $x \ge 0$, $y \ge 0$; $z(8, 7) = 590$

31. To minimize the cost, an expectant mother should take 1 Baby Bliss Prenatal Pill and 3 Natural Gift Prenatal pills. Objective function: $z = 0.4x + 0.34y$; Constraints: $x \ge 0$, $y \ge 0$, $0.1x + 0.1y \ge 0.4$, $200x + 400y \ge 1000$, $9x + 6y \ge 27$; Test points: $z(5,0) = \$2$, $z(3, 1) = \$1.54$, $z(1, 3) = \$1.42$, $z(0, 4.5) = \$1.53$.

5.6 Exercises

1. $ax^2 + bx + c = 0$

3. standard form

5. vertex

7. $x = -1, 7$

9. $x = \frac{3}{4} \pm \frac{\sqrt{17}}{4}$

11. $x = -\frac{1}{3} \pm \frac{\sqrt{7}}{3}$

13. $x = -2 \pm \sqrt{6}$

15. $x = -3, \frac{1}{2}$

17. 117 feet; 42 feet

19. 47.27 feet; 3.44 seconds

21. 8.84 seconds

23. Between inner cables: from –2 to 2, distance is 4 ft; Between outer cables: from –4 to 4, distance is 8 ft.

25. Distance: 2 meters; Length: 8 meters

27. a. 7 meters **b.** 4 meters

29. a. 6 centimeters

b. 9 centimeters

31. a. Normal conditions: 4.5 meters, When windy: 3.92 meters. When rainy: 2.33 meters

b. Normal conditions: 2.8 meters, When windy: 2.22 meters, When rainy: 0.63 meters

c. Normal conditions: 14 seconds, When rainy: 10.58 seconds

d. The ball falls the fastest when it is rainy, followed by the windy weather, and then the normal conditions. However, at the beginning of the fall, the ball falls slightly slower in the windy weather.

33. a. $N(t) = N_0 e^{2t}$

b. $A_l(t) = 10{,}000 + 243t$

c. 9.84×10^9 cells/L

d. 20.8 minutes

35. a. $h(t) = -5t^2 + 3.9t + 6$

b. $d(t) = 2.32t$

c. 5.92 meters;

d. 1.55 seconds

e. 6.76 meters;

f. 0.39 seconds

5.7 Exercises

1. $f(x) = ab^x$

3. Frequency

5. 10

7. 16,341 people

9. a. Approximately 1066 people

b. Approximately 1211 people

c. Approximately 1498 people

11. pH = 3.52; acidic

13. [H+] = 0.001

15. 100 times more

17. One octave: 440 Hz; two octaves: 880 Hz; three octaves: 1760 Hz

19. Two octaves higher

21. 438.948 Hz

23. a. Brother: $B(x) = 50 + 5x$; Sister: $S(x) = 50(1.1)^x$

b. Brother: \$65; Sister: \$66.55

c. 1 day;

d. Brother, because after being 1 day late paying back the money, you would need to pay your sister more money.

25. $I = 1015.5$
$\approx 3{,}162{,}277{,}660{,}168{,}379$

27. 8.70

29. Rather get the pennies.

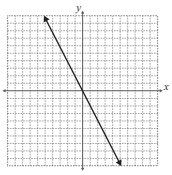
Chapter 5 Exercises

1.

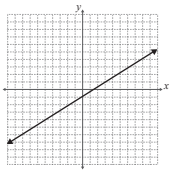
x	$f(x) = x^2 + 2x - 3$
–3	0
–2	–3
0	–4
1	–3
2	0
3	12

3. indep var = miles driven; dep var = cost of rental; $y = 0.25x + 35$ for $x \geq 0$; \$97.50

5.



7.



9. $m = \frac{3}{4}$; y-intercept $= (0, -3)$

11. a. $C(x) = 5400 + 1400x$

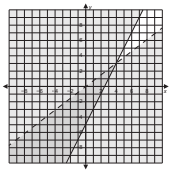
b. $R(x) = 2000x$

c. $P(x) = 600x - 5400$

d. $x = 9$

e. \$6600

13.



15. The maximum value of z is 95 and occurs at the point (10,15).

17. a. 19 miles per gallon

b. 30 miles per gallon

c. About 44 miles per gallon

19. a. About 24,367 people

b. May 2220

21. About 6.85 seconds

Chapter 6: Finance

6.1 Exercises

1. borrowing

3. principal

5. False **7.** \$1000

9. \$233.75 **11.** \$126

13. \$3712

15. a. \$5484.47 **b.** \$1984.47

17. a. \$18,325.20 **b.** \$12,675.20

19. a. \$65,758.59 **b.** \$50,758.59

21. a. \$387,717.95

b. \$380,417.95

23. a. 5.12% **b.** 5.12% **c.** 5.13%

d. 7.76% **e.** 7.78% **f.** 7.79%

25. \$360 **27.** \$12,500

29. \$402.30 **31.** \$722.44

33. \$13,960.32 **35.** \$16,518.97

37. The second bank, its value after 7 years is \$14,079.92, as opposed to \$13,947.11 for the first.

39. a. \$5287.23;

b. About 4 years

41. \$16,912.57 **43.** \$1398.84

45. About 3.5 years

47. About 6.67 years, or 6 years and 8 months

49. 2.14% **51.** 1.66%

53. \$88,956.99

55.

First Bank of Lending Loan APY	
Loan Amount	APY
< \$20,000	11.73%
\$20,000–\$99,000	9.30%
≥ \$100,000	5.88%

6.2 Exercises

1. present value **3.** shareholder

5. True **7.** \$7574.87

9. \$18,621.74 **11.** \$26,701.34

13. \$17,229.90 **15.** \$10.22

17. \$31.39 **19.** \$10,071.48

21. \$48,733.54 **23.** \$2925.07

25. \$782.48

27. a. \$17.91 **b.** \$39.39 **c.** \$91.19

29. a. \$317.41

b. \$57,133.80; I = \$17,866.20

31. \$1673 **33.** \$1776

35. a. \$6012;

b. \$288,576 deposited and \$211,424 in interest

37. \$471.17 **39.** \$5877.10

41. \$107,395.23

43. a. \$87,971.89;

b. \$40,000 deposited and \$47,971.89 in interest

45. a. \$56,556.06; **b.** \$163,566.51; **c.** \$72,846.51; **d.** \$545.22

47. a. NKE; **b.** –\$0.74; **c.** \$173.78;

d. Decreased by \$1.46;

e. About 1.6 billion shares;

f. \$77

49. While there are no guarantees, the rule of thumb was developed based on the historical data of the interest rates on different types of saving strategies. Therefore, unless the market experiences a crisis more severe than in the past, the four percent rule of thumb will likely ensure that the funds will last 30 years.

6.3 Exercises

1. down payment

3. collateral **5.** open-ended

7. a. $331.15 **b.** $7947.60

9. a. $501.81 **b.** $6021.72

11. a. $168.89 **b.** $6080.04

13. a. Please see math.hawkeslearning.com for full answer.;

b. $215,761.92; 163 payments; the 164th payment is the first one where more than half of the payment goes to the principal.

15. $1168.97 **17.** $880.29

19. a. $960.10; **b.** $172,818

21. a. $2599.60 **b.** $124,780.80 **c.** $10,780.80

23. a. $125.36 **b.** $3008.64 **c.** $358.64

25. $506.65 **27.** $361.63

29. The five-year loan with an APR of 7.5% ($400.76 monthly payment, $4045.60 in interest paid)

31. Option A: PMT =$319.61, Total cost = $7670.64; Option B: PMT = $424.76, Total cost = $7645.68

33. Option A: PMT =$360.44, Total amount =$30,276.96; Option B: PMT =$496.21, Total amount =$29,772.60

35. a. $32,200; **b.** $197,800

37. $44,262.71

39. a. $823.33; **b.** $1124.58

41. $324,776.49

43. a. Approximately 114 payments; **b.** Approximately 27 payments; **c.** $1260

45. 260% **47.** 910%

49. a. $8718.72; **b.** $9146.16; **c.** The better credit score would save $427.44.

51. Option 1

53. When the interest rate is fixed and the loan has a long term in a country with a high inflation rate, the future worth of the total sum of regular payments may be less than the present worth of the loan.

6.4 Exercises

1. Form 1040 **3.** deductions

5. False **7.** $58,133.55

9. $65,913.23 **11.** $32,866.32

13. a. $78,490.88 **b.** $72,440.88

15. $2778.50 **17.** $12,334.00

19. $52,040.00 **21.** $34,057.10

23. $67,023.00 **25.** $65,459.40

27. $51,966.50

29. The taxpayer received a federal tax refund of $3895.59 – $3763.14 = $128.45.

31. a. $26.10 **b.** $111.60 **c.** $344.29 **d.** $1455.71

33. $81,048 **35.** $47,956

37. $96,045 **39.** $58,851

6.5 Exercises

1. income, expenses

3. prorate **5.** $38.16

7. $67.80 **9.** $40

11. $41.38

13. a. $4500 **b.** $1500

15. a. $2425 **b.** $808.33

17. Necessities: $2560; Wants: $1536; Savings: $1024

19. Necessities: $1236; Wants: $741.60; Savings: $494.4

21. a. $2132 **b.** $132

23. a. $4245.50 **b.** $1125.50

25. $730 **27.** $561

29. $506

31. Answers will vary. A budget can help you get a solid understanding of the amount of money you make and how you spend your money. A budget can also help prevent you from overspending each month.

Chapter 6 Exercises

1. $390

3. a. $7061.72; $7451.05; $390,674.35

b. $1561.72; $2696.05; $383,374.35

5. a. $24,839.67 **b.** $24,854.70

7. a. $38,481.59 **b.** $8481.59

9. a. $153,926.38

b. $33,926.38

11. $2387.67

13. a. $330.61

b. $71,411.24; $I = \$63,588.76$

15. a. $1150 **b.** 97.5%

17. $243,932.24

19. The 5-year loan, because the 4-year loan has a higher payment than you can afford ($638.39/month) and the 6-year loan has you paying more interest ($6246 instead of $4775).

