

We now substitute these points into the objective function for profit to find any maximum that may occur.

$$z = 105x + 57$$

$$(10, 15): z = 105(10) + 57(15) = 1905$$

$$(6, 23): z = 105(6) + 57(23) = 1941$$

$$(0, 27.5): z = 105(0) + 57(27.5) = 1567.5$$

$$(16, 0): z = 105(16) + 57(0) = 1680$$

$$(0, 0): z = 105(0) + 57(0) = 0$$

Therefore, the maximum of $z = 1941$ occurs at the point $(6, 23)$. This means that the entrepreneur can realize their maximum profit of \$1941 when they make and sell 6 framed art pieces and 23 sets of placemats.

Skill Check Answers

1. a. 2. $(0, 0)$, $(70, 0)$, $(0, 35)$

5.5 Exercises

✓ CONCEPT CHECK

- Linear programming is a process that optimize certain quantities in a mathematical model constrained by _____.
- The _____ expresses the quantity that is to be optimized.
- Constraints express the _____ of each variable or combination of variables.
- The Fundamental Theorem of Linear Programming states that the maximal and minimal values are found at the _____ of the region.
- The _____ is the set of all points that satisfy the constraints of an optimization problem.

💡 PRACTICE

- Suppose you plan to build a rectangular pool on your property. You want the pool to have a volume of 350 square feet with the depth of 5 feet. To save on materials for the pool walls and pool bottom, you need to minimize the corresponding surface area. Let x represent the length of the pool and y represent the width of the pool. Write the objective function that describes the surface area of the open pool.

7. A construction company took a job for a fee of \$80,000. If the project is finished ahead of schedule, the customer will pay an additional \$2000 for each day saved. There are a limited number of employees in the company who can work on this project, so they are offered overtime. For each overtime day, the employees will receive a bonus of \$200. The company wants to maximize the profit on the project. Let x represent the number of days saved and y represent the number of days of overtime work. Write the objective function that describes the revenue for the project.
8. A florist has 200 roses a day and uses them to make two types of bouquets. The small bouquets consist of 10 roses and sell for \$35. The large bouquets consist of 25 roses and sell for \$80. The florist wants to maximize his revenue. Let x represent the number of small bouquets and y represent the number of large bouquets. Write the objective function that describes the revenue for selling the bouquets.
9. Suppose you need a task to be done as soon as possible. You can hire two types of workers, the full-time ones working 10 hours a day and the part-time ones working 6 hours a day. You have a limited budget for the task, and the full-time workers will need to be paid more per hour than the part-time ones. You want to maximize the number of working hours per day depending on the number of workers of each type. Let x represent the number of full-time workers and y represent the number of part-time workers. Write the objective function that describes the number of working hours per day.
10. Daniel has a job in cargo transportation and wants to maximize his earnings. He needs to transport several 200-pound and 600-pound loads. The carrying capacity of his truck is 45,000 pounds. He is paid \$50 for transporting a lighter load and \$100 for a heavier one. The number of lighter loads should be no more than one-third of the total number transported. Let x represent the number of lighter loads and y represent the number of heavier loads. Write the objective function that describes the amount Daniel earns for transporting the cargo.
11. A factory produces three types of cardboard boxes. The amount of cardboard needed to make each type of box is 6 ft^2 , 4 ft^2 , and 9 ft^2 , respectively. The first type of box sells for \$1.50, the second for \$1.25, and the third for \$2.10. You need to maximize the revenue for selling these types of boxes. Let a represent the number of boxes of the first type, let b represent the number of boxes of the second type, and let c represent the number of boxes of the third type. Write the objective function that describes the revenue.
12. Suppose you need to determine the minimum point of the objective function $z = 380x - 415y$. After graphing the constraints, you find the following three vertices of the feasible region polygon.
 - a. $(100, -45)$
 - b. $(-215, 310)$
 - c. $(0, 70)$

Which of the points minimizes the objective function?

13. Consider the objective function $z = -220x + 185y$. The following three vertices of the feasible region polygon were found after graphing the constraints.

