Skill Check 5.3.3

Suppose you are planning to manufacture a new board game that requires an initial cost of $5000 and an additional $11 for each game produced. You plan to then sell each board game for $28. Write the cost function and the revenue function for this venture. Then, find the break-even point for your board game business.

$$P(x) = 16x - 18,000$$
$$P(x) = 16(2500) - 18,000$$
$$P(x) = 22,000$$

Therefore, Harley will make a profit of $22,000 when she sells 2500 scanners.

The break-even point might not always be an integer. Note that for many businesses, it is only possible to sell in whole units, and not in partial units. This means that if the break-even point is a fraction, you should always round up to the next whole number so that the company is actually covering *all* costs and not making a loss of even a small fraction.

Skill Check Answers

1. $x = -\frac{19}{2}$ **2.** (2, 4) is not the point of intersection **3.** $C(x) = 5,000 + 11x$; $R(x) = 28x$; The break-even point is approximately 294.12, which rounds up to 295.

5.3 Exercises

CONCEPT CHECK

1. When drawing a system of linear equations, one of three things will occur:

a. The lines will _____.

b. The lines will _____.

c. The lines will _____.

2. The first step in solving a system of linear equations is to _____ one of the equations for one of the _____.

3. The break-even point is the amount of units that need to be produced and sold so that the _____ by the business is equal to the amount of _____.

4. The _____ function is equal to the price per unit multiplied by the number of units sold.

5. The _____ function is equal to the original fixed cost plus the production cost for each unit multiplied by the number of units.

PRACTICE

Determine if the given point is the point of intersection for the pair of lines.

6. $(3,-8)$; $\begin{cases} y = -5x + 7 \\ 3x + y = 1 \end{cases}$

7. $(6,-3)$; $\begin{cases} 0.5x - 2y = 9 \\ x + 3y = 13 \end{cases}$

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8. $(-4,-5)$; $\begin{cases} 3x+4y=-32 \\ 2x+y=-13 \end{cases}$

9. $\left(\frac{1}{4},2\right)$; $\begin{cases} -8x=-2y \\ 6x-3y+6=0 \end{cases}$

Solve the system of equations.

10. $\begin{cases} 4x+3y=1 \\ 2x-2y=4 \end{cases}$

11. $\begin{cases} 2x+4y=8 \\ 6x-8y=4 \end{cases}$

12. $\begin{cases} y=3x-1 \\ y=3x+2 \end{cases}$

13. $\begin{cases} -\frac{2}{3}x+\frac{1}{2}y=\frac{4}{7} \\ \frac{2}{12}x-\frac{1}{8}y=-\frac{1}{7} \end{cases}$

14. $\begin{cases} \frac{3}{4}x+3y=\frac{4}{3} \\ -\frac{1}{4}x-y=\frac{7}{3} \end{cases}$

15. $\begin{cases} -4x+y=1 \\ 4x+4y=-16 \end{cases}$

16. $\begin{cases} 6x+y=30 \\ 6x+y=-19 \end{cases}$

17. $\begin{cases} x+y=-1 \\ -3x+y=-1 \end{cases}$

18. $\begin{cases} 9x+3y=15 \\ 3x+y=5 \end{cases}$

19. $\begin{cases} y=4x+12 \\ 2y=8x+24 \end{cases}$

20. $\begin{cases} -5x+y=-12 \\ 5y=10x-15 \end{cases}$

21. $\begin{cases} 6x-y=11 \\ -6y=-8x+10 \end{cases}$

22. The profit function for a company selling x teapots is given by $P(x)=2.4x-3230.50$. Find the company's profit if 17,000 teapots are sold.

23. If a company sells 3000 units of their product, what profit can they expect to make if their profit function is given by $P(x)=0.54x-841.75$?

APPLICATIONS

24. Twelve gallons of a salt solution consists of 30% salt. This solution was created by mixing a 40% solution with a 25% solution. How many gallons of each of the solutions was used to create the 30% solution? Let x = the number of gallons of the 40% solution and y = the number of gallons of the 25% solution. The corresponding model is the following system of equations.

$$\begin{cases} x+y=12 \\ 0.4x+0.25y=0.3(12) \end{cases}$$

25. A student bought a calculator and a textbook for an algebra course. He told his friend that the total cost was \$170 (without tax) and that the calculator cost \$20 more than four times the cost of the textbook. What was the cost of each item? Let x = the cost of a calculator and y = the cost of the textbook. The corresponding modeling system is $\begin{cases} x+y=170 \\ x=4y+20 \end{cases}$. Solve the system by using the method of substitution.