21. a. $251.70 **b.** $6040.80

23. $57,592.27

25. a. $4937.50; **b.** $1645.83

27. $1916/month

Chapter 7: Numeration and Measurement Systems

7.1 Exercises

1. 10, 2, 3 **3.** 20

5. $(2\cdot10^3)+(3\cdot10^2)+(5\cdot1)$ or $(2\cdot10^3)+(3\cdot10^2)+(0\cdot10^1)+(5\cdot1)$

7. $(6\cdot10^2)+(2\cdot10^1)+(1\cdot1)+(9\cdot10^{-1})$

9. $(8\cdot10^1)+(2\cdot10^{-2})$ or $(8\cdot10^1)+(0\cdot1)+(0\cdot10^{-1})+(2\cdot10^{-2})$

11. 951 **13.** 300,062

15. 10,200.07 **17.** 46

19. $(11\cdot60^1)+(55\cdot1)=715$

21. $(48\cdot60^2)+(8\cdot60^1)+(20\cdot1)=173{,}300$

23. 𒌋𒐗𒐗

25. 𒐗 𒌋𒌋𒌋 𒐗𒐗𒐗 𒐗𒐗𒐗

27. 𒐗 𒌋 𒐗 𒌋

29. 8

31. $(10\cdot20)+(4\cdot1)=204$

33. $(3\cdot20^3)+(0\cdot20^2)+(3\cdot20)+(3\cdot1)=24{,}063$

35. 

37. 

39. 

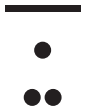
41. Answer will vary depending on the current year. Babylonian

2022: 𒌋𒌋𒌋𒐗𒐗𒐗 𒌋𒌋𒌋𒐗𒐗

2023: 𒌋𒌋𒌋𒐗𒐗𒐗 𒌋𒌋𒌋𒐗𒐗𒐗

2024: 𒌋𒌋𒌋𒐗𒐗𒐗 𒌋𒌋𒌋𒐗𒐗𒐗𒐗

Mayan

2022: 

2023: 

2024:

43. Answers will vary. Possible answers might include that it's an advantage to have a symbol for zero. One disadvantage is that it takes up a lot of space to write.

45. Answers will vary. Encourage students to think about the fact that without any a place holder, the numbers 1 and 60 look the same. One way to remove the ambiguity is to consider the context of how the number is being used. For instance, 60 of anything (for example, tablespoons or pints) doesn't seem to make sense in a recipe. Likewise, a length of 1 might seem too small for a distance to travel.

7.2 Exercises

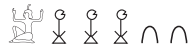
1. hieroglyphics **3.** M

5. changed **7.** 6050

9. 700,000 **11.** 1,405,003

13. 

15. 

17. 

19. 444 **21.** 301

23. 8555 **25.** 90,809

27. ρ ϛ ϛ **29.** , αοη

31. η υκδ M **33.** 48

35. 651 **37.** 975

39. XLII **41.** CLXIII

43. CDXCI **45.** 54

47. 1008 **49.** 25,575

51. 一百零三 **53.** 七十五

55. 二千零二

57. Answers will vary depending on the current year. For example,

2022: , βκβ

2023: , βκγ

2024: , βκδ

59. Answers will vary depending on the current year. For example,

2022: 二千零二十二

2023: 二千零二十三

2024: 二千零二十四

61. **a.** 52;

b. Answers will vary. Can include the look of the number or the fact the Super Bowl is in the year after the main portion of the football season;

c. Answers will vary.

63. Possible answers may include: Drawing the numerals takes up a lot of space and time. The formation of some of the numerals is complex. A new character is needed for each power of 10.

65. Answers will vary.

7.3 Exercises

1. 10 **3.** letters

5. 1000s, 8s **7.** 624

9. 5460 **11.** 513

13. 1752

15. $BB_{16}=187$ and $603_2=192$; Thus, 603_2 is larger.

17. 1101001_3 **19.** 40_{12}

21. 1010111100_2 **23.** $21E_{16}$

25. $3B_{16}$ **27.** $72C_{16}$

29. $B038_{16}$ **31.** 111101_2

33. 111111101101_2

35. 1111010000100101_2

37. 144_7 **39.** 1100_2

41. 51103_6 **43.** 273_9

45. $7E5_{16}$ **47.** 1202_5

49. 220_3 **51.** 1032_4

53. 148_{16}

55.

+ Base 8	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

57.

× Base 4	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	10	12
3	0	3	12	21

59.

+ Base 7	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	11	13	15
3	0	3	6	12	15	21	24
4	0	4	11	15	22	26	33
5	0	5	13	21	26	34	42
6	0	6	15	24	33	42	51

61. Base 6 **63.** Base 8

7.4 Exercises

1. 1 **3.** liter
5. are **7.** 156 in.
9. 7 lb 8 oz **11.** 10 pt
13. 255 minutes **15.** 18,480 ft
17. 64 fl oz **19.** 1.5625 mi^2
21. 0.23 km **23.** 1.23 m
25. 0.356 kL **27.** 1200 g
29. 72 600 dm^2
31. 6 272 000 cm^2
33. 99 000 000 cm^2
35. 3.45 a **37.** 104 fl oz
39. A king-size mattress is 16 inches wider.
41. 32.5 gallons
43. 1.36 km
45. $2.40
47. Trey can put 22 tacks along each chair (or approximately 22.25 tacks)
49. 0.9 hectares **51.** 19 000 cm^3
53. meters or kilometers
55. millimeters or centimeters
57. 0.0000006 kg, 0.61mg, 100.6 mg, 1021 g

7.5 Exercises

1. dimensions
3. approximations
5. 2.54 **7.** 241.5 km
9. 45.75 meters **11.** 29.64 acres
13. 52 km^2 **15.** 1.225 oz
17. 1190.7 g **19.** 1.584 gal
21. 6.6667 tbsp
23. 29,976,308 km
25. 0.5167 mi/min
27. 354.375 g/L
29. 0.31 lb/ft **31.** 13.56 yd^2/lb
33. 350.6 °F **35.** 135 °C
37. 77 °F **39.** –11.111 °C
41. 100 cm, 100 in., 10 ft, 100 yd, 100 m, 10 km, 10 mi
43. 1206.11 lb/ft^3 **45.** 0.19 L/sec
47. 62 miles per hour
49. 13 min
51. 4 fl oz; 374 °F
53. 19.4 °F
55. 12.125 gallons; $452.02
57. 5.45 mi/hr
59. No, they would be underbaked or not cooked at all because of the oven temperature. Also, there is twice as much cinnamon as called for, so they might not taste good.
350 g ≈ 12.25 oz
400 g ≈ 14 oz
22.5 mL ≈ 1.5 tbsp
140 °C ≈ 284 °F

Chapter 7 Exercises

1. $(1\cdot10^2)+(7\cdot10^1)+(5\cdot10^0)+(8\cdot10^{-1})+(3\cdot10^{-2})$
3. 1449 **5.** 7206
7. 120,052 **9.** 847
11. 1047 **13.** 6039
15. 76 **17.** $15B_{16}$
19. 1004_5 **21.** 2112_3
23. 216 oz **25.** 2400 min
27. 152 400 cL
29. 57 750 000 m^2
31. 56.68 m **33.** 15.9 qt
35. 29.444 °C **37.** 0.504 g/mL

Chapter 8: Number Theory

8.1 Exercises

1. 6 **3.** False
5. Composite **7.** Prime
9. Composite **11.** Composite
13. Composite
15. 2 · 2 · 2 · 2
17. 2 · 3 · 3 · 3 · 3
19. 5 · 5 · 5 · 5 **21.** 3 · 5 · 7
23. 7 **25.** 45
27. 36 **29.** 35
31. 7 **33.** 1
35. 8 **37.** 6
39. Yes **41.** No
43. No **45.** No
47. **a.** 8 groups **b.** 4 violinists **c.** None

49. 4 boxes

51. a. 12 ft

b. Troop A: 15 ft × 12 ft; Troop B: 17 ft × 12 ft

53. All prime numbers smaller than $\sqrt{1291} \approx 35.9305$: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, and 31

8.2 Exercises

1. check-sum digit

3. False **5.** True

7. True **9.** True

11. True **13.** 1

15. 0 **17.** $8 \pmod{10}$

19. $3 \pmod 4$ **21.** $5 \pmod{12}$

23. 2 **25.** 1

27. 1:00 p.m. **29.** 12:00 a.m.

