2. First, find the multiples of the first 9 digits.

$$(10\cdot 0)+(9\cdot 3)+(8\cdot 0)+(7\cdot 6)+(6\cdot 4)+(5\cdot 0)+(4\cdot 6)+(3\cdot 1)+(2\cdot 5)=130$$

Next, add 2 for the final digit of the ISBN. $130+2=132$

Finally, find $132 \pmod{11}$.

$$132\equiv 0 \pmod{11}$$

Because the sum is congruent to 0 (mod 11), the ISBN is valid.

8.2 Exercises

CONCEPT CHECK

1. The 10th digit of a 10-digit ISBN is referred to as the _______________.

2. True or False: If a number is a multiple of n, that number will be congruent to 0 in modulus n.

3. True or False: The numbers in an ISBN are selected at random.

4. True or False: The check-sum digit in an ISBN must always be equal to 0.

PRACTICE

Determine whether each statement is true or false.

5. $2^3 \equiv 3 \pmod{5}$

6. $\sqrt{81} \equiv 3 \pmod{2}$

7. $(3\cdot 4)\equiv 0 \pmod{6}$

8. Any two even numbers x and y are equivalent to each other mod 2; that is, $x\equiv y \pmod{2}$.

9. Any two odd numbers a and b are equivalent to each other mod 2; that is, $a\equiv b \pmod{2}$.

10. $645,234\equiv 111,111,111 \pmod{3}$ (**Hint:** Two numbers are congruent mod 3 if both are divisible by 3. Use Section 8.1 methods to help you decide.)

11. $9,436,278,463,920\equiv 764,283,237,885 \pmod{5}$

Compute each value.

12. $12\equiv$ ___ $\pmod{5}$

13. $13\equiv$ ___ $\pmod{4}$

14. $120\equiv$ ___ $\pmod{11}$

15. $84\equiv$ ___ $\pmod{3}$

16. $5^2\equiv$ ___ $\pmod{2}$

17. 4328 mod 10

18. 60,002 mod 6

19. 311 mod 4

20. 44 mod 12

21. 113 mod 12

22. $3^4 \equiv$ __ (mod 3)

23. 812 mod 6

24. $72 \equiv$ ___(mod 11)

25. 113 mod 8

Convert each 24-hour clock time to an equivalent 12-hour clock time.

26. 0800 hours

27. 1300 hours

28. 2100 hours

29. 0000 hours

Convert each 12-hour clock time to an equivalent 24-hour clock time.

30. 5:00 p.m.

31. 12:00 p.m.

32. 1:00 p.m.

33. 7:00 a.m.

Compute each value.

34. $(56+87) \bmod 13$

35. $[12(34+6)] \bmod 7$

36. $[128-(15+8)\cdot 23] \bmod 11$

37. $(2^5-17) \bmod 3$

38. $(11 + 39) \bmod 5$

39. $(91 - 16) \bmod 6$

40. $(27 \cdot 18) \bmod 7$

41. $[6(11 + 5) - 21] \bmod 13$

42. $[3(15 + 17) + 2(56-14)] \bmod 8$

43. $(9^2 - 3^4) \bmod 3$

APPLICATIONS

Determine whether each 10-digit ISBN is valid. If it is not, state what the correct check-sum number should be.

44. 0-392-31123-2

45. 1-103-24582-6

46. 0-332-15573-0

47. 1-02-345678-8

48. 0-022-44668-8

49. 2-013-31943-2

Determine the missing digit for each 13-digit ISBN.

For a 13-digit ISBN, multiply the first digit by 1, the second digit by 3, the third digit by 1, the fourth digit by 3, and so on, until you get the 12th digit, which you multiply by 3. These products are added and the 13th digit is chosen so that the total of all these numbers is 0 modulo 10.

50. *Emma* by Jane Austin: 978048640648**?**

51. *Harry Potter and the Sorcerer's Stone* by J.K. Rowling: 978059035342**?**

52. *The Hunger Games* by Suzanne Collins: 97**?**0439023528

53. *The Girl with the Dragon Tattoo* by Stieg Larsson: 9**?**80307949486

54. *The Way of Kings* by Brandon Sanderson: 978-0765376?71

55. *Furies of Calderon* by Jim Butcher: 978-?441012688

Decide whether each barcode is valid. If it is not, state what the correct check-sum number should be.

Each of the following bar codes consists of 12 digits. To find the check-sum digit, multiply the first digit by 3, the second digit by 1, the third digit by 3, the fourth digit by 1, and so on, until you get the 11th digit, which you multiply by 3. The final digit is chosen so that the sum of the first 11 products and the final digit is 0 modulo 10.

56. 1 00230 42041 ?

57. 3 74034 61005 ?

58. 1 02010 40302 ?

Calculate the correct first digit for each barcode.

59. ? 00200 30X45 0

60. ? 21045 72933 6

61. ? 21323 43536 2

Credit card companies use the Luhn algorithm to help construct secure numbers. Use the following steps to determine whether each credit card number is valid. If it is not, find the correct check-sum number.

1. The check-sum digit is the last digit in the number, whether the card number is 13, 15, or 16 digits long. Working right to left, starting with the digit to the left of the check-sum number, double the value of every other digit.
2. If a number becomes a 2-digit number after doubling, treat each digit as an individual digit. Finally, sum the digits of all doubled numbers as well as the undoubled numbers including the check-sum number.
3. If the total is congruent to 0 $(\text{mod } 10)$, then the number is valid according to the Luhn formula; otherwise it is not valid.

62. 3780 2850 1184 225

63. 300 9255 939 6891

64. 389 841 621 516 22

65. 4929 1175 0198 3180

8.2 PROJECT

AFFINE SHIFT CIPHERS

In Section 8.2, you learned about modular arithmetic. In this project, you will use modular arithmetic to encode and decode messages.

One method of encoding messages is to simply convert the letters of the alphabet to numbers by identifying A with 0, B with 1, and so on.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

A message encoded in this manner would be a string of numbers. We can make this encryption slightly more advanced by using a shift cipher, also called a Caesar cipher, which converts the message back to a string of letters. To use a shift cipher on the numerically encoded alphabet, add a fixed value (the "shift" value) to the number and then calculate the result modulo 26, then substitute the corresponding value into the encoded string.

For example, suppose we want to encode the letter T using a shift cipher with a shift of 9. First, we need to know that T corresponds to 19. Adding the shift value results in a value of 28, which is not a number modulo 26. We can calculate that $28 \text{ mod } 26 = 2$. The letter we'd use in the encoded message is C.

1. Encode the word NOTE using a shift cipher with a shift value of 11.

If the value of the shift is known, a shift cipher is easy for anyone to decode. In the example with a shift of 9, the encoded letter was C, which corresponds to a value of 2. To decode this letter, we subtract the shift value to get -7. Since this number is negative, we can add 26 to get a value between 0 and 25. In this case, we get 19, which corresponds to T.

2. Decode DLQP that was coded using a shift cipher with a shift of 11.

Since shift ciphers are relatively easy to decode, more advanced coding methods are used to keep information safe. One slightly more advanced method is an