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26. On a round-trip to the south of the state, the pilot of a small personal plane recorded the average speed of the plane at 159 mph when flying with a tailwind. On the return trip, the plane averaged 133 mph flying back into the same wind. What was the speed of the plane in still air and the speed of the wind? Let x = the speed of the plane, and y = the speed of the wind. The corresponding model is the following system of equations. Solve the system to find the speed of the plane and the wind.

$$\begin{cases} x + y = 159 \\ x - y = 133 \end{cases}$$

27. The theater department brought in \$3072 from the sale of 723 tickets to the spring play. Unfortunately, the record of how many types of tickets were sold was lost. The department would like to know how many student tickets were bought for \$2 and how many nonstudent tickets were bought at \$5. Let x = the number of non-student tickets and y = the number of student tickets. The corresponding model is the following system of equations. Determine how many of each type of ticket was sold.

$$\begin{cases} 5x + 2y = 3072 \\ x + y = 723 \end{cases}$$

28. Ten liters of an acid solution contains 42% sulfuric acid. This mixture was obtained by combining a 30% solution with a 50% solution. How many liters of each solution was used? Let x = the number of liters of the 30% solution and y = the number of liters of the 50% solution. The corresponding model is the following system of equations. Solve the system to find the number liters used for each type of solution.

$$\begin{cases} 0.3x + 0.5y = 0.42(10) \\ x + y = 10 \end{cases}$$

29. Suppose that \$10,000 was invested in two accounts. In one year, the 7.5% mutual fund account earned \$585.45 more interest than the 3.47% CD. How much was originally invested in each account? Let x = the amount deposited in the mutual fund and y = the amount deposited in the CD. The corresponding model is the following system of equations. Determine the amount deposited in each account.

$$\begin{cases} x + y = 10,000 \\ 0.075x = 585.45 + 0.0347y \end{cases}$$

30. A farmer plans to sell harvested tomatoes at a nearby market. The cost of growing tomatoes is \$165 plus \$0.50 for each pound of the product. The farmer plans to sell the tomatoes at \$2 per pound. He wants to know how many pounds he'll need to sell in order to recoup the costs of growing tomatoes.

Write two linear functions, one for the cost of growing the tomatoes, $C(x)$, and one for the amount of money the farmer will earn, $R(x)$. Use the functions to find out how many pounds of tomatoes need to be sold in order for the farmer to not lose any money.

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31. In some countries, there is a limit on water consumption per person per day. If this rate is exceeded, water supply is paid at increased rates. Suppose that a family spends 270 liters of water per day for general household needs, and each member of the family also spends 50 liters of water individually. The water consumption rate per person in this region is 140 liters per day, and this family does not exceed the allowed rate.

Write two linear functions, one for the amount of water the family spends, $C(x)$, and one for the amount of water specified by the consumption rate, $R(x)$. Use the functions to find the maximum number of people in this family given that they do not overpay for the water supply services.

32. You are investing in a new outdoor game similar to a corn-toss game, but for golf. The start-up cost includes an initial lump sum of $9200. The production cost for each game is $17.43 and they will be sold for $84.99. Let x represent the number of games produced.

a. Write the cost function, C.

b. Write the revenue function, R.

c. Write the profit function, P.

d. Determine the break-even point and describe what this point means.

33. A shoe company that manufactures waterproof running shoes has a fixed cost of $100,900. Each pair of shoes sells for $37.50 wholesale and costs $27.60 to produce, $1.00 to ship, $0.30 for shipping insurance, and $3.00 in custom taxes. Let x = the number of pairs of running shoes produced.

a. Write the cost function, C.

b. Write the revenue function, R.

c. Write the profit function, P.

d. Determine the break-even point and describe what this point means.

34. A technology company is developing a new data scanner for use in hospitals. Overhead costs for manufacturing the data scanners requires a fixed cost of $328,600. Each scanner will sell for $10,000 and cost $4700 to produce. Let x represent the number of scanners produced.

a. Write the cost function, C.

b. Write the revenue function, R.

c. Write the profit function, P.

d. Determine the break-even point and describe what this point means.

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35. A ride share company is recruiting drivers. They advertise that each driver will earn on average, \$18.26 per hour. The company charges each driver a fee of 25% of each fare. Before deciding if you want to become a driver, you calculate the hidden costs for the car you already own. Insurance, gas, maintenance, and depreciation will cost you \$8.16 per hour. Let x = the number of hours you drive for the ride share company. Answer the following questions based on the average earnings per hour as advertised by the company.

a. Determine the cost function if you decide to drive for the company.

b. Determine your revenue function.

c. Determine your profit function.

d. Suppose you drive 5 hours a day for 6 days the first week and 6.5 hours a day for 4 days the second week. How much did you make your first two weeks as a driver?

e. Some drivers say that the average profit that they make per hour is actually higher than you calculated because they would be paying insurance, maintenance and have depreciation on their car anyway. Do you agree? Explain your reaction.

36. Maggie decides to become a make-up artist. To do this, she is taking a course for \$3300 and purchases the necessary materials for \$1500. She also estimates having to spend \$20 per client on supplies. Maggie expects to receive an average of \$80 per client. Let x be the number of clients.

a. Write the cost function for Maggie's business.

b. Write the revenue function.

c. Write the profit function for the business.

d. Determine the x-value of the break-even point.

e. If Maggie serves 100 clients, what profit can she expect to make?

37. A company sells its product for \$40 each. Variable costs are \$12 per item, and the total fixed costs are \$56,000. Let x be the number of products sold.

a. Write the cost function for the company.

b. Write the revenue function.

c. Write the profit function for the company.

d. Determine the x-value of the break-even point for the company.

e. If the company sells 3000 items, what profit can it expect to make?

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