31. 1200 **33.** 0700

35. $4 \pmod 7$ **37.** $0 \pmod 3$

39. 3 **41.** 10

43. 0

45. No, the check-sum digit should be 1.

47. Valid

49. No, the check-sum digit should be 6.

51. 7 **53.** 7

55. 0 **57.** 5

59. 1 **61.** 0

63. Yes

65. No, the check-sum digit should be 5.

8.3 Exercises

1. Fermat's Little Theorem

3. False

5.
$$\begin{aligned} 2^{23} - 2 &= 2^{16} \cdot 2^7 - 2 \\ &\equiv 9 \cdot 13 - 2 \pmod{23} \\ &\equiv 2 - 2 \pmod{23} \\ &\equiv 0 \pmod{23} \end{aligned}$$

7.
$$\begin{aligned} 3^{79} - 3 &= \left(3^4\right)^{19} \cdot 3^3 - 3 \\ &\equiv (2)^{19} \cdot 27 - 3 \pmod{79} \\ &\equiv 2^{17} \cdot 2^2 \cdot 27 - 3 \\ &\equiv 11 \cdot 4 \cdot 27 - 3 \pmod{79} \\ &\equiv 3 - 3 \pmod{79} \\ &\equiv 0 \pmod{79} \end{aligned}$$

9.
$$\begin{aligned} 2^{17} - 2 &= 2^5 \cdot 2^5 \cdot 2^5 \cdot 2 \cdot 2 - 2 \\ &\equiv (15 \cdot 15)(15 \cdot 4) - 2 \pmod{17} \\ &\equiv 4 \cdot 9 - 2 \pmod{17} \\ &\equiv 2 - 2 \pmod{17} \\ &\equiv 0 \pmod{17} \end{aligned}$$

11.
$$\begin{aligned} 4^{31} - 4 &= 4^3 \cdot 4^3 \cdot 4^3 \cdot 4^3 \cdot 4^3 \cdot 4^3 \cdot 4^3 \cdot 4^3 \\ &\quad \cdot 4^3 \cdot 4^3 \cdot 4 - 4 \\ &\equiv 2^5 \cdot 2^5 \cdot 4 - 4 \pmod{31} \\ &\equiv 1 \cdot 1 \cdot 4 - 4 \pmod{31} \\ &\equiv 4 - 4 \pmod{31} \\ &\equiv 0 \pmod{31} \end{aligned}$$

13.–31. Answers will vary.

8.4 Exercises

1. prime **3.** False

5.
$$\begin{aligned} & 3^{2(13-1)(7-1)+1} - 3 \\ &= 3^{2(12)(6)+1} - 3 \\ &= 3^{145} - 3 \\ &= \left(3^6\right)^{24} \cdot 3 - 3 \\ &\equiv (1)^{24} \cdot 3 - 3 \pmod{91} \\ &\equiv 1 \cdot 3 - 3 \pmod{91} \\ &\equiv 3 - 3 \pmod{91} \\ &\equiv 0 \pmod{91} \end{aligned}$$

7. 49 **9.** 33

11. 39 **13.** 6

15. 64 **17.** 38

19. 12 **21.** 31

23. $d = 151$

25. $p = 17$, $q = 11$, and $d = 59$

Chapter 8 Exercises

1. 4 groups **3.** 170

5. GCD = 21; because the GCD is not 1, the numbers are not relatively prime.

7. 0 **9.** 2

11. No, the check-sum digit should be 5.

13. 1 **15.** 3

17. 4,459,580 **19.** 12345

Chapter 9: Geometry

9.1 Exercises

1. circumference

3. tessellation

5. \$600 **7.** 25 bags

9. 77.04 cm^2 **11.** 37.68 $in.^2$

13. 246 ft^2 **15.** \$21.60

17. 15 boxes

19. 38 shrubs; 165 flowers

21. Rotate wheels forward 6.4 rotations; turn 90° left; rotate wheels forward 1.9 rotations; turn 90° left; rotate wheels forward 1.1 rotations.

23. 13 inches **25.** \$641.30

27. \$37.15

29. $C = 18.84$ in.; $A = 28.26$ $in.^2$

31. \$11.42 **33.** 18 m^2

35. 9.29 $in.^2$ **37.** 4 m^2

9.2 Exercises

1. three; two **3.** triangular

5. False **7.** Volume

9. Volume **11.** Surface area

13. 120 ft^3

15. a. 270 m^3 **b.** 18 m^3

17. 162 cubes **19.** 96 ft^2

21. 486 in.^{2}

23. a. 14,657,414.63 mi^2

b. 3,517,779,524 mi^2

25. a. $144\pi \approx 452.39$

b. 49 sheets

27. a. 33,912 ft^3

b. 27,129.6 bushels

c. 169.56 min (about 2.8 hours)

29. a. Vase A: 490.87 cm^3
Vase B: 502.65 cm^3

b. Vase B holds 11.78 cm^3 more than Vase A.

31. a. 169.65 cm^3

b. About 86 minutes

33. a. 85.3 ft^3

b. About 3.2 ft^3

35. 0.32 m^3

37. Answers will vary. At a height of 20 cm and a volume of 400 cm^3, the radius should be 2.5 cm.

39. The radius cannot be smaller than 2.3 ft.

41. a. $SA = 65.81$ m^2

b. Cost: $477.13

9.3 Exercises

1. 180

3. sine; cosine; tangent

5. False **7.** Acute

9. Acute **11.** Obtuse

13. 56° **15.** 81°

17. $x = 4$

19. $m\angle 1 = 96°$ **21.** $m\angle 3 = 58°$

23. 47° and 43° **25.** 84° and 96°

27. No, $\dfrac{360°}{140°} \approx 2.57$

29. $\sin A = 0.38$;
$\cos A = 0.92$;
$\tan A = 0.42$;
$\sin B = 0.92$;
$\cos B = 0.38$;

31. Law of Cosines

33. Law of Cosines

35. $\sqrt{5} \approx 2.24$

37. a. 106.5 in. or 8.9 ft;

b.

$$\text{Maximum Cabinet Height} < \sqrt{\text{Ceiling Height}^2 - \text{Cabinet Depth}^2}$$

39. Calvin will need to make a right triangle with sides measuring 60 feet and 71.5 feet with a hypotenuse measuring 93.3 feet to get the 50° angle. He would then need to measure 30 feet along the hypotenuse to mark the next corner.

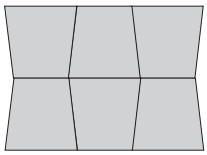
41. 148.25 feet **43.** 47.57 feet

45. a. 5.99 ft **b.** 14.83 ft

47. 61.6° **49.** 38.7°

51. 49.7 feet tall **53.** 75 mi

55. Answers will vary. You can create the following tessellation.



Chapter 9 Exercises

1. 145.96 in.^{2} **3.** $528

5. 48.33 in.^{3}, 42.96 in.^{3}, 31.24 in.^{3}

7. a. 1.13 in.^{3}

b. 3.75 in. by 4.5 in. by 2.75 in.

9. 3.75 in.

11. 13° and 77° **13.** 78° and 102°

15. a. 9.54 ft **b.** 15.26 ft

17. 117.57 ft

Chapter 10: Probability

10.1 Exercises

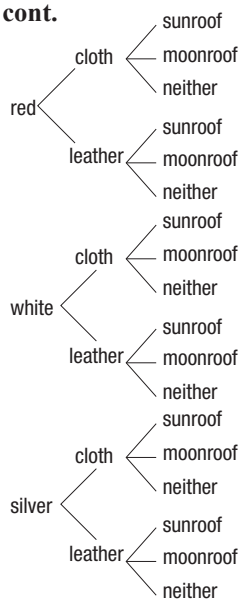
1. trial

3. tree diagram

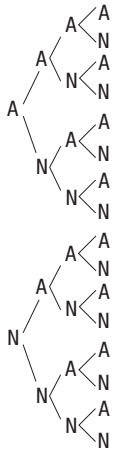
5. False

7. {RCS, RCM, RCN, RLS, RLM, RLN, WCS, WCM, WCN, WLS, WLM, WLN, SCS, SCM, SCN, SLS, SLM, SLN}

7. cont.



9. {AAAA, AAAN, AANA, AANN, ANAA, ANAN, ANNA, ANNN, NAAA, NAAN, NANA, NANN, NNAA, NNAN, NNNA, NNNN}



11. $\{11, 22, 33, 44, 55, 66, 77, 88, 99\}$

13. {WFS, WFR, WFC, WFM, WDS, WDR, WDC, WDM, BFS, BFR, BFC, BFM, BDS, BDR, BDC, BDM, PFS, PFR, PFC, PFM, PDS, PDR, PDC, PDM}

15. {PCM, PCN, PGM, PGN, LCM, LCN, LGM, LGN}

17. {COT, COS, CMT, CMS, POT, POS, PMT, PMS, NOT, NOS, NMT, NMS}

19. Classical

21. Experimental

23. Classical

25. $\frac{70}{200} = 0.35$

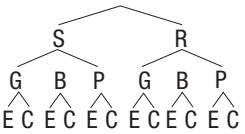
27. a. $\frac{82}{500} = 0.164$

b. $\frac{405}{500} = 0.81$

29. $\frac{2}{15} \approx 0.133333$

31. $\frac{38}{115} \approx 0.330435$

33.



$\frac{6}{12} = 0.5$

35. $\frac{18}{38} \approx 0.473684$

37. $\frac{11}{321} \approx 0.034268$

39. $\frac{64}{213} \approx 0.300469$

41. $\frac{1}{3} \approx 0.333333$

43. $\frac{1}{4} = 0.25$

45. a. $\frac{8}{19} \approx 0.421053$;

b. $\frac{38}{38} = 1$;

c. $\frac{15}{19} \approx 0.789474$

47. Answers will vary. With classical probability, the entire sample space is known. With experimental probability, only part of the sample space can be determined by observing or collecting data. For both types of probability, the probability is calculated by dividing the size of the event space (or number of times the event occurs) by the size of the sample space (or number of times the experiment is performed).

