Dr. Williams could put together is the number of subsets of the 16 items on the buffet. We can use the formula for the number of subsets to determine how many different meals he could make. Using the formula for the number of subsets, we have the following.

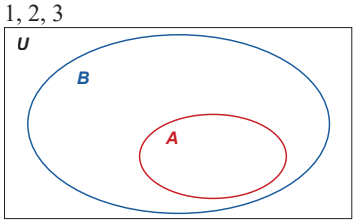
$$\text{Number of Plates} = 2^{16} = 65{,}536.$$

This means that if Dr. Williams ate at the buffet once a day every day, he could eat for almost 180 years without duplicating a meal. Certainly, it would seem the sign is not misleading.

Skill Check Answers

1. 1, 2, 3

2.



3. $\varnothing, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$

2.2 Exercises

✔ CONCEPT CHECK

1. A _____ is a visualization of the relationships that exist between sets.

2. If set A is a subset of set B, that means every element of set ___ is also an element of set ___.

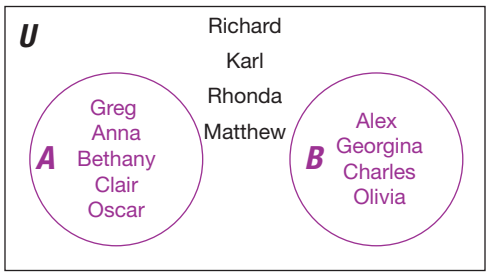
3. In a proper subset, there is at least one _____ of the set that is not in the subset.

4. True or False: The empty set is a subset of every set.

5. True or False: If set X is a subset of set Y and set X is also equivalent to set Y, then set X is not a proper subset of set Y.

PRACTICE

Use the Venn diagram to solve each problem.

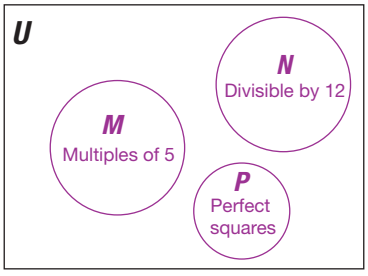


6. List A and B using roster notation.

7. Find A'.

8. List U using roster notation.

The following Venn diagram represents the sets M, N, and P within the universal set $U = \{x \mid x \text{ is a positive integer and } x \leq 25\}$ The sets are defined as $M = \{x \mid x \text{ is a multiple of 5}\}$, $N = \{x \mid x \text{ is divisible by 12}\}$, and $P = \{x \mid x \text{ is a perfect squuare}\}$. Use the diagram to answer the following questions.



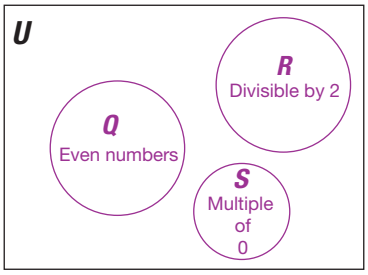
9. List the elements of sets M, N, and P in roster notation.

10. Find $|M|$, $|N|$, and $|P|$.

11. Find M'.

12. Is $M = P$? Is $M \sim P$? Explain your answers.

The following Venn diagram represents the sets Q, R, and S within the universal set $U = \{x \mid x \text{ is a positive integer and } x < 20\}$. The sets are defined as $Q = \{x \mid x \text{ is an even number}\}$, $R = \{x \mid x \text{ is divisible by 2}\}$, and $S = \{x \mid x \text{ is a multiple of 0}\}$. Use the diagram to answer the following questions.



13. List the elements of sets Q, R, and S in roster notation.

14. Find $|Q|$, $|R|$, and $|S|$.

15. Find R'.

16. Is $Q = R$? Is $Q \sim R$? Explain your answers.

17. Let $A = \{0, 1, 2, 3, 4, 5\}$ and $B = \{5, 4, 3, 2, 1\}$. Is $B \subseteq A$?

18. Let $A = \{\{a, b\}, c, d, e\}$ and $B = \{a, b, c\}$. Is $B \subseteq A$?

19. The set B contains the names of three of the most expensive paintings ever sold. If $B = \{$*The Card Players* by Cézanne, *No. 5 1948* by Pollock, *Woman III* by de Kooning$\}$, list all the subsets of B.

