$$\begin{array}{r} \overset{1}{1}02_3 \\ \times\ 21_3 \\ \hline 102_3 \\ 2110_3 \end{array}$$

$2 \times 1 = 2_3$
No conversion necessary.

Now we can add the two products together to find the answer, converting any values back into base 3, if necessary.

$$\begin{array}{r} 102_3 \\ \times 21_3 \\ \hline 102_3 \\ +2110_3 \\ \hline 2212_3 \end{array}$$

So 102_3 multiplied by 21_3 is equal to 2212_3.

✔ Skill Check 7.3.5

Multiply $D1_{16}$ and 8_{16}

Skill Check Answers

1. 108

2. $111100100011000000_2 = (1 \cdot 2^{17}) + (1 \cdot 2^{16}) + (1 \cdot 2^{15}) + (1 \cdot 2^{14}) + (1 \cdot 2^{11}) + (1 \cdot 2^{7}) + (1 \cdot 2^{6})$
$= 131{,}072 + 65{,}536 + 32{,}768 + 16{,}384 + 2048 + 128 + 64$
$= 248{,}000$

$3C8C0_{16} = (3 \cdot 16^4) + (12 \cdot 16^3) + (8 \cdot 16^2) + (12 \cdot 16^1)$
$= 196{,}608 + 49{,}152 + 2048 + 192$
$= 248{,}000$

3. 1000_2 **4.** 20_3 **5.** 688_{16}

7.3 Exercises

✔ CONCEPT CHECK

1. The Hindu-Arabic numeration system is in base ______.

2. The number 1348 is written in base ______.

3. The hexadecimal system uses ______ to represent the digits 10 through 15.

4. Numbers in base 2 and base 16 are often used in ______ ________.

5. In the Hindu-Arabic numeration system, the 4th digit represents the ______ place. In the base 2 system, the 4th digit represents the ___ place.

PRACTICE

Convert each number to base 10.

6. 201_3

7. 4444_5

8. 11011_2

9. 1010101010100_2

10. 5301_6

11. 1001_8

12. AB_{16}

13. $6D8_{16}$

14. Which is larger, 1111001000110001100_2 or $A9CFE_{16}$?

15. Which is larger, BB_{16} or 60_{32}?

Convert each base 10 number to the indicated base.

16. 52 to base 2

17. 1000 to base 3

18. 2000 to base 5

19. 48 to base 12

20. 66 to base 12

21. 700 to base 2

22. 256 to base 8

23. 542 to base 16

Convert each base 2 number to base 16.

24. 10001_2

25. 111011_2

26. 1010101101_2

27. 11100101100_2

28. 1000100010101100_2

29. 1011000000111000_2

Covert each base 16 number to base 2.

30. 17_{16}

31. $3D_{16}$

32. CAB_{16}

33. FED_{16}

34. $A79B2_{16}$

35. $3D095_{16}$

Perform the indicated operations.

36. Add the numbers 155_8 and 72_8.

37. Add the numbers 63_7 and 51_7.

38. Add the numbers 221_5 and 24_5.

39. Add the numbers 101_2 and 111_2.

40. Add the numbers 1222_3 and 201_3.

41. Add the numbers 45231_6 and 1432_6.

42. Subtract the number 10_8 from 34_8.

43. Subtract the number 41_9 from 324_9.

44. Subtract the number 110011_2 from 110110110_2.

45. Subtract the number $42A_{16}$ from $C0F_{16}$.

46. Subtract the number 506_7 from 2135_7.

47. Subtract the number 2243_5 from 4000_5.

48. Multiply 155_7 and 3_7.

49. Multiply 22_3 and 10_3.

50. Multiply 510_8 and 10_8.

51. Multiply 12_4 and 31_4.

52. Multiply 24_5 and 11_5.

53. Multiply $3A_{16}$ and 4_{16}.

54. An addition table for numbers in base 5 is shown. Fill in the missing numbers. Assume all numbers in the table are in base 5.