10.2 Exercises

1. multiply

3. product

5. True

7. 362,880

9. 1

11. 75,600

13. 210

15. 6188

17. 840

19. 24

21. 30

23. 6

25. 3

27. $\frac{1}{6}$

29. 1099

31. $9 \cdot 10 \cdot 10 = 900$

33. $5 \cdot 2 \cdot 2 \cdot 3 = 60$

35. $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

37. $16 \cdot 14 \cdot 7 \cdot 12 = 18{,}816$

39. $9 \cdot 10 \cdot 10 \cdot 5 = 4500$

41. a. 67,600 **b.** 58,500

43. 10,000 **45.** 2730

47. Permutation; ${}_{10}P_2 = 90$ ways

49. Permutation; ${}_{84}P_3 = 571{,}704$ ways

51. Permutation; $\frac{8!}{3!2!2!1!} = 1680$ codes

53. Combination; ${}_{123}C_6 = 4{,}249{,}404{,}082$ ways

55. Permutation; $\frac{9!}{1!4!2!2!} = 3780$ ways

57. Permutation; ${}_{12}P_3 = 1320$

59. Combination; ${}_8C_4 = 70$

61. Permutation; ${}_{20}P_7 = 390{,}700{,}800$

63. Permutation; $\frac{10!}{6!4!} = 210$

65. Permutation; $\frac{12!}{4!4!2!2!} = 207{,}900$

67. Answers will vary; should be ${}_{(\text{number of class members})}C_3$

10.3 Exercises

1. complement

3. against, for

5. False

7. $\frac{3}{1000} = 0.003$

9. $\frac{1}{26 \cdot 25 \cdot 24} = \frac{1}{15600} \approx 0.000064$

11. $\frac{1}{{}_{48}C_2} \approx 0.000887$

13. a. Permutation

b. $10^4 = 10{,}000$

c. $\frac{10}{10{,}000} = 0.001$

15. $\frac{1}{{}_{20}C_5} = \frac{1}{15{,}504} \approx 0.000064$

17. $\frac{1}{{}_{10}P_5} = \frac{1}{30{,}240} \approx 0.000033$

19. $\frac{1}{{}_{52}C_2} = \frac{1}{1326} \approx 0.000754$

21. $\frac{1}{{}_6P_6} = \frac{1}{720} \approx 0.001389$

23. a. ${}_{46}C_4 = 163{,}185$

b. $\frac{{}_4C_4}{163{,}185} = \frac{1}{163{,}185} \approx 0.000006$

c. $\frac{{}_{41}C_4}{163{,}185} = \frac{101{,}270}{163{,}185} \approx 0.620584$

25. a. $10^4 = 10{,}000$

b. ${}_{10}P_4 = 10 \cdot 9 \cdot 8 \cdot 7 = 5040$

c. $\frac{1}{4!} = \frac{1}{24} \approx 0.041667$

d. $\frac{1}{3 \cdot \left(\frac{4!}{2!1!1!}\right)} = \frac{1}{36} \approx 0.027778$

e. Repeat a digit

27. $\frac{{}_3C_2}{{}_{14}C_2} = \frac{3}{91} \approx 0.032967$

29. a. ${}_{11}C_5 = 462$;

b. $\left({}_6C_3\right)\left({}_5C_2\right) = (20)(10) = 200$;

c. $\frac{200}{462} \approx 0.4329$

31. $1 - \frac{5}{26} \approx 0.807692$

33. $1 - \frac{(5803.7 + 15{,}004.0)}{137{,}147.0}$

$\approx 1 - 0.151718$

≈ 0.848282

35. $1 - \frac{6}{36} \approx 0.833333$

37. 75%

39. $\frac{3{,}874{,}000}{26{,}233{,}000} \approx 0.147677$

41. $\frac{1}{7}$ **43.** $\frac{2}{5}$

45. $\frac{5}{7}$

47. a. 0.822 **b.** \$80.24

49. a. $\frac{8}{9} \approx 0.8889$, $\frac{9}{10} = 0.9$, $\frac{14}{15} \approx 0.9333$, $\frac{16}{17} \approx 0.9412$, $\frac{20}{21} \approx 0.9524$

b. Dustin Johnson was most likely to win.

51. The set of even numbers larger than 0.

53. The complement consists of players aged 8 or 9, so there are nine players in the complement of *A*.

55. All employees who have worked there for less than or equal to five years.

10.4 Exercises

1. subtracting **3.** independent

5. False

7. Mutually exclusive

9. Mutually exclusive

11. These are not independent events. Because we are not allowed to repeat characters, choosing a character omits it from being chosen the next time.

13. Yes

15. $\frac{24}{36} = \frac{2}{3} \approx 0.666667$

17. $\frac{16}{52} = \frac{4}{13} \approx 0.307692$

19. $\frac{9}{30} = 0.3$

21. $\frac{10}{19} \approx 0.526316$

23. $\frac{13}{36} \approx 0.361111$

25. $\frac{22}{52} \approx 0.423077$

27. $\frac{8}{36} = \frac{2}{9} \approx 0.222222$

29. $\frac{8}{52} = \frac{2}{13} \approx 0.153846$

31. a. $\frac{102}{137} \approx 0.744526$

b. $\frac{76}{137} \approx 0.554745$

33. 0.46 **35.** $\frac{9}{16} = 0.5625$

37. $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = 0.25$

39. $\frac{13}{52} \cdot \frac{4}{52} = \frac{52}{2704} \approx 0.019231$

41. $\frac{3}{20} \cdot \frac{5}{19} \approx 0.039474$

43. $\frac{6}{12} \cdot \frac{2}{11} \approx 0.090909$

45. $\frac{6}{10} = \frac{3}{5} = 0.6$

47. $\frac{3}{51} \approx 0.058824$

49. 0.666667

51. $\frac{31}{35} \approx 0.8857$

53. 0.8077 **55.** 0.1774

10.5 Exercises

1. successes, failures

3. True **5.** Yes; binomial

7. No; there is no fixed number of trials.

9. No; the probability of each success does not remain the same.

11. No; there are three outcomes instead of two.

13. No; there are more than two possible outcomes.

15. Yes; binomial

17. 0.2734 **19.** 0.2592

21. 0.2330 **23.** 0.0289

25. 0.3331 **27.** 0.1876

29. 0.2447 **31.** 0.9417

33. 0.0345 **35.** 1.29×10^{-12}

37. 0.1445 **39.** 0.3223

41. 0.5331 **43.** 0.4086

45. 0.0706

10.6 Exercises

1. probability experiment

3. False

5. Expected value = 2.15

7. Expected value = 16

9. \$2.33

11. Expected value = 1

13. 1.201 strings

15. a. \$1300 **b.** \$200

17. \$115

19. a. \$7.20 **b.** \$576

21. \$23,500

23. a. Expected value = –\$9.73

b. Lose \$243.25

25. a. Expected value = –\$40

b. Lose \$600

27. Your sister's offer. The expected value from her offer is \$10, while the expected value from your mother's offer is only \$7.

29. a. Expected value = 1.14 points

b. Expected value = 1.03 points

c. Yes, it is since he is expected to make fewer points with that strategy.

Chapter 10 Exercises

1. Experimental

3. The sample space is the set of $14 \cdot 14 = 196$ ordered pairs. where the first of each pair one of the 14 possible outcomes on the first spin and the second is one of the 14 possible outcomes for the second spin. That is, \$250 would be paired with each of the 14 possible outcomes on the second spin, as would \$300, \$350, and so on.

5. $\frac{150}{1237} \approx 0.121261$; higher than expected based on the classical probability of about 0.083333 that we calculated in Exercise 4b.

7. $\frac{5!}{2!1!1!1!} = 60$

9. a. $\frac{{}_5C_4}{{}_6C_4} = \frac{5}{15} \approx 0.333333$

b. $6! = 720$

c. $\frac{5!}{6!} = \frac{120}{720} \approx 0.166667$

d. $\frac{2 \cdot 5!}{6!} = \frac{1}{3} \approx 0.333333$

11. {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

13. $\frac{10}{100} \cdot \frac{10}{100} = \frac{1}{100} = 0.01$

15. $\frac{50}{100} \cdot \frac{25}{99} = \frac{25}{198} \approx 0.126263$

17. $\frac{25}{100} \cdot \frac{24}{99} + \frac{10}{100} \cdot \frac{9}{99} + \frac{15}{100} \cdot \frac{14}{99} + \frac{25}{100} \cdot \frac{24}{99} = \frac{5}{33} \approx 0.151515$

19. $\frac{15}{36} = \frac{5}{12} \approx 0.416667$

21. $\frac{13}{36} \approx 0.361111$

23. $\frac{1}{52} \approx 0.019231$

25. 0.0745 **27.** 0.237668

29. 0.3504

31. a. \$2409.09 **b.** \$4000

33. –\$48 **35.** 4.2 days

Chapter 11: Statistics

11.1 Exercises

1. counts, measurements, observations

3. bias

5. Population of interest: women with PCOS; Variables of interest: insulin level, symptoms of PCOS

7. Population of interest: all people; Variables of interest: noise level, cortisol level

9. Sample statistic

11. Sample statistic

13. Population: cancer patients; Sample: 675 cancer patients; Sample statistics: 76% and 63%.

15. Population: Middle-aged women; Sample: 36,000 health records of women; Sample statistics: 18% and 11%; Population parameter: 40%

17. Population: All coffee consumers; Sample: 6195 customers who complete the survey; Parameter: 77%; Statistics: 45%, 32%, and 23%

19. Observational study

21. Observational study

23. Experiment

25. Systematic sampling

27. Cluster sampling

29. Convenience sampling

31. Random sampling

33. Random sample, since there are not clear divisions in the sample group of 1200 patients. Possible biases include age, race, and stage of development of the skin disease in the patients. Answers will vary.

35. Convenience sample, since the population is all children with autism, this is a very large group that may be difficult to study. It may be best to start with a convenience sample to get an idea of where to start. A possible bias to consider is that the effects may be different on children at different levels of functionality, so it may be best to make sure that a range of children are represented. Answers will vary.

37. a. Explanatory variable: type of antiviral agent; Response variable: viral load;

b. Treatment: new antiviral agent;

c. Treatment group: Group A; Control group: Group B;

d. Single-blind. Yes, because the viral load is not a subjective measure.

39. a. Explanatory variable: vitamin C; Response variable: iron level

b. Treatment: iron + vitamin C

c. Treatment group: group that takes iron with vitamin C; Control group: group that takes only iron

d. Neither single-blind nor double-blind. Yes, because iron level is not a subjective measure.

