Solution

The statement we are given is a conditional statement, $a \Rightarrow b$, where a is the statement "I save enough money" and b is the statement "I will go on a vacation." Now we can write each of the variations.

Converse ($b \Rightarrow a$): If I go on vacation, then I saved enough money.

Inverse ($\sim a \Rightarrow \sim b$): If I do not save enough money, then I will not go on vacation.

Contrapositive ($\sim b \Rightarrow \sim a$): If I do not go on vacation, then I did not save enough money.

The contrapositive is logically equivalent to the original statement.

✔ Skill Check 3.3.3

Write the converse, inverse, and contrapositive of the following statement.

If I cannot find my phone, then it is in the car.

Skill Check Answers

1. Truth Table for a and $a \vee a$

a	a	$a \vee a$
T	T	T
F	F	F

2. The weather gets worse and we will not leave. **3.** Converse: If my phone is in the car, then I cannot find it. Inverse: If I can find my phone, then it is not in the car. Contrapositive: If my phone is not in the car, then I can find it.

3.3 Exercises

✔ CONCEPT CHECK

1. _____ statements have exactly the same truth values in all corresponding circumstances.

2. De Morgan's Laws define the negation of two _____.

3. The negation of a conditional statement is a _____.

4. True or False: The statement "if the dog is a good boy, then he gets a treat" is logically equivalent to "if the dog did not get a treat, then he is not a good boy."

5. True or False: The statement "if the calculator gives me the wrong answer, then it is broken" is logically equivalent to "if the calculator is broken, then it gives me the wrong answer."

PRACTICE

Complete each truth table for the given tautology and then decide which two statements within the compound statement are logically equivalent, if applicable.

6. $(p \Rightarrow \sim q) \Leftrightarrow \sim(p \wedge q)$

Truth Table

p	q	$\sim q$	$p \Rightarrow \sim q$	$p \wedge q$	$\sim(p \wedge q)$	$(p \Rightarrow \sim q) \Leftrightarrow \sim(p \wedge q)$
T	T					
T	F					
F	T					
F	F					

7. $\sim(p \Rightarrow q) \Rightarrow (p \vee q)$

Truth Table

p	q	$p \Rightarrow q$	$\sim(p \Rightarrow q)$	$p \vee q$	$\sim(p \Rightarrow q) \Rightarrow p \vee q$
T	T				
T	F				
F	T				
F	F				

8. $(\sim p \wedge \sim q) \Leftrightarrow \sim(p \vee q)$

Truth Table

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \vee q$	$\sim(p \vee q)$	$(\sim p \wedge \sim q) \Leftrightarrow \sim(p \vee q)$
T	T						
T	F						
F	T						
F	F						

9. $[p \wedge (\sim p \vee q)] \Leftrightarrow (p \wedge q)$

Truth Table

p	q	$\sim p$	$\sim p \vee q$	$p \wedge (\sim p \vee q)$	$p \wedge q$	$[p \wedge (\sim p \vee q)] \Leftrightarrow (p \wedge q)$
T	T					
T	F					
F	T					
F	F					

Show that each pair of statements is logically equivalent or explain why they are not.

10. **a.** $a \Rightarrow b$ **b.** $b \Rightarrow a$

11. **a.** $n \Rightarrow m$ **b.** $m \vee \sim n$

12. **a.** $p \wedge \sim q$ **b.** $\sim(\sim p \vee q)$

13. **a.** $r \wedge \sim s$ **b.** $\sim(r \Rightarrow s)$

14. **a.** $w \Rightarrow z$ **b.** $\sim w \Rightarrow \sim z$

15. **a.** $p \vee q$ **b.** $p \vee (q \wedge \sim p)$

Write the negation of the given compound statement using De Morgan's laws.

16. The band will go on tour, or the band will release a new album and film a new music video.

17. The dog will wear a bow tie and will not wear a hat, and the dog will have his picture taken.

De Morgan's Laws state that $\sim(p \wedge q) \equiv \sim p \vee \sim q$ and $\sim(p \vee q) \equiv \sim p \wedge \sim q$. Construct a truth table to show that each pair of compound statements is logically equivalent.

18. $\sim(p \wedge q)$ and $\sim p \vee \sim q$ **19.** $\sim(p \vee q)$ and $\sim p \wedge \sim q$

Use De Morgan's Laws to write an equivalent statement without using parentheses.

20. $\sim(\sim p \wedge q)$ **21.** $\sim(\sim p \vee \sim q)$

Use De Morgan's Laws to write an equivalent statement.

22. $\sim p \wedge q$ **23.** $p \vee q$

Negate each conditional statement.

24. $\sim p \Rightarrow a$ **25.** $a \Rightarrow (c \vee d)$

26. $(c \wedge d) \Rightarrow b$ **27.** $(w \vee z) \Rightarrow (w \wedge z)$

28. If Brooke comes to my room before 2:00 p.m. then I do not finish my homework before 10:00 p.m.

29. My computer goes to sleep whenever I leave it alone for 15 minutes.

30. Having a ticket from the Sunday paper is sufficient to get a free ice cream at Dairy Dip.

31. Being in Charleston, South Carolina, implies that I am on Eastern Standard Time.

32. Consider the conditional statement "If the weather is cold enough, we will go ice skating." Determine if the following conditional statements are logically equivalent to the given conditional statement.

a: If we go ice skating, then the weather is cold enough.

b: If we do not go ice skating, then the weather is not cold enough.

c: We will go ice skating if and only if the weather is cold enough.

33. Consider the conditional statement "If the puppy is not asleep, then the puppy is causing mischief." Determine if the following conditional statements are logically equivalent to the given conditional statement.

a: If the puppy is not causing mischief, then the puppy is asleep

b: If the puppy is asleep, the puppy is not causing mischief.

c: The puppy is asleep or the puppy is causing mischief.

Use De Morgan's Laws to write an equivalent statement.

34. There is a space available in the 8:00 a.m. Biology class or I am not able to make the perfect schedule for next semester.

35. In Biology the nucleotide bases adenine and thymine pair together in DNA and the bases do not pair together in RNA.

Write the converse, inverse, contrapositive, and biconditional for each conditional statement.

36. If I go to the movies, then I will get popcorn.

37. Getting an 89 on the final is sufficient for me to get an A in biology.

38. It is dark whenever I get out of my last class.

39. Seeing a puppy will lead to me smiling.

Find each conditional variation.

40. Find the inverse of $p \Rightarrow q$.

41. Find the converse of $\sim q \Rightarrow p$.

42. Find the contrapositive of $\sim p \Rightarrow q$.

43. Find the inverse of $\sim q \Rightarrow \sim p$.