- a. (365, 450)
- b. (-160, 95)
- c. (-5, -270)

Which of the points minimizes the objective function?

14. The following three points are the vertices of the feasible region polygon for the objective function $z = -80x - 325y$.

- a. (130, 90)
- b. (320, 0)
- c. (-170, 120)

Which of the points minimizes the objective function?

15. Laura needs to find the maximum point of the objective function $z = 135x + 295y$. After graphing the constraints, she identifies the following three vertices of the feasible region polygon.

- a. (0, 400)
- b. (400, 0)
- c. (250, 250)

Which of the points maximizes the objective function?

16. Consider the objective function $z = 340x - 230y$. The following three vertices of the feasible region polygon were found after graphing the constraints.

- a. (60, 10)
- b. (100, 100)
- c. (0, -100)

Which of the points maximizes the objective function?

17. The following three points are the vertices of the feasible region polygon for the objective function $z = -55x + 410y$.

- a. (-125, 300)
- b. (0, 405)
- c. (-15, 395)

Which of the points maximizes the objective function?

18. The objective function $z = -5x - 7y$ is subject to the following constraints.

$$y - x \geq 6$$

$$3x + 2y \leq 12$$

$$x \geq 0$$

$$y \geq 0$$

Find the maximum value of z .

19. The objective function $z = -12x + 8y$ is subject to the following constraints.

$$x + y \leq 20$$

$$5x - y \geq 10$$

$$x \geq 0$$

$$y \geq 0$$

Find the maximum value of z .

20. The objective function $z = 3x - 19y$ is subject to the following constraints.

$$x + y \leq 2$$

$$8y - 4x \leq 10$$

$$x \geq 0$$

$$y \geq 0$$

Find the maximum value of z .

21. The objective function $z = 9x + 4y$ is subject to the following constraints.

$$x + y \geq 12$$

$$15y - 3x \leq 18$$

$$x \leq 24$$

$$y \geq 0$$

Find the minimum value of z .

22. The objective function $z = -x - 6y$ is subject to the following constraints.

$$x + y \leq -15$$

$$12x - 8y \geq 20$$

$$y \geq -13$$

Find the minimum value of z .

23. The objective function $z = 13x - y$ is subject to the following constraints.

$$2x + 5y \leq 20$$

$$-x + 5y \geq 5$$

$$x \geq 0$$

$$y \geq 0$$

Find the minimum value of z .

 APPLICATIONS

24. A plane carrying food and water to a resort island can carry a maximum of 30,000 pounds and is limited in space to carrying no more than 575 cubic feet. Each container of food weighs 175 pounds and takes up 5 cubic feet. Each container of water weighs 60 pounds and takes up 1 cubic foot of cargo space. Hotels on the island will buy the food for 14 dollars a pound and the water for 3 dollars per pound. Let x represent the number of food containers and y represent the number of water containers.
- Determine the objective function to represent the revenue generated by the plane.
 - Write the constraint inequality that describes the amount of weight the plane can carry.
 - Write the constraint inequality that describes the amount of cargo the plane can hold.
 - List any other constraints.
 - What is the optimum number of containers of each item that will maximize the revenue generated by the plane?
 - What is the maximum revenue?
25. To prevent pests, an orchard can have no more than 6 times as many apple trees as peach trees. Also, 2 times the number of apple trees plus 3 times the number of peach trees must not exceed 180. The revenue from a single apple tree is \$157 and the revenue from a single peach tree is \$423. Let x represent the number of apple trees and y represent the number of peach trees.
- Determine the objective function to represent the revenue earned.
 - Write the constraint inequality that describes that an orchard can have no more than 6 times as many apple trees as peach trees.
 - Write the constraint inequality that describes that the number of apple trees plus 3 times the number of peach trees must not exceed 190.
 - List any other constraints.
 - Determine the number of each type of tree that will maximize revenue.
 - What is the maximum revenue?