41. Answers will vary. For example, income level, health of residents, happiness of residents, average annual temperature, number of activities in the area.

43. Both begin with separating the population into groups, but cluster sampling surveys every member of certain randomly chosen groups, whereas stratified sampling takes a random sample from each group.

11.2 Exercises

1. frequency distribution

3. frequency histogram

5. True

7. a.

Class	Freq.	**b.** Rel. Freq.
0–9	0	0%
10–19	6	17.1%
20–29	8	22.9%
30–39	5	14.3%
40–49	4	11.4%
50–59	3	8.6%
60–69	5	14.3%
70–79	4	11.4%

c. 11.4% **d.** 0%

e. 20–29 calls

9.

Class	Freq.	Rel. Freq.
1800–2199	1	$\frac{1}{15} \approx 7\%$
2200–2599	4	$\frac{4}{15} \approx 27\%$
2600–2999	6	$\frac{6}{15} = 40\%$
3000–3399	3	$\frac{3}{15} = 20\%$
3400–3799	1	$\frac{1}{15} \approx 7\%$

11. a. Relative frequency; the amount spent on a particular category as a percentage of Sofie's total expenses

b. Housing and Utilities

c. $2946.30

13. a.

Hours	Number of Students	Relative Frequency
0.0–4.4	9	16.4%
4.5–8.9	14	25.5%
9.0–13.4	21	38.2%
13.5–17.9	11	20%

b. 38.2%

c. 41.8% or 41.9%

15. a. iPhone **b.** 19% **c.** 28%

d. No, because the percentage calculated in part **c.** is not above 50%.

17. a. Approximately 6 hours

b. Approximately 7 hours

c. There is essentially no difference in the number of hours wives spend sleeping.

d. A pair of side-by-side bar graphs: one for husbands and one for wives.

19. a. 106 **b.** 43 **c.** 33%

d. 750–799 words and 850–899 words

e. Yes, because the percentage of essays with 700 or more words is 51.9%. It is close enough to 50%.

21. a. Although it's easy enough to see the peak, the way the horizontal tick marks are displayed on the graph makes it difficult to easily identify the date of the peak. A good guess would be somewhere around June 13th, 2021

b. Based on the graph shown, it is true that the average usage time has decreased during this period. However, we need to consider other factors along with a larger window of time.

23. The class widths are not equal.

25. A pie chart is an inappropriate way to display this data. Possible answers could include reference to the fact that not all states are represented in the graph. The pie chart gives percentages based on only these 8 states. The percentages in the pie chart make it seem that the percentage of women who had mammograms is much less than it really is. It could appear that Massachusetts contains 14% of all women age 50 and older who reported mammograms. Furthermore, the percentages add up to more than 100% because each percentage is the total for each category, which means this graph is not a representation of a whole.

11.3 Exercises

1. sum, divided **3.** left

5. True

7. Mean: $19.59; median: $18.35; mode: $32.00; range: $21.92; standard deviation: $8.739; unimodal

9. Mean: 310; median: 310; mode: no mode; range: 0; standard deviation: 0

11. Mean: 22.9; median: 24.5; mode: 24, 26, 35; range: 29; standard deviation: 9.0; multimodal

13. Mode **15.** Mode

17. 86

19. At least 1040 g each

21. Yes

23. No, percentiles give the relative position in terms of percentages. Without knowing more information about the size of the sample, we cannot compute the size of the sample.

25. Georgia

27. The child weighs less than the median weight.

29. a. Min = $21,590, Q_1 = $25,030, Q_2 = $33,385, Q_3 = $35,960, Max = $38,680

b. 15%

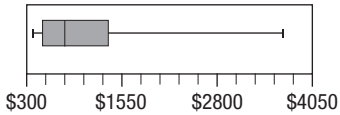
c. [$21,590,$38,680]

31. Min = 20, Q_1 = 56, Q_2 = 84, Q_3 = 114, Max = 168

33. a. Min: $100,960; Q_1: $120,900; Q_2: $145,850; Q_3: 159,635; Max: $182,500

b. 75% **c.** $81,540

35. Min: 310; Q_1: 510; Q_2: 705; Q_3: 1225; Max: 3700



37. No, you should not operate with means. We cannot assume that all five classes are the same size. For instance, suppose one section has 100 students with a mean of 72 and another section has only 11 students, but with a mean of 79. We cannot give equal emphasis to the two means. You will not get an accurate answer.

39. Amelia scoring an 82 is just her raw score; it means she got 82% of the questions on the test correct. Amelia scoring in the 82nd percentile tells you that she scored at least as well as 82% of the people who took the test. It tells you more about her relative performance on the test.

41. Yes, the five-number summary can be put back together in numerical order: 9, 11, 13.5, 17.5, 19. So, $Q_1 = 11$.

11.4 Exercises

1. discrete

3. standard deviations

5. True

7. a. X3 has the largest mean and X1 has the smallest.

b. X3 has the largest standard deviation and X1 has the smallest.

c. BMW X1 generally costs less than BMW X2.

9. a. $z = 0.67$ **b.** $z = -0.93$

c. $z = 1$ **d.** $z = 0$

11. a. $z = -2.11$ **b.** $z = 1.81$

c. $z = 0.43$

13.

	z	x	μ	σ
a.	−0.87			
b.			277.12	
c.		130.30		
d.				8.99

15. 92.79% **17.** 1.32%

19. 15.15% **21.** 99.90%

23. 50.00% **25.** 95%

27. 28.64% **29.** 27.13%

31. 33.81% **33.** 98.80%

35. $0.378 \approx 0.38$ **37. a.** −2.1

39. a. 5.16% **b.** 95.76%

c. 99.94% **d.** 88.88%

41. a. 68% **b.** 280 and 748

c. Below 163 and above 865

43. a. 99.7%

b. 495 mL and 499 mL

c. Bottles that contain at least 501 mL of water are the top 2.5%. Bottles that contain at most 493 mL of water are the bottom 2.5%.

45. Her second race

47. Sofia is taller when compared to her class.

49. 17.92% **51.** 27.14%

53. 25.25%

55. $z = 0$, since half the z-scores lie above this value, and half the z-scores lie below this value.

11.5 Exercises

1. point estimate

3. interval estimate

5. \$18 **7.** $(16.30, 19.70)$

9. a. About (820, 1010). We can be 95% confident that the mortality rate in 2010 was from 820 to 1010 deaths per 100 000 population.

b. Although the sample mean for 2012 falls below 900, the confidence interval still contains 900. Therefore, we cannot conclude with 95% confidence that the actual value is less than 900.

c. Because these are 95% confidence intervals, it is not possible to be 100% certain for any of the years. However, the interval for 2016 is completely below 850, so it is the most likely year to have the mortality below 850.

d. Although the sample mean for 2016 is smaller than that of 2015, their confidence intervals overlap. Therefore, we cannot say that there is a statistically significant difference between the two years' mortality rates.

11. We can be 90% confident that the percentage of patients whose pain is completely relieved by the anesthetic drug is between 76.2% and 81.4%.

13. $(14.6, 15.4)$; The professor can be 90% confident that the mean amount of time that her students spend studying is between 14.6 and 15.4 hours per week.

15. $(18.6, 19.8)$; The writer can be 95% confident that the mean computer usage time for American households is between 18.6 and 19.8 hours per week.

17. $(691, 749)$; We can be 98% confident that the mean amount of money that homeowners spend on lawn service each year is between \$691 and \$749.

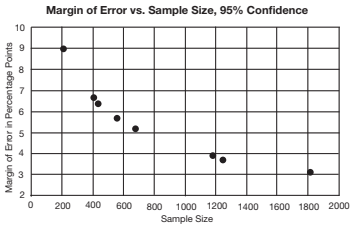
19. $(21.1, 24.1)$; The physical therapist can be 99% confident that the mean recovery time for patients using the new therapy after ACL surgery is between 21.1 and 24.1 days.

21. $(0.60\%, 17.09\%)$; With 95% confidence, we can say that the percentage of all faculty members at the community college who know sign language is between 0.60% and 17.09%.

23. $(14.41\%, 31.31\%)$; We can say with 95% confidence that the percentage of all kindergartners who say pancakes are their favorite breakfast food is between 14.41% and 31.31%.

25. $(51.70\%, 68.30\%)$; With 95% confidence, the percentage of all students at that college who do not regularly check their campus e-mail accounts is between 51.70% and 68.30%.

27. a.



b. The graph shows us that as the sample size increases, the margin of error gets smaller. This means that with the same level of confidence, we can create a tighter confidence interval, which gives a better approximation of the population parameter.

c. A confidence interval for a population parameter is constructed using a sample statistic and a margin of error. Since we are only given the margin of error and not the sample statistics, we do not have enough information to construct the confidence intervals.

29. A margin of error tells how many percentage points the result of the poll may differ from the true result for the whole population. Thus, from the estimated sample parameters, we can conclude that in 95% of such surveys, the percentage of US adults who find dating is now harder than it was 10 years ago will be from 44.9% to 49.1%. Similar reasoning is used for the other parameters.

31. a. $68\% \pm 2.2\% = (65.8\%, 70.2\%)$

b. 95% Confidence Interval for the Population Percentage:

Rep/Lean Rep, Too Much: $74\% \pm 3.3\% = (70.7\%, 77.3\%)$

Dem/Lean Dem, Too Much: $64\% \pm 3.0\% = (61.0\%, 67.0\%)$

The confidence intervals for the two groups do not overlap. Therefore, we can say that we are 95% confident that Republicans are more likely to say technology companies have too much power than Democrats.

33. a. 2.7%

b. With 95% confidence, the percentage of US households owning a pet is from 56.6% to 62.0%.

Chapter 11 Exercises

1. Population: all US college freshmen; Sample: 203,967 freshmen who responded; Sample statistic: 27.6% describe themselves as "liberal;" Sample statistic: 20.7% describe themselves as "conservative;" Sample statistic: 47.4% describe themselves as "middle of the road."