+ Base 5	0	1	2	3	4
0	0	1	2	3	4
1	1				
2	2				11
3	3		10		
4	4				

55. An addition table for numbers in base 8 is shown. Fill in the missing numbers. Assume all numbers in the table are in base 8.

+ Base 8	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1			3		5			
2	2							
3		4					11	
4		5		7	10			
5	5							
6		7						
7		10						

56. An addition table for numbers in base 12 is shown. Fill in the missing numbers. Assume all numbers in the table are in base 12, which uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B.

+ **Base 12**	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	2	3	4	5	6	7	8	9	A	B
1		2										
2			4									
3			5									
4			6				A					
5			7		9							
6			8									15
7		8			B							
8				B								
9									15			
10	A											
11								16				

57. A multiplication table for numbers in base 4 is shown. Fill in the missing numbers. Assume all numbers in the table are in base 4.

× **Base 4**	0	1	2	3
0	0	0	0	0
1	0		2	
2	0			12
3	0	3		

58. The following table is a multiplication table for numbers in base 5. Fill in the missing numbers. Assume the numbers in the table are in base 5.

× **Base 5**	0	1	2	3	4
0	0	0	0	0	0
1	0				
2	0		4		13
3	0	3			
4				22	

59. A multiplication table for numbers in base 7 is shown. Fill in the missing numbers. Assume all numbers in the table are in base 7.

× Base 7	0	1	2	3	4	5	6
0	0	0			0	0	0
1			2	3			
2							
3	0				15		
4		4					
5		5				34	
6	0						

WRITING & THINKING

60. Describe one way to convert a number in base 2 to base 16.

61. A piece of an addition table that was in the process of being completed is shown. Based on the information that you have, determine which base is being used for the addition. Assume all numbers in the table are in the necessary base.

		2	3	4		
		2	3	4		
		3	4			
2	0		4	5	10	
3	0	3	5	10	11	
			10	11	12	

62. A piece of an addition table that was in the process of being completed is shown. Based on the information that you have, determine which base is being used for the addition. Assume all numbers in the table are in the necessary base.

		5	6	7	
			6	7	
		6	7	8	
				10	
		8		11	
		10		12	
			12		

63. A piece of a multiplication table that was in the process of being completed is shown. Based on the information that you have, determine which base is being used for the multiplication. Assume all numbers in the table are in the necessary base.

		4	5	6	
			0	0	
		4	5	6	
		10	12	14	
		20			

7.3 PROJECT

THE BAKER'S DOZEN

In the sixteenth century, bakers in the United Kingdom who sold their goods by the dozen (12 items) were obligated to meet specific weight and quality standards. Failing to do so was considered a crime. To avoid punishment, it became a common practice to include an additional item with the dozen purchased to assure the law was properly obeyed. This became known as the baker's dozen.

One particular baker served six customers: Anne, Elinor, Giles, Lancelot, Rose, and Florence. The baker placed their orders on the counter and asked them to verify that their orders were correct and that the orders were, indeed, a baker's dozen each.

Anne nodded her head and confirmed, "Yes. 13." The other five nodded as well. Elinor said, "15 exactly!" Giles smiled, "23 for me. Perfect." Lancelot added, "1101 here. Thank you!" Rose happily counted, "10 for me! Yum!" Finally, Florence, about to sample a morsel, gave a thumbs up and said, "11. I love a baker's dozen!"

The baker looked confused. He knew he had put exactly the same number of items in each customer's box. As the pleased clients left the bakery, the solution to the puzzle dawned on him.

1. Explain the apparent contradiction with the numbers and how each customer arrived at their count. Be sure to support your explanation with useful mathematical expressions and equations.

Later that day, the baker asked his assistant to count the remaining inventory and determine how many items they sold during the day. The assistant counted that there were 22 items left in the inventory. The baker knew they started with 175 items and said, "We've sold 159 items today!" His assistant replied with "No! We sold 243 today."

2. Explain how both the baker and his assistant are correct.