26. On your birthday, your great aunt gave you \$19,000. You would like to invest at least \$4000 of the money in municipal bonds yielding 4% and no more than \$2000 in Treasury bills yielding 6%. How much should be placed in each investment in order to maximize the interest earned in one year? Assume simple interest applies. Let x represent the amount of money in municipal bonds and y represent the amount of money in Treasury bills.
- Write the objective function to represent the interest earned in one year.
 - Write a constraint that describes the money being invested in municipal bonds.
 - Write a constraint that describes the money being invested in Treasury bills.
 - Write any additional constraints.
 - How much should be placed in each investment in order to maximize the interest earned in one year?
27. A volunteer has been asked to drop off some supplies at a facility housing victims of a hurricane evacuation. The volunteer would like to bring at least 132 bottles of water, 82 first aid kits, and 45 security blankets on his visit. The relief organization has a standing agreement with two companies that provide victim packages. Company A can provide packages of 6 water bottles, 4 first aid kits, and 3 security blankets at a cost of \$2.50. Company B can provide packages of 9 water bottles, 5 first aid kits, and 1 security blanket at a cost of \$2.00. Let a represent the number of packages provided by Company A and b represent the number of packages provided by Company B. (**Note:** The companies cannot provide partial packages.)
- Write the objective function to represent the relief organization's cost of supplies.
 - Write the constraint inequality that describes the number of bottles of water.
 - Write the constraint inequality that describes the number of first aid kits.
 - Write the constraint inequality that describes the number of security blankets.
 - List any other constraints.
 - How many of each package should the volunteer pick up to minimize the cost?
 - What amount does the relief organization pay?

28. An outdoor recreation supply company's two best-selling hammocks are made from cotton and nylon rope. The best-selling hammock requires 2.5 yards of cotton and 100 feet of rope. The second best-selling hammock needs 4 yards of cotton and 150 feet of rope. The company has 450 yards of cotton and 3000 feet of rope in stock. The best-selling hammock makes a profit of \$21 for the company and the second best-selling hammock makes a profit of \$33. Let x be the number of best-selling hammocks and y be the number of second best-selling hammocks.
- Write the objective function to represent the profit from the hammocks.
 - Write the constraint inequality that describes the number of yards of cotton.
 - Write the constraint inequality that describes the number of feet of rope.
 - List any other constraints.
 - Based on the stock the company has, how many of each hammock should the company make to maximize their profit?
29. A transport company can purchase at most 9 buses with 30 seats and at most 7 buses with 50 seats. The selling price of a 30-seat bus is \$45,000 and the selling price of a 50-seat bus is \$60,000. The company can allocate \$780,000 for the purchase of the buses. How many buses of each type should the company buy to maximize the total number of bus seats available?
30. To produce tables and cabinets, a furniture factory uses two types of wood: oak and pine. A minimum of 6 cubic feet of oak and 5 cubic feet of pine are required to produce one table. A minimum of 4 cubic feet of oak and 10 cubic feet of pine are required to produce one cabinet. Due to limited stock, the factory can use no more than 1400 ft³ of oak and 2300 ft³ of pine. Determine how many tables and cabinets the factory should produce to maximize the revenue if a table sells for \$1300 and a cabinet sells for \$1900.
31. For optimum prenatal care, an ob-gyn doctor recommends that expecting mothers taking a combination of supplements which included folic acid, calcium, and iron. The following table shows the recommended daily dosage for each essential nutrient along with two possible brands of multi-vitamins that contain various amounts of each. Each Baby Bliss Prenatal pill costs \$0.40 and each Natural Gift Prenatal pill costs \$0.34. How many pills of each brand should an expecting mother take each day to meet the recommended dosage and minimize the cost of supplements? Let x be the number of Baby Bliss Prenatal pills and y be the number of Natural Gift Prenatal pills.

	Recommended daily dosage	Baby Bliss Prenatal (\$0.40)	Natural Gift Prenatal (\$0.34)
Folic Acid	0.4 mg	0.1 mg	0.1 mg
Calcium	1000 mg	200 mg	400 mg
Iron	27 mg	9 mg	6 mg