3. Not necessarily; it is convenient, but may not be representative of the campus as a whole.

5. a. Answers will vary. For example, choose one store and survey people as they enter the store; choose one mall entrance and survey people as they enter the mall; set up a booth in the middle of the mall and ask people to participate as they walk by, etc.

b. Answers will vary.

7. a. Explanatory variable: the drug for insomnia; Response variable: side effects

b. Treatment: the drug for insomnia

c. Treatment group: group that was taking the drug; Control group: group that was taking placebo

d. Single-blind. No, because the researchers can be biased when interpreting the patients' answers along with unintentionally pushing them for certain answers.

9. a. 74% **b.** 5940 **c.** 8100

d. Yes, it is categorical data. However, it may not include ALL types of jobs, in which case the graph could be misleading.

11. Mean = 3.22; median = 3.21; mode = 3.29; range = 0.44; stdev = 0.13; min = 3.05; $Q_1 = 3.07$; med = 3.21; $Q_3 = 3.29$; max = 3.49

13. a. Min = 322, $Q_1 = 326$, $Q_2 = 335$, $Q_3 = 342$, Max = 345

b. 40% **c.** [322, 345]

15. a. $z_1 = 6.25$ **b.** $z_2 = -1.5$

c. $z_3 = 12.5$

17. Hasef: $z = 0.889$; Kimberly: $z = 0.885$; Hasef's score was better.

19. 38.59% **21.** 99.93%

23. 12.10% **25.** 99.94%

27. 95.45% **29.** 13.59%

31. True; $z > 0$, so it is higher than the mean.

33. a. 40.13% **b.** 45.03%

c. 14.84%

Chapter 12: Data Science

12.1 Exercises

1. Domain knowledge
3. False
5. Answers will vary.
   a. Does the use of caffeine affect the productivity rates of night-shift workers?
   b. Data can be collected by surveying employees of local businesses that have a night-shift.
   c. A scatterplot can be used to see if there is a trend with productivity and caffeine usage between employees who do and do not consume caffeine.
   d. A bar graph can be created using data for employees who do not consume caffeine versus employees who do.
   e. A social media post could be used.
7. Answers will vary.
   a. What are the most commonly purchased items during the Thanksgiving holiday and how much of each are purchased?
   b. Previous years' sales data could be used along with latest trends in Thanksgiving meals along with decorations.
   c. The sales data across multiple years can be analyzed to see if sales of different items are consistent or are increasing or decreasing.
   d. Line graphs can be used to show sales trends across the years.
   e. An infographic illustrating the trends (whether increasing, decreasing, or steady) for various popular products along with a list of suggested items to keep in stock for the Thanksgiving season.
9. Answers will vary. A weakness is that the question is too vague; it could refer to software, features such as the camera, or price. Alternative questions include "Which main features should the next iteration of the flagship smartphone include?", "Which features do users wish to see in the next iteration of the flagship smartphone?", and "What are the market trends for main features of smartphones?"
11. Answers will vary. A weakness is that the question does not define what "better" means. Alternative questions include "Does flossing before brushing your teeth result in less overall plaque growth between dental cleanings than brushing before flossing?," "Does flossing before brushing your teeth result in less overall gum inflammation between dental cleanings than brushing before flossing?", and "Does flossing before brushing your teeth result in fewer cavities per year than brushing before flossing?"
13. Answers will vary. A weakness is that the question is too vague; it does not define what kind of focus is under consideration nor does it mention a frequency of meditation. Alternative questions include "Does meditating for 20 minutes a day increase a person's ability to focus on work?", "Does meditating for 30 minutes each morning increase a person's decision-making ability?", "Does meditating for 20 minutes in the middle of the workday increase an employee's ability to focus on tasks during the afternoon?"
15. Answers will vary. Questions include "What is the most popular time of day for people to visit the bakery?", "What are the most popular bakery items?", and "What is the age group of the typical customer?"
17. Answers will vary. Questions include "What do employees enjoy most about working for the company?", "What type of training is offered to new employees?", "and "What do employees who leave within a year of being hired have in common?"
19. Answers will vary. Questions include "How many people visit the museum per day?", "What are the main points of congestion in the current lobby?", and "What are the restrictions with redesigning the lobby?"

12.2 Exercises

1. raw, user-friendly
3. LOOKUP
5. True
7. Delimiter is semicolon, six columns
9. Delimiter is a comma, seven columns
11. Lines 4 and 5 seem to be duplicates, Chapter 5 doesn't have an Approved status, and "Antonio" seems to be misspelled in cell E8. The duplicate line can be removed with the Remove Duplicates tool, the missing approval and misspelled name can be fixed with Find and Replace.
13. **a.** and **c.**
15. twelve and
17. Okra Express
19. **a.** Cape cod, colonial, farmhouse, and ranch
    b. 30 **c.** 80 **d.** 20
21. Book Title

12.3 Exercises

1. possible relationship

3. regression line

5. False

7. a. 148.05 **b.** 16.05

c. –346.95

9. No linear correlation

11. No linear correlation

13. No linear correlation

15. No linear relationship

17. Strong positive relationship

19. Yes **21.** No

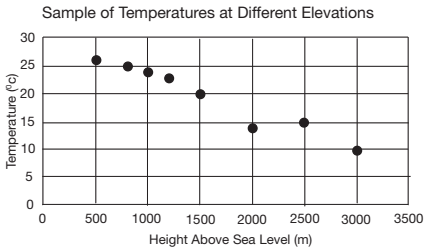
23. a. Positive **b.** $r = 0.897$

c. Yes

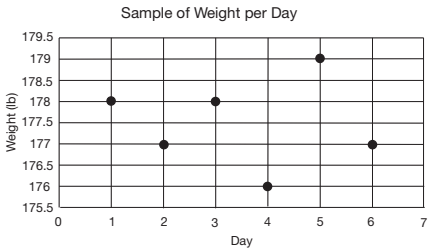
25. a. Positive **b.** $r = 0.981$

c. Yes

27. There appears to be a negative linear correlation.



29. There appears to be no correlation.



31. a. $\hat{y} = 0.418x + 47.535$

b. Yes, $r = 0.695$

c. 76.8

33. a. $\hat{y} = 0.993x + 0.121$

b. Yes, $r = 0.990$

c. 5.6

12.4 Exercises

1. word cloud

3. Infographics

5. Answers will vary. The theme is about individuals, friends, and feelings.

7. Answers will vary. There is less of a focus on the monster than on the writer's feelings.

9. Answers will vary. A love story

11. Answers will vary.

13. Chang **15.** Morales

17. Necklaces

19. Necklaces and dresses seem to tie for least successful.

21. No different than the national average

23. Answers will vary. Information about tourism numbers in 2018 could be added to the map for states that show statistically lower marriage rates compared to the national rate.

25. Existing customers make up a majority of the sales because their bar is consistently higher.

27. Answers will vary. The corresponding data from the same quarter in the previous year could also be displayed so there is a benchmark for comparison. Additional information could be added to detail the success of individual products being sold.

Chapter 12 Exercises

1. Answers will vary. A weakness is that the question is too vague and doesn't explain what "best" means for the overall skin health. Alternative questions include "Which sunscreen blocks the most UVA rays over a period of 2 hours?", "Which sunscreen causes the least amount of irritation to sensitive skin in the form of rashes or clogged pores?", "Which sunscreen has the longest lasting protection against UVB rays?"

3. Delimiter is a comma, five columns

5. Negative linear correlation

7. Weak positive relationship

9. Statistically significant

11. a. Negative **b.** $r = -0.708$

c. Not statistically significant

13. a. 555.8 **b.** 657.12

c. 809.1

15. Answers will vary. Dark colors could be used. An eerie shape related to the story could be formed by the words.

Chapter 13: Voting and Apportionment

13.1 Exercises

1. plurality

3. preference ballot

5. majority rule

7.

	Rankings							
1st	Stephen	Noah	Oliver	Jack	Oliver	Noah	Stephen	Stephen
2nd	Oliver	Oliver	Stephen	Stephen	Noah	Lars	Lars	Jack
3rd	Jack	Jack	Jack	Lars	Lars	Stephen	Oliver	Lars
4th	Lars	Lars	Noah	Noah	Jack	Jack	Noah	Noah
5th	Noah	Stephen	Lars	Oliver	Stephen	Oliver	Jack	Oliver
# Votes	212	133	543	24	179	8	201	11

9.

	Rankings										
1st	Mori	Nguyen	Dey	Smith	Mori	Smith	Abbadi	Smith	Mori	Smith	Abbadi
2nd	Abbadi	Dey	Smith	Abbadi	Nguyen	Nguyen	Mori	Dey	Abbadi	Dey	Mori
3rd	Dey	Smith	Mori	Nguyen	Abbadi	Dey	Nguyen	Abbadi	Nguyen	Mori	Dey
4th	Nguyen	Mori	Abbadi	Dey	Dey	Abbadi	Dey	Nguyen	Dey	Abbadi	Nguyen
5th	Smith	Abbadi	Nguyen	Mori	Smith	Mori	Nguyen	Dey	Smith	Nguyen	Dey
# Votes	3	7	3	6	5	2	1	1	2	1	2

11. a. 720 **b.** 9194

c. *Nothing Compares to You* by Sinead O'Connor

d. 4598 **e.** No

13. Any number smaller than 36

15. Witt must be in second place and Longsdon in first place in column one

17. 10

19. Robert Griffin III: 1687; Andrew Luck: 1407; Trent Richardson: 978; Montee Ball: 348; Tyrann Mathieu: 327

21. a. Leesburg, VA

b. Louisville, CO

c. Answers will vary.

23. a. Candidate B

b. Candidate B

c. Candidate B

d. Candidate B; he or she won using all methods discussed.