20. Given $A = \{x \mid x \in$ positive whole numbers less than $100\}$ and $B = \{x \mid x \in$ even non-negative integers less than $100\}$, is either set a subset of the other? Explain.

List all subsets of the given set.

21. $L = \{\text{lemon, lime}\}$

22. $T = \{1, 2, 3\}$

23. $Q = \{\text{four, vier, cuatro, quatre}\}$

24. $W = \{\text{one}\}$

Draw each Venn diagram.

25. Let U consist of all artists. Draw a Venn diagram to represent the two sets violinist and musicians.

26. Let U consist of all four legged animals. Draw a Venn diagram to represent the two sets male dogs and female dogs.

27. Draw a separate Venn diagram to illustrate each of the following.

a. $W \subseteq Y$

b. $Y \subseteq W$

28. Let $U = \{x \mid x \in \mathbb{R}\}$, $W = \{x \mid x \text{ is a counting number less than } 20\}$, and $Y = \{2, 4, 6, 8, 10\}$. Draw a Venn diagram to represent U, W, and Y with the elements in the proper regions.

29. Let $U = \{\text{Red, Orange, Yellow, Green, Blue, Indigo, Violet}\}$, $A = \{\text{Green, Orange, Yellow}\}$, and $B = \{\text{Indigo, Violet}\}$. Draw a Venn diagram to represent U, A, and B with the elements in the proper regions.

Solve.

30. Let $P = \{\text{baklava, cannoli, eclair, polvorones, strudel}\}$. Determine if the following sets are proper subsets of P.

a. $A = \{\text{baklava, cannoli, eclair}\}$

b. $B = \{\ \}$

c. $C = \{\text{baklava, strudel, cannoli, eclair, polvorones}\}$

d. $D = \{\text{polvorones, pie, paxlava}\}$

31. Let $M = \{\text{c, a, l, i, p, e, r}\}$. Determine if the following sets are proper subsets of M.

a. $A = \{\text{r, e, p, l, i, c, a}\}$

b. $B = \{\text{r, e, c, a, p}\}$

c. $C = \{\text{l, e, a, p}\}$

d. $D = \{\text{r, a, i, s, e}\}$

List all proper subsets of the given set.

32. The set C contains the names of the top three grossing films of all time as of Summer 2014, $C = \{\textit{Avatar}, \textit{Titanic}, \textit{Marvel's The Avengers}\}$.

33. $F = \{2, 4, 6\}$

34. $P = \{\text{pen, pencil}\}$

35. $W = \{\text{cloudy, rainy, sunny}\}$

36. $T = \{10, 20, 30, 40\}$

37. $L = \{0\}$

Solve.

38. W contains The New York Times' top five fiction books for 2013. If $W = \{\textit{The Goldfinch}, \textit{Americanah}, \textit{The Flamethrowers}, \textit{Life After Life}, \textit{Tenth of December}\}$, how many subsets does W contain? How many proper subsets?

39. Given that $B = \{$ ⚡, ✠, ♡, ⏲, ◎, ☎, 🚲, ☺ $\}$. How many subsets does B have? How many proper subsets does B have?

40. Determine the number of subsets contained in Y if $Y = \{x \mid x \text{ is an odd positive integer and } x < 25\}$.

41. A set has 32 subsets. How many elements are in the set?

42. A set has 127 proper subsets. How many elements are in the set?

43. The national fast food chain Wendy's advertises that there are 256 ways to personalize a Wendy's hamburger. How many condiments must Wendy's carry for their customer to have this many choices?

