

**Solution**

Since  $A$  consists of American adults who have at least one tattoo, the complement of  $A$  consists of American adults who do *not* have at least one tattoo. We know that *at least one* means greater than or equal to one, so *does not have at least one* implies has less than one or simply no tattoos at all. We can write the complement of  $A$  as follows.

$$A' = \{x \mid x \in U \text{ and } x \text{ has no tattoos}\}$$

This could also be written as  $A' = \{x \mid x \text{ is in } U \text{ and } x \text{ has no tattoos}\}$ . Note that based on the poll data,  $A'$  represents approximately 70% of American adults.

$B$  is the set of American adults with exactly two tattoos. The complement of  $B$  includes all of the elements in our universal set that are not contained in  $B$ . This translates to American adults that do *not* have exactly two tattoos. The complement of  $B$  therefore contains adults with 0 tattoos, 1 tattoo, or 3 or more tattoos. We can write the complement of  $B$  in either of the following ways.

$$B' = \{x \mid x \text{ is an American adult who does not have exactly two tattoos}\}$$

Or

$$B' = \{x \mid x \in U \text{ and } x \text{ has either 0 tattoos, 1 tattoo, or 3 or more tattoos}\}$$

**✓ Skill Check 2.1.4**

Let

$$U = \{x \mid x \text{ is a book published in the US in 2021}\}$$

$$A = \{x \mid x \in U \text{ and you read } x \text{ as an e-book}\}.$$

1. Find  $A'$ .
2. Is it possible for you to have read a book in  $A'$ ?

**Skill Check Answers**

1.  $|W| = 4$ ,  $|Y| = 5$ ;  $W$  and  $Y$  are neither equal nor equivalent.
2. Roster notation:  $J = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ ; Set-builder notation: possible answers include  $J = \{x \mid x \text{ is a natural number less than } 10\}$  and  $J = \{x \mid x \in \mathbb{N}, x < 10\}$
3. Yes,  $Y = \emptyset$ .
4.  $A' = \{x \mid x \in U \text{ and you did not read } x \text{ as an e-book}\}$ ; Yes, but not as an e-book; perhaps as a printed or audio book.

## 2.1 Exercises

**✓ CONCEPT CHECK**

Determine whether each statement is true or false. If the statement is false, explain why.

1. It is always possible to list every element of a set using roster notation.
2.  $\emptyset \sim \{0\}$
3. Let  $U = \{\text{set of all students enrolled at Xavier University}\}$  and  $A = \{x \mid x \in U \text{ and } x \text{ is a student with less than 30 earned credit hours}\}$ . Then  $A' = \{x \mid x \in U \text{ and } x \text{ is a student with at least 30 earned credit hours}\}$ .
4.  $0 \in \emptyset$

5.  $x \in \{x, y, z\}$
6.  $\{x\} \in \{x, y, z\}$
7.  $|\emptyset| = 0$
8. Let  $Y = \{\text{Tim, Emilia, Aleesa, Whit}\}$ , then  $|Y| = 4$ .
9. Let  $A = \{\text{Sounds, Angels, Ravens, Titans}\}$  and  $B = \{201, 38, 46, 23\}$ , then  $A \sim B$ .

 **PRACTICE**

Write each set using roster notation.

10.  $A$  is the set of months of the year that have exactly 30 days.
11.  $B$  is the set of states whose names begin with the letter N.
12.  $C$  is the set of positive numbers smaller than 100 that have 2 digits that are the same.
13.  $D$  is the set of last names of people who teach this course at your university.
14.  $E$  is the set of planets in our solar system.
15.  $F$  is the set of weekdays.
16.  $A = \{x \mid x \text{ is an odd number and } 20 < x < 30\}$
17.  $B = \{x \mid x \leq 15 \text{ and } x \text{ is a positive multiple of } 3\}$
18.  $C = \{x \mid 2x = 4\}$
19.  $D = \{x \mid x \text{ is a state that shares a common border with Colorado}\}$

Determine the cardinal number of each set.

20.  $W = \{3, 4, 5, 6, 7, 8, 9, 10, 0\}$
21.  $X = \{x \mid x \text{ is a positive integer and } x \leq 10\}$
22. The empty set
23.  $Y = \{x \mid x \text{ is a United States president, past or present}\}$

Determine if the given pairs of sets are equal, equivalent, or neither.