25. a. Nixon; no **b.** Nixon; yes

c. Nixon won both the popular vote and the electoral vote. The popular vote is much closer than the electoral vote shows.

d. No. Even with all of Wallace's electoral votes, Humphrey would only have had 237 electoral votes. A majority of 270 electoral votes were required to win the election.

13.2 Exercises

1. Condorcet criterion

3. monotonicity criterion

5. Borda count method

7. a. Susan **b.** Courtney

c. Courtney won all head-to-head comparisons, so the Condorcet criterion applies and is not satisfied.

9. a. Mechanics

b. Heat and Optics

c. No, the Borda count method declares Heat and Optics the winner, but Mechanics has a majority of first-place votes, so the majority criterion is not satisfied.

11. a. New Orleans, LA

b. New Orleans, LA

c. Yes, New Orleans is the preferred destination both before and after Chicago was taken out of the selected options, so the irrelevant alternatives criterion is satisfied.

13. a. H. Kennedy

b. T. Parchment

	Rankings			
1st	H. Kennedy	T. Parchment	H. Kennedy	T. Parchment
2nd	T. Parchment	H. Kennedy	T. Parchment	H. Kennedy
# Votes	8	14	10	11

c. No, Kennedy was the winner before Jones pulled out of the election, but not after, so the Borda count method does not satisfy the alternative criterion here.

15. The majority criterion is satisfied because Johnny Depp wins using both the Borda count method or the majority method.

17. Any x such that x is less than or equal to 11

19. Any x between 42 and 100, inclusive

21. a. Blake

b. The pairwise comparison method does not produce a winner. Luke, Lauren, and Blake all receive 2 points and therefore there is a tie.

c. The Condorcet criterion does not apply here since no candidate wins every head-to-head comparison against the other candidates.

23. No; the winner using the Borda count method is Paladin, but Knight received the majority of the first-place votes and would win using the majority method.

13.3 Exercises

1. standard quota

3. Alabama paradox

5. new states paradox

7. $11,071.79

9. Davis: $474,166,057; Santa Cruz: $104,378,746

11. a. 229.016

b. Riverside: 20.475; Santa Barbara: 26.553

13. The apportionments are different.

Campus	Electric Vehicles
Berkeley	62
Davis	91
Irvine	55
Los Angeles	124
Merced	3
Riverside	20
San Diego	80
San Francisco	18
Santa Barbara	27
Santa Cruz	20

15. a. SD = 21; SQ: I = 1.667; II = 2.048; III = 2.286

b. They should offer two sections of each course with about 21 students per section.

17. They should offer two sections of each course with about 21 students per section.

19. a. Northern: 5; Southern: 6; Eastern: 3; Western: 2

b. Northern: 5; Southern: 6; Eastern: 3; Western: 2

c. No, the apportionments are the same.

21. $13 + 3 + 3 + 4 + 2 = 25$

23. a. Sciences: 5; humanities: 5; professional and trade schools: 1

b. Sciences: 5; humanities: 5; professional and trade schools: 1

c. Sciences: 5; humanities: 5; professional and trade schools: 1

d. Sciences: 5; humanities: 5; professional and trade schools: 1

25. Campus 1: 21 police officers; campus 2: 19 police officers; campus 3: 1 police officer; The Alabama paradox occurs because when the total number of police officers to be apportioned increased, the number of police officers appointed to campus 3 decreased.

27. a. Freshmen: 5 members; sophomore: 4 members; junior: 4 members; senior: 4 members

b. Freshmen: 5 members; sophomore: 4 members; junior: 4 members; senior: 4 members

c. Freshmen: 5 members; sophomore: 4 members; junior: 4 members; senior: 4 members

13.4 Exercises

1. coalition **3.** dictator

5. $[13: 9, 6, 5, 3, 2]$

7. a. 25 **b.** 2

c. No **d.** No

9. 46 **11.** 45

13. No dictator; no dummy player

15. a. 5 **b.** 42 **c.** No

d. $\{P_1, P_2, P_3\}$, $\{P_1, P_2, P_4\}$, $\{P_1, P_2, P_3, P_4\}$, $\{P_1, P_2, P_4, P_5\}$, $\{P_1, P_2, P_3, P_5\}$, $\{P_1, P_2, P_3, P_4, P_5\}$

e. P_1 and P_2 are critical in all winning coalitions.

f. P_1 and P_2 have veto power.

17. P_2

19. $\text{BPI}(P_1) = \frac{10}{26} \approx 0.385 = 38.5\%$;

$\text{BPI}(P_2) = \text{BPI}(P_3) = \text{BPI}(P_4)$

$= \text{BPI}(P_5) = \frac{4}{26} \approx 0.154 = 15.4\%$;

no dictators; none have veto power

21.

$\text{SSPI}(P_1) = \frac{12}{24} = 0.5 = 50\%$;

$\text{SSPI}(P_2) = \text{SSPI}(P_3) = \text{SSPI}(P_4)$

$= \frac{4}{24} \approx 0.167 = 16.7\%$;

no dictators; none have veto power

23. a. Any coalition of at least 218 members

b. Any coalition of at least 291 members

25. P_1: 25%; P_2: 25%; P_3: 25%; P_4: 25%

Chapter 13 Exercises

1.

	Rankings									
1st	H	P	S	S	K	P	P	J	S	K
2nd	K	J	H	H	J	H	J	P	H	P
3rd	P	K	J	K	P	S	S	H	J	J
4th	J	S	P	P	H	J	H	K	K	S
5th	S	H	K	J	S	K	K	S	P	H
# Votes	5	4	6	3	7	1	4	1	2	2

3. $\frac{7(6)}{2} = 21$

5. a. 5! = 120 **b.** 793

c. 388 **d.** Candidate A

e. 397 **f.** No

7. a.

Harris; SGA	20
Icin; Social Work Club	73
Green; ΑΛΠ	86
Lawson; FCA	6
Albert; GSA	132
Roman; ΓΒΘ	89
Switzer; Galois Club	37
Belton; Hispanic Culture Center	10
Finley; History Club	38
Issen; NAEA	33
San Marie; NBS	19
Centus; NTSS	82
Dennis; ΩΨΦ	56
Russan; ΦΑ	130
Hunter; ΦΜΑ	45
Austin; ΣΓΡ	8
Molda; Student Design Group	92
Irene; Voices of Praise	88

b.

1	Albert; GSA	132
2	Russan; ΦΑ	130
3	Molda; Student Design Group	92
4	Roman; ΓΒΘ	89
5	Irene; Voices of Praise	88
6	Green; ΑΛΠ	86
7	Centus; NTSS	82
8	Icin; Social Work Club	73
9	Dennis; ΩΨΦ	56
10	Hunter; ΦΜΑ	45

9. a. Clinton; no **b.** Clinton; yes

c. No; answers will vary. For example, although it is possible that the popular vote may have changed, the electoral vote would not (since Perot did not receive any electoral votes) and Clinton would still win the election.

11. a. *Django Unchained*

b. *Zero Dark Thirty*

c. No; the Condorcet criterion is not satisfied because the plurality method chose *Django Unchained* as the winner, but *Zero Dark Thirty* won the head-to-head comparisons against every other candidate.

13. The irrelevant alternatives criterion would be met using the plurality with elimination method because even if Jacksonville, FL were removed, the winner is still Boston, MA.

15. The Condorcet criterion is satisfied because outdoor living spaces wins using the plurality method and also wins all head-to-head matchups with the other categories.

17. The missing ranking for honesty is 2nd and the missing ranking for sense of humor is 1st.

19.

County	Officers
A	45
B	52
C	55
D	40
E	24
F	34

21. $62,816.18

23. Austin: $4,014,970,172; Dallas: $1,084,773,995

25. a. 183.515 students/vehicle

b. San Antonio: 126.409 vehicles; Permian Basin: 14.691 vehicles

27. Yes, the apportionments are different.

Campus	Allocation
Arlington	112
Austin	253
Brownsville	51
Dallas	68
El Paso	89
San Antonio	126
Tyler	26
Permian Basin	15
Pan American	85

29.

Course	Allocation
Phys Sci	2 Sections
Biology	1 Section
Anatomy	2 Sections

31.

Course	Allocation
Comp I	8 Grad Stud
Comp II	5 Grad Stud
Surv of Lit	2 Grad Stud
Poetry	4 Grad Stud

33. a.

Activity	Allocation
Swimming	$1,715,249
Library	$18,167,643
Baseball	$8,051,171
Softball	$6,846,996
Theater	$5,271,767
Comp Hob	$3,339,485
Music	$11,607,689

b.

Activity	Allocation
Swimming	$1,715,249
Library	$18,167,643
Baseball	$8,051,171
Softball	$6,846,996
Theater	$5,271,767
Comp Hob	$3,339,486
Music	$11,607,688

c.

Activity	Allocation
Swimming	$1,715,249
Library	$18,167,643
Baseball	$8,051,171
Softball	$6,846,996
Theater	$5,271,767
Comp Hob	$3,339,486
Music	$11,607,688

d.