APPLICATIONS

Every year, thousands of deaths happen as the result of a drunk driver causing a car crash. The number of deaths varies state to state, with higher numbers occurring in states with higher populations. In 2019, the five states with the highest number of deaths due to drunk drivers were California, Florida, Georgia, North Carolina, and Texas. Let

$U = \{x \mid x \in$ death as the result of drunk driving in the United States$\}$

$C = \{x \mid x \in$ death as the result of drunk driving in California$\}$

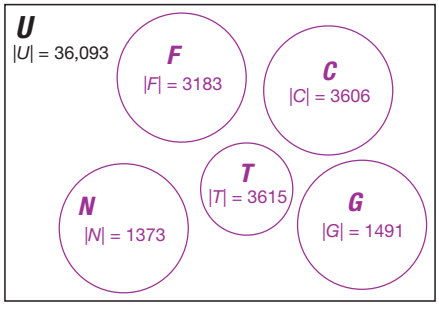
$F = \{x \mid x \in$ death as the result of drunk driving in Florida$\}$

$G = \{x \mid x \in$ death as the result of drunk driving in Georgia$\}$

$N = \{x \mid x \in$ death as the result of drunk driving in North Carolina$\}$

$T = \{x \mid x \in$ death as the result of drunk driving in Texas$\}$

The following Venn diagram depicts the five states with the most deaths that occur as the result of drunk driving in 2019 within the universal set of all deaths that occurred as a result of drunk driving in the United States. The cardinal number for each set is shown inside the appropriate circle.



44. Which state had the most deaths caused by drunk driving in 2019?

45. Which state had the second highest number of deaths caused by drunk driving in 2019?

46. How many total deaths caused by drunk driving happened in these five states in 2019?

47. How many deaths caused by drunk driving happened in the United States in 2019 but did not happen in one of these five states?

WRITING & THINKING

48. Explain why Red $\not\subset$ {Red, Blue, Green}.

49. A new keyless car security system allows the owner to create a "handprint" to start the car by selecting up to five places on the six-space keypad on which to place their fingers. How many possible handprints can the user choose from if at least one space must be selected?

50. Pizza Jet promotes a special lifetime offer—once you order all possible pizza combinations from Pizza Jet, you can get free pizza for life! Pizza Jet offers a selection of 12 toppings (including cheese) for their pizzas. How many different pizzas must you order from Pizza Jet to qualify for their lifetime offer? Is this possible? Explain.

2.2 PROJECT

ONES, ZEROS, AND THE NUMBER OF REGIONS IN A VENN DIAGRAM

A computer chip is basically a collection of transistors, which are electronic components that work as switches. A transistor can be in one of two states: on or off. In computer engineering, the number 1 is used to represent the *on* state while 0 is used to represent the *off* state. Numbers composed only of zeros and ones are called binary numbers.

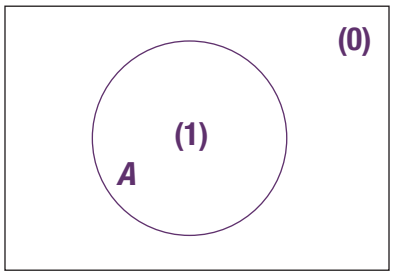


Figure 1: One-Set Venn Diagram

In this activity, you will investigate the relationship between binary numbers and the regions of a Venn diagram. Consider the one-set Venn diagram in Figure 1. We have labeled each region of the diagram by asking the question, "Does this region contain elements of set A?" If the answer is yes, we labeled the region with (1); if the answer is no, we labeled the region with (0). Table 1 shows a summary of the regions.

Table 1: Regions in Figure 1

Elements of Set *A*?	Region
Yes	(1)
No	(0)

As you can see, we have exactly 2 regions: (1) and (0). We are labeling the Venn diagram using single digit binary numbers. Let's use the same idea to label a two-set Venn diagram. (Note that some regions in the figures may end up containing no elements when actual sets are considered, but the diagrams take into account all possible regions when considering the relationship between a fixed number of sets.) The region in Figure 2 that contains no elements of set A but some of the elements of set B is labeled (10), and the region that contains no elements of set