f. To determine how high above the fence the ball was when it went over the fence, we first need to determine the height of the ball as it went over the fence. Then, we need to subtract the height of the fence from the height of the ball.

From part e., we know that the ball went over the fence after 2.81 seconds. We can use that value for t in the height function to calculate how high the ball was when it went over the fence. Using the formula, we have the following.

$$h(t) = -16t^2 + 87.75t + 4$$
$$h(2.81) = -16(2.81)^2 + 87.75(2.81) + 4$$
$$h(2.81) = 124.2399$$

Thus, Acuña's homerun ball was approximately 124.24 feet off of the ground when it went over the fence.

Now we subtract the height of the fence, 8 feet 8 inches, from the ball's height. Before we can subtract, both measurements need to be in the same format. We'll convert the fence height to a decimal form of feet: $8 \text{ in.}\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) \approx 0.67 \text{ ft.}$

This gives that the fence is 8.67 feet tall. Now we can subtract.

$$124.24 - 8.67 = 115.57$$

Thus, the homerun ball was 115.57 feet above the fence when it went over it for a homerun.

✔ Skill Check 5.6.4

Suppose a rocket is fired vertically upward from 1200 feet above the ground, with an initial velocity of 620 ft/sec. Write the quadratic model for its height $h(t)$ in feet above the ground after t seconds and determine how long the projectile will be in flight.

Skill Check Answers

1. $a = 3, b = -5, c = 2$ **2.** $x = -\frac{3}{2}$ and $x = \frac{9}{2}$ **3.** $(3, -16)$

4. $h(t) = -16t^2 + 620t + 1200$; approximately 40.6 seconds

5.6 Exercises

1. A quadratic equation in x is written in standard form as _______.

2. The ______________ is used to determine the solutions of a quadratic equations.

3. To use the quadratic formula, the quadratic equation needs to be written in ______________.

4. A parabola is a symmetrical U-shaped curve around a ______________ line.

5. The _______ of a parabola is the extreme point that represents the quadratic equations minimum or maximum.

PRACTICE

Use the quadratic formula to solve each equation.

6. $x^2 + 2x - 3 = 0$

7. $x^2 - 6x - 7 = 0$

8. $x^2 = -14x - 49$

9. $2x^2 + 5x - 3 = 0$

10. $2x^2 = 2x + 40$

11. $2x^2 - 3x - 1 = 0$

12. $3x^2 + 2x - 2 = 0$

13. $16x^2 + 8x - 1 = 0$

14. $(x + 5)(x - 1) = -3$

15. $(x + 4)(x - 2) = -4$

APPLICATIONS

16. The function $C(x) = 0.0086x^2 + 1.11x - 1.37$ represents the stopping distance in feet while talking on a cell phone and driving at a speed of x mph. What distance will it take you to stop while talking on a cell phone if you are driving 65 mph? 75 mph?

17. The function $W(x) = 0.02x^2 + 0.5x$ represents the stopping distance in feet on a wet road when driving at a speed of x. What distance will it take you to stop while driving on a wet road if you are driving 65 mph? 35 mph?

Use the fact that a projectile can be modeled by the function $h(t) = -16t^2 + v_0t + y_0$ to solve the following problems.

18. A projectile is fired vertically upward from a height of 200 feet above the ground, with an initial velocity of 1100 ft/sec.

a. Write a quadratic equation to model the projectile's height in feet above the ground after t seconds.

b. During what time interval will the projectile be more than 8000 feet above the ground? Round your answer to the nearest hundredth.

c. What is the total flight time of the projectile? Round your answer to the nearest hundredth.

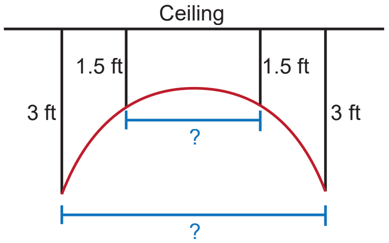
19. A ball is thrown straight up, from ground zero, with an initial velocity of 55 feet per second. Find the maximum height attained by the ball and the time it takes for the ball to return to ground zero.

20. From the top of a 250-foot-tall building, a ball is thrown straight up with an initial velocity of 25 feet per second. Find the maximum height attained by the ball and the time it takes for the ball to hit the ground.