Activity	Allocation
Swimming	$1,715,249
Library	$18,167,643
Baseball	$8,051,171
Softball	$6,846,996
Theater	$5,271,767
Comp Hob	$3,339,486
Music	$11,607,688

35. No **37.** 13

39. a. 6 **b.** 96

c. $\{P_1, P_2, P_3, P_4, P_5, P_6\}$, $\{P_1, P_2, P_3, P_4, P_5\}$, $\{P_1, P_2, P_3, P_4, P_6\}$

d. Yes, all players are critical in some coalition.

e. Yes, P_1, P_2, P_3, P_4

f. No

g. $\text{BPI}(P_1) = \text{BPI}(P_2) = \text{BPI}(P_3) = \text{BPI}(P_4) = \frac{3}{14} \approx 0.214;\ = 21.4\%$

$\text{BPI}(P_5) = \text{BPI}(P_6) = \frac{1}{14} \approx 0.071 = 7.1\%$

41. a. $\langle P_2, P_1, P_3, P_4\rangle, \langle P_2, P_1, P_4, P_3\rangle, \langle P_3, P_4, P_1, P_2\rangle, \langle P_4, P_3, P_1, P_2\rangle, \langle P_2, P_3, P_1, P_4\rangle, \langle P_3, P_2, P_1, P_4\rangle, \langle P_2, P_4, P_1, P_3\rangle, \langle P_4, P_2, P_1, P_3\rangle$

b. $\langle P_1, P_2, P_3, P_4\rangle, \langle P_1, P_2, P_4, P_3\rangle, \langle P_3, P_4, P_2, P_1\rangle, \langle P_4, P_3, P_2, P_1\rangle, \langle P_1, P_3, P_2, P_4\rangle, \langle P_3, P_1, P_2, P_4\rangle, \langle P_1, P_4, P_2, P_3\rangle, \langle P_4, P_1, P_2, P_3\rangle$

c. $\text{SSPI}(P_1) = \text{SSPI}(P_2) = \frac{8}{24} = \frac{1}{3} \approx 0.333;\ = 33.3\%$

$\text{SSPI}(P_3) = \text{SSPI}(P_4) = \frac{4}{24} = \frac{1}{6} \approx 0.167 = 16.7\%$

d. No **e.** No

Chapter 14: Graph Theory

14.1 Exercises

1. degree **3.** edge

5. loop

7. chromatic number

9. cycle

11. Vertices: USA, UK, Algeria, Canada, Mexico; edges: 10, 1, 2, 60, 3

13. Vertices: Hannah, Lee, Jalen, Anna, Mike, Kiara; Edges: e_1, e_2, e_3, e_4, e_5, e_6, e_7

15. a. v_1, v_2, v_3

b. No, it does not start and end at the same vertex.

c. Answers will vary. The walk must start and end with the same vertex.

17. a. Answers will vary. For example, *a*, *b*, *c*, *b*, *e*, *a*, *e*, *c*, *d*, *e*, *f*, *d*, *f*, *a*.

b. Answers will vary. For example, *u*, *w*, *v*, *x*, *u*, *y*, *z*, *u*.

19. a. Answers will vary. For example, *k*, *m*, *n*.

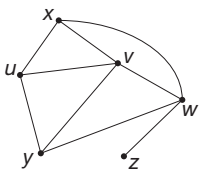
b. Answers will vary. For example, *r*, *q*, *t*, *n*.

c. Answers will vary. For example, *j*, *t*, *m*, *k*, *s*, *q*, *p*.

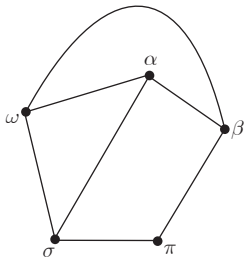
21. G_1 is connected; G_2 is disconnected

23. Yes. Explanations will vary. For example, both graphs consist of the same vertices and edges.

25. Answers will vary. For example,



27. Answers will vary. For example,



29. Answers will vary because the cycle may be written beginning with any of the vertices. However, the cycle must include all vertices in their correct order. The first vertex in the cycle list should not be repeated at the end of the list. For example, *r*, *t*, *v*, *w*, *u*, *s* is of length 6.

31. a. Answers will vary. For example, *ab*.

b. Answers will vary. For example, *a* and *e*.

c. Answers will vary. For example, *ae* and *de*.

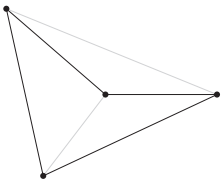
d. Answers will vary. For example, *c* and *r*.

e. Answers will vary. For example, *a*.

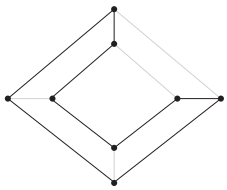
f. Answers will vary. For example, *a*, *b*, *c*, *d*, *e*.

33. Answers will vary. For example,

a.



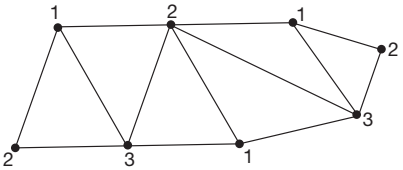
b.



c. This graph doesn't have a Hamilton cycle.

35. Answers will vary. Cameras can be placed in rooms *i*, *g*, *b*, and *d* or in rooms *j*, *g*, *b*, and *d*.

37. Minimum number of colors = 3;



14.2 Exercises

1. connected

3. spanning tree

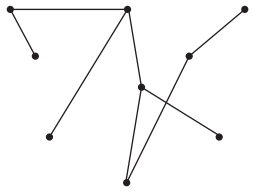
5. one

7. No, the graph has cycles in it, and is therefore not a tree.

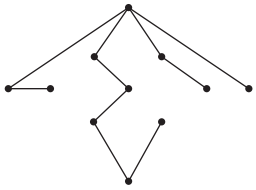
9. Yes, the graph is a tree.

11. 4 **13.** 3

15. Answers will vary.



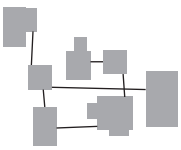
17. Answers will vary.



19. 4 **21.** 14

23. 8

25.



27. 22 cities

14.3 Exercises

1. bipartite graph

3. matching

5. neighborhood

7. No, the vertices cannot be divided into two groups without edges joining vertices in the same group.

9. $N(B) = \{x_2, x_4, x_6, y_1, y_3, y_5\}$

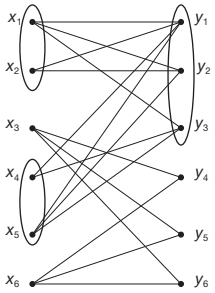
11. $N(D) = \{\text{all vertices in } Q\}$

13. No, there aren't enough vertices to have a matching.

15. No, there is a vertex without an edge, and hence it cannot have a matching.

17. The graph is regular bipartite, so it has a matching.

19. No, if $A = \{x_1, x_2, x_4, x_5\}$, $N(A)$ has only three vertices.

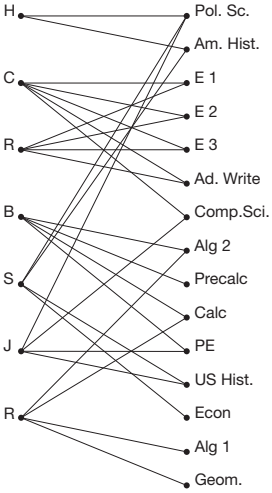


21. No, not every vertex has the same degree.

23. No, because the graph is not bipartite.

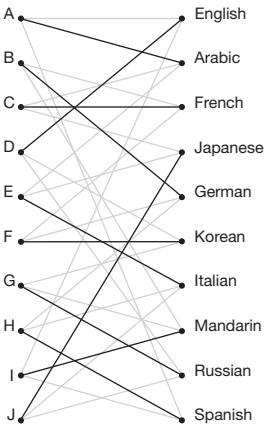
25. Answers will vary. For example,

27.



No matching possible.

29. Answers will vary.



31. No two edges of the same color can share a vertex. Therefore, every edge of the same color has a distinct pair of vertices it joins together

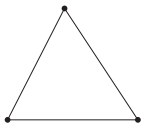
14.4 Exercises

1. edges crossing

3. $v + f - 3 = 2$

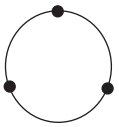
5. complete

7.



Graph S

9.



11. 4

13. $v = 6$, $e = 10$, $f = 6$; $6 + 6 - 10 = 2$, so Euler's formula is satisfied.

15. $v = 5$, $e = 5$, $f = 2$; $5 + 2 - 5 = 2$, so Euler's formula is satisfied.

17. 21 edges **19.** 4 faces

21. Yes, the graph is planar.

23. No, the graph is not planar (it is $K_{3,3}$).

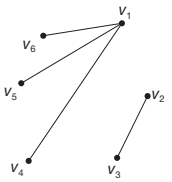
25. No, the graph is not planar (it is $K_{3,3}$).

27. $\chi(K_3) = 3$; $\chi(K_5) = 5$; $\chi(K_n) = n$

29. Any planar drawing when drawn on a sphere will have its exterior face changed from an infinite face to a finite one.

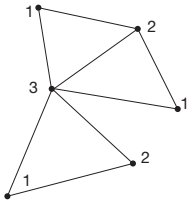
Chapter 14 Exercises

1. No, the graph is not connected.



3. Yes, they are the same graph.

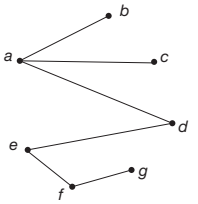
5. Chromatic number = 3; Answers will vary. For example,



7. No, the graph is not a tree because it contains at least one cycle.

9. 11

11. Answers will vary. For example,



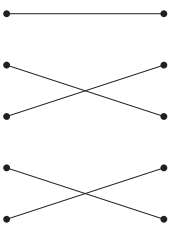
13. a. Yes, it is a spanning tree. A spanning tree will ensure that all cities are connected with optical fiber.

b. Yes, it is a minimum-weight spanning tree.

15. $N(A) = \{\text{Jeff, Isaac, Peg, Sarah}\}$

17. The graph has a matching.

19. Answers will vary. For example,



21. 8

23. #21: $f+v-e=2$

$$8+9-15\stackrel{?}{=}2$$
$$17-15\stackrel{?}{=}2$$
$$2=2$$

#22: $f+v-e=2$

$$8+12-18\stackrel{?}{=}2$$
$$20-18\stackrel{?}{=}2$$
$$2=2$$

25. No, the graph is not planar because it contains K_5 as a minor.