24.  $A = \{1, 2, 3, 5, 7, 11, 13, 17, 19\}$

$B = \{2, 4, 6, 10, 14, 22, 26, 34, 38\}$

25.  $C = \{\text{banana, tomato, grape, eggplant, watermelon, pumpkin}\}$

$D = \{\text{blackberry, raspberry, strawberry, sloeberry}\}$

26.  $E = \{\text{c, o, n, s, t, r, u, e}\}$

$F = \{\text{r, e, c, o, u, n, t, s}\}$

27.  $G = \{\text{r, e, s, c, u, e, d}\}$

$H = \{\text{s, e, c, u, r, e}\}$

Write each set using set-builder notation.

28. Let  $G$  be the set of whole numbers less than five.

29. Let  $H$  be the set of natural numbers less than or equal to 50.

30. Let  $U$  be the set of all the states in the United States of America and  $J$  be the set of all states that border an ocean.

31. Let  $U$  be the set of all students enrolled at a public school of higher education and  $K$  be the set of all collegiate athletes.

32.  $M = \{2, 3, 4, 5, 6, 7\}$

33.  $Q = \{3, 6, 9, 12, 15, 18, 21, \dots\}$

34.  $U = \{-10, -8, -6, -4, -2\}$

35.  $P = \{1, 2, 4, 9, 16, 25, 36, \dots\}$

Determine whether each set is the empty set.

36.  $W = \{x \mid x \text{ is a positive integer less than } 0\}$

37. The set  $X$  is the set of months that have fewer than 30 days.

38.  $Y = \{\emptyset\}$

39.  $Z = \{x \mid x \text{ is divisible by } 10\}$

Use the set  $A = \{\text{b, a, s, k, e, t}\}$  to solve each problem.

40. Find  $|A|$ .

41. If  $U = \{\text{a, b, c, d, } \dots, \text{x, y, z}\}$ , find  $A'$ .

42. If  $U = \{\text{a, b, c, d, } \dots, \text{x, y, z}\}$ , find  $|A'|$ .

43. If  $U = \{\text{a, b, c, d, } \dots, \text{x, y, z, A, B, C, D, } \dots, \text{X, Y, Z}\}$ , find  $A'$ .

44. If  $U = \{a, b, c, d, \dots, x, y, z, A, B, C, D, \dots, X, Y, Z\}$ , find  $|A'|$ .

Use the set  $B = \{1, 2, 3, 4\}$  to solve each problem

45. Find  $|B|$ .

46. If  $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ , find  $B'$ .

47. If  $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ , find  $|B'|$ .

48. If  $U = \{-9, -8, -7, -6, \dots, 6, 7, 8, 9\}$ , find  $B'$ .

49. If  $U = \{-9, -8, -7, -6, \dots, 6, 7, 8, 9\}$ , find  $|B'|$ .

Use the given sets to answer each question.

$P = \{\text{lasagna, rotini, orzo, tortellini, penne}\}$

$Q = \{x \mid x \text{ is a pasta shape}\}$

$R = \{\text{penne, tortellini, orzo, rotini, lasagna}\}$

$S = \{\text{marinara, pesto, alfredo, Bolognese, carbonara}\}$

50. Is  $P = Q$ ? Why or why not?

51. Is  $P = R$ ? Why or why not?

52. Is  $P = S$ ? Why or why not?

53. Are any of  $P$ ,  $Q$ ,  $R$ , and  $S$  equivalent? Explain.

Use the given sets to answer each question.

$A = \{\text{Business, Physics, Psychology, Kinesiology, Graphic Design, History}\}$

$B = \{\text{History, Graphic Design, Kinesiology, Physics, Business}\}$

$C = \{\text{Art History, Education, Nursing, Biology, Statistics}\}$

$D = \{x \mid x \text{ is a university major}\}$

54. Is  $A = B$ ? Why or why not?

55. Is  $B = C$ ? Why or why not?

56. Is  $A = D$ ? Why or why not?

57. Are any of  $A$ ,  $B$ ,  $C$ , and  $D$  equivalent? Explain.

### WRITING & THINKING

58. Give an example of a set that cannot be represented using roster notation. Explain your reasoning.