$$\begin{aligned}3^{4,294,967,297}-3&=3^{4,294,967,296+1}-3\\&=3^{4,294,967,296}\cdot 3^{1}-3\\&\equiv 3,029,026,160\cdot 3-3\pmod{4,294,967,297}\\&\equiv 9,087,078,480-3\pmod{4,294,967,297}\\&\equiv 9,087,078,477\pmod{4,294,967,297}\\&\equiv 497,143,883\pmod{4,294,967,297}\end{aligned}$$

Thankfully, this is not 0. So 4,294,967,297 is indeed not prime, but composite.

This is quite a lot of work to do by hand with a calculator, but certainly far less than the hundreds of millions of divisions we would otherwise have had to check by the divisor method. Even computers can't handle the divisor method for numbers this big, but they can use modular arithmetic to apply Fermat's Little Theorem. Now we have a healthy appreciation for what goes on behind the scenes for this computation.

***Skill Check Answers***

**1.** $$\begin{aligned}10^{11}-10&=(10\cdot 10)\cdot(10\cdot 10)\cdot(10\cdot 10)\cdot(10\cdot 10)\cdot(10\cdot 10)\cdot 10-10\\&\equiv 1\cdot 1\cdot 1\cdot 1\cdot 1\cdot 10-10\pmod{11}\\&\equiv 1\cdot 10-10\pmod{11}\\&\equiv 0\pmod{11}\end{aligned}$$

**2.** **a.** $3^7$ **b.** $4^{10}$ **c.** $2^6$

# 8.3 Exercises

## CONCEPT CHECK

1. One can test for prime numbers using ____________________________.

2. True or False: If a statement is true, then the contrapositive is also true.

3. True or False: If you calculate a 0 from the contrapositive of Fermat's Little Theorem, then the power must be a prime number.

## PRACTICE

Verify that Fermat's Little Theorem holds true for each prime number using the value of *x* given.

4. The prime number 11 and $x = 6$

5. $x = 2$ with the prime 23

6. The third prime number and $x = 10$

7. $p = 79$ and $x = 3$

8. The prime number 13 and $x = 3$

9. The prime number 17 and $x = 2$

**10.** $x = 3$ with the prime 19

**11.** $p = 31$ and $x = 4$

**12.** $p = 7$ and $x = 2$

Use the contrapositive of Fermat's Little Theorem to show that each given number is composite.

**13.** 63,571 using $x = 3$

**14.** 12,731 using $x = 3$

**15.** 65,476,751 using $x = 2$

**17.** 7802 using $x = 2$

**18.** 10,431 using $x = 3$

**19.** 103,322 using $x = 3$

**20.** 123,456,782 using $x = 2$

Use technology to confirm that each given number is composite using the contrapositive of Fermat's Little Theorem.

**21.** 2427

**22.** 24,271

**23.** 245,127

**24.** 10,050,207

**25.** 1963

**26.** 1649

**27.** 903,219

**28.** 10,203

**29.** 565,841

**30.** 106,057

**31.** 35,066

**32.** 14,772,361

## 8.3 PROJECT

## FERMAT AND MERSENNE PRIMES

In Section 8.3, you learned about Fermat's Little Theorem and testing whether numbers are prime. In this project, you will explore two special types of numbers and test whether they are prime.

Very large prime numbers play a vital role in keeping information secure. There are two special types of numbers that have played a role in searching for large primes. They are called Fermat numbers and Mersenne numbers. A Fermat number is a number of the form $2^{2^n} + 1$, where $n = 0, 1, 2, \ldots$. For example, when $n = 2$, we have the Fermat number $2^{2^2} + 1 = 2^4 + 1 = 17$. Since this resulting number is prime, we say that 17 is a Fermat prime.

1. Find the Fermat numbers that correspond to $n = 0$, $n = 1$, and $n = 3$. Are these numbers Fermat primes? (Note that the Fermat number 65,537, which corresponds to $n = 4$, is a Fermat prime.)
2. Find the Fermat number that corresponds to $n = 5$.
3. Evaluate the expression $\left(2^9 + 2^7 + 1\right)\left(2^{23} - 2^{21} + 2^{19} - 2^{17} + 2^{14} - 2^9 - 2^7 + 1\right)$. Is the Fermat number that corresponds to $n = 5$ a prime number? Why or why not?