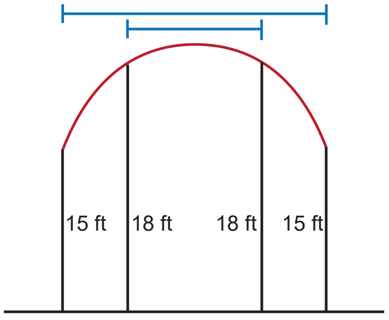
21. A ball is dropped from the top of a 1250-foot-tall building. How long does it take the ball to hit the ground? (Note that $v_0 = 0$.)

22. Grayson drops a rock into a well in which the water surface is 275 feet below ground level. How long does it take the rock to hit the water surface? (Note that $v_0 = 0$.)

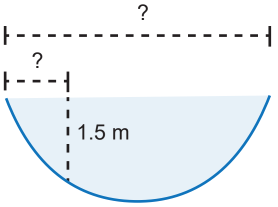
23. A parabola-shaped floodlight is attached to the ceiling with four cables. The outermost support cables are 3 feet long and the inner cables are 1.5 feet long. The shape of the floodlight can be modeled by the quadratic equation $y=-\frac{x^2}{8}-1,$ where x is the horizontal distance from the center of the floodlight and y is the vertical distance from the ceiling to the floodlight. Let the center of the floodlight be placed at 0 on the x-axis. What does the distance between the inner cables and the outer cables need to be? A simplified rendering of the floodlight is shown in the following figure.



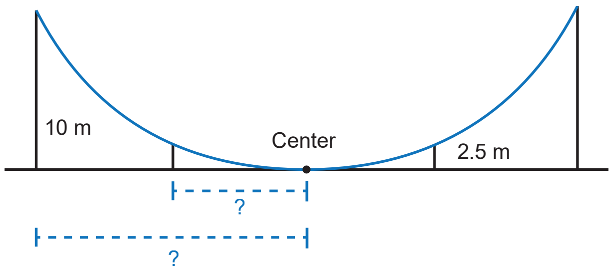
24. A domed roof is mounted on two types of columns. The outermost support columns are 15 feet high, and the interior decorative columns are 18 feet high. A simplified rendering of the construction is shown in the following figure. The shape of the roof can be modeled by the quadratic equation $y=-\frac{(x-10)^2}{25}+19,$ where x is the horizontal distance from the leftmost column and y is the vertical height from the floor to the roof. Let the leftmost column base be placed at the origin. What is the distance between the two inner columns? What is the distance between the two outer columns?



25. The shape of the bottom of a pool can be modeled by the quadratic equation $y=\frac{(x-4)^2}{8}-2,$ where x is the horizontal distance from the leftmost pool edge and y is the depth of the pool. Let the leftmost pool edge be placed at the origin. What is the distance from the edge to the place where the depth is 1.5 meters? What is the length of the pool? A simplified rendering of the pool is shown in the following figure.



26. Due to their reflectivity, parabolas are used in the construction of amphitheaters so that the audience can clearly hear the actors. Suppose the amphitheater is built based on the parabola where x is the horizontal distance from the center of the amphitheater and y is the vertical height from the scene to the seats on the specific row. Let the center of the theater be placed at 0 on the x-axis. What is the distance between the center and the seats at the 2.5-meter height? What is the distance between the center and the seats at the 10-meter height? A simplified rendering of the amphitheater is shown in the following figure.



27. After a golfer hits a ball that is lying on the ground, it flies along the trajectory given by the quadratic equation $y = -\frac{x^2}{4} + \frac{3x}{2} + \frac{7}{4}$, where y is the height of the ball above the ground and x is the horizontal coordinate of the ball, both in meters. Assume the land from the initial location of the ball to the hole is flat.

a. Determine how far the hole must be from the golfer for the ball to land in it.

b. What is the maximum height the ball will reach?

28. A suspension bridge has the shape of a parabola that can be described by the quadratic equation $y = \frac{1}{20}\left(x^2 - 4x - 21\right)$, where x and y are the horizontal and vertical coordinates of the points on the bridge, both in meters, and the bridge connects to its supports at the level of $y = 0$.

a. Determine the distance between the bridge supports.

b. What is the maximum "depth" of the bridge?

29. The cross section of a glass is a parabola described by the quadratic equation $y = x^2 + x - \frac{35}{4}$, where x and y are the horizontal and vertical coordinates of the points on the glass, both in centimeters, and the edge of the glass is on the level of $y = 0$.

a. Determine the diameter of the glass.

b. What is the maximum depth of the glass?

30. An arch in the form of a parabola is installed at the entrance to the city. It can be described by the quadratic equation $y=-\frac{x^2}{4}+\frac{7x}{2}$, where y is the height of the arch and x is the horizontal distance from one of the arch bases, both in meters.

a. Determine the distance between the arch bases.

b. What is the maximum height of the arch?

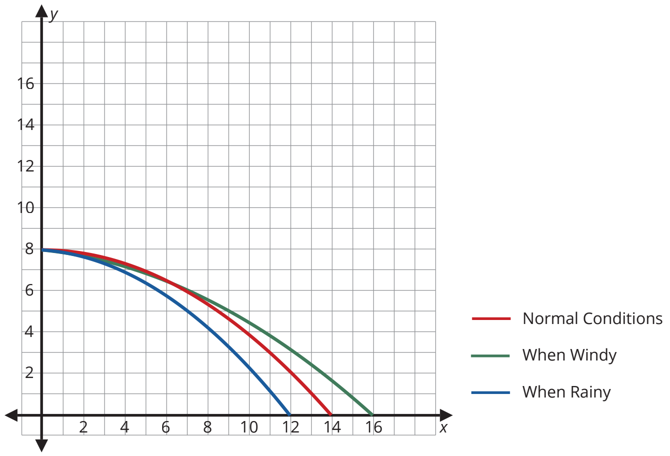
31. The flight paths of a ball thrown from a hill under various conditions are modeled with the following quadratic functions, where x is the time in seconds, and the value of the function is the height, in meters, of the ball above the base of the hill.

Height under Normal Conditions: $f(x)=-\frac{x^2}{40}-\frac{x}{10}+8$

Height When It's Windy: $g(x)=-\frac{2x^2}{49}+8$

Height When It's Rainy: $h(x)=-\frac{x^2}{20}-\frac{x}{15}+8$

Each of the curves are depicted in the following figure.



a. Determine the height of the ball for each condition after 10 seconds.

b. If a person who is 1.7 meters tall stands at the base of the hill, how far above the person will the ball be after 10 seconds?

c. If a person can catch the ball when it is 1.7 meters above the ground, how long will the ball fly before being caughtunder normal conditions and when it's rainy.

d. Use the graphs in the figure to compare the heights of the ball for each of the weather conditions.

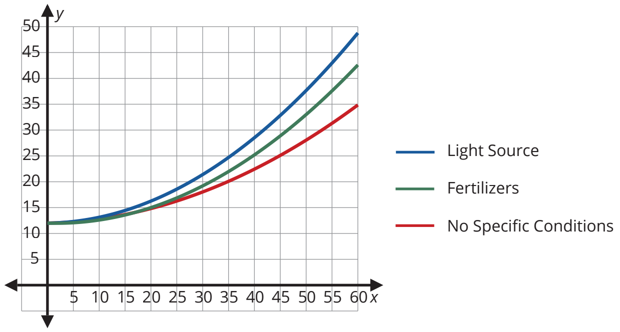
32. A biologist studied the growth rate of geraniums under various conditions for 60 days. His results are modeled with the following quadratic functions, where x is the time in days, and the value of the function is the height of the plant, in centimeters, corresponding to that time.

No Specific Conditions: $f(x) = 0.006x^2 + 0.021x + 12$

With an Additional Light Source: $g(x) = 0.01x^2 + 0.013x + 12$

With Fertilizers: $h(x) = 0.009x^2 - 0.03x + 12$

Each of the curves are depicted in the following figure.



a. Determine the height of the plant for each condition after 30 days.

b. At the beginning of the experiment, all three plants had the same height, 12 centimeters. By how many times did the plants grow from their original size in 30 days?

c. Find the time required for the plant to grow twice its original size when there is an additional light source and when there are fertilizers.

d. Use the graphs in the figure to compare the heights of the plant for each growth condition.

33. You may have noticed that sometimes the water in ponds or aquariums can turn blue-green. This is due to the increased reproduction of cyanobacteria. The reproduction rate of cyanobacteria is described by an equation of the form $N(t) = N_0e^{rt}$, where N_0 is the initial population size (given in millions of cells per liter), r is the growth rate, and t is the time in hours.

Among other factors, the growth rate is dependent on the concentration of phosphates in the water. This dependency is modeled by the linear function $r = 2p$, where p is the concentration of phosphates given in milligrams per liter.

a. Write the exponential function of the population size $N(t)$ if the concentration of phosphates is normal; that is, equals 1 mg/L.

b. Write the exponential function of the population size $N(t)$ if the concentration of phosphates is twice the norm and the initial population size is 3.3×10^6 cells/L.

c. Use the function from part b. to calculate the population size after 2 hours.

d. Use the function from part a. to calculate the time required for the population to double.

34. When you put your money into a savings account, its future value depends, among other things, on the way the interest is calculated. Let's consider two options.

Simple Interest: The future value of an account with a simple interest is calculated by the formula $A = P(1 + rt)$, where P is the initial deposit, r is the annual percentage rate (APR) written as a decimal, and t is the length of the deposit in years.

Continuous Compound Interest: The future value of an account with a continuous compound interest is calculated by $A = Pe^{rt}$,with the variables having the same meanings.

a. Write the linear function for the future value of an account using the simple interest rate if the initial deposit is \$10,000 and the APR is 2.43%.

b. Use the same values for the initial deposit and the APR to write the exponential function for the future value of an account using the continuous compound interest.

c. Compare the future values of the accounts after 5 years using the functions found in parts a. and b.

d. How long does it take for the accounts to earn the first \$1000 of interest? Use the functions obtained in parts a. and b. to determine how long it takes the accounts to earn the first \$1000 of interest.

35. Consider a diver jumping from a platform into the pool. His height above the water can be modeled by the quadratic equation $h(t) = -5t^2 + v_0t + y_0$, where t is the number of seconds after the jump, v_0 is the initial vertical velocity of the diver, and y_0 is the height of the platform in meters. At the same time, the horizontal distance from the diver to the platform is described by the function $d(t) = v_1t$, where v_1 is the initial horizontal velocity.

Assume that the height of the platform is 6 meters, the initial vertical velocity is 3.9 m/s, and the initial horizontal velocity is 2.32 m/s. Answer the following questions.

a. Write the quadratic function for the height of the diver above the water after t seconds.

b. Write the linear function for the horizontal distance the diver will travel after t seconds.

c. How high above the water will the diver be after 0.8 seconds?

d. How long will it take for the diver to hit the water?

e. What is the maximum height above the water the diver can reach?

f. At what time t will the height in part e. be reached?

36. Suppose the revenue of a company is described by the linear function $R(p,Q) = pQ$, where p is the price of an item produced and Q is the number of items sold. The quantity Q is linearly dependent on the price and can be modeled by $Q(p) = -2000p + 134{,}000$. Suppose 15,000 items were sold last month. Answer the following questions.

- **a.** Write the linear function for last month's revenue using the given number of items sold.
- **b.** Write the quadratic function for the revenue, assuming that the quantity is a function of price.
- **c.** What is the revenue if the price of the item is \$23? Use the function obtained in part b.
- **d.** What is the maximum revenue the company can achieve based on the function obtained in part b?
- **e.** What price corresponds to the maximum revenue found in part d?

5.6 PROJECT

THE WEIGHTLESSNESS OF PARABOLIC ARCS

A reduced-gravity aircraft is an aircraft that can simulate weightlessness of its passengers and contents by following a parabolic flight path. While zero gravity (zero-g) is not perfectly attained, the simulation is close enough to zero-g to train astronauts and film movie scenes. This type of aircraft has lovingly been given the nickname "vomit comet" due to two-thirds of all passengers experiencing airsickness during the 40 to 60 parabolic maneuvers of the flight.

According to NASA, the function $f(t) = -4.9t^2 + 87.21t + 9144$ can be used to describe the altitude in meters of a certain reduced-gravity aircraft t seconds after the start of the parabolic maneuver. Reduced gravity occurs during the entire parabolic arc of the maneuver.

1. Determine the altitude when reduced gravity starts and ends.
2. How long does the reduced-gravity period last? Round your answer to the nearest tenth of a second.
3. What is the maximum height attained by the aircraft during the parabolic maneuver? At what time into the parabolic maneuver is this height attained? Round your answer to the nearest tenth.

Suppose an 80-second movie scene takes place in zero-g. The production crew needs to plan the film sequence to minimize the cost of renting a reduced-gravity aircraft.

4. The 80-second scene would need to be split up across multiple periods of weightlessness and then stitched together in editing. What is the minimum number of parabolic arcs the movie crew would need to film the entire scene once?
5. If it takes the aircraft approximately 5 minutes from the end of one parabolic arc to set up to start another parabolic arc, how long would it take to film the 80-second scene one time?