14.4 Exercises

✔ CONCEPT CHECK

1. Graphs that can be drawn on a plane without _____ are called planar graphs.

2. In a planar graph, a _____ is a region inside a cycle of edges or the infinite exterior region on the outside of the graph.

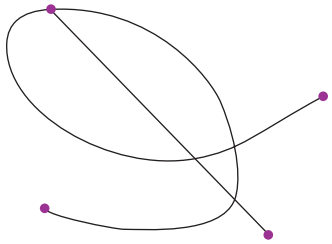
3. Euler's formula states that if G is a connected planar graph with v vertices, e edges, and f faces, then _____.

4. A planar graph G with v vertices has at most _____ edges.

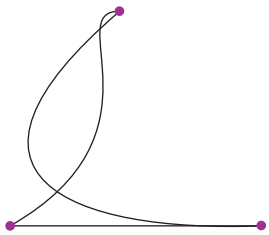
5. A _____ graph is defined to be a graph in which each vertex is connected to all other vertices.

PRACTICE

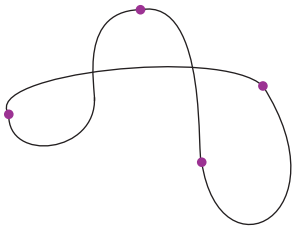
Draw each planar graph without any crossing edges.

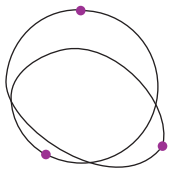
6. 

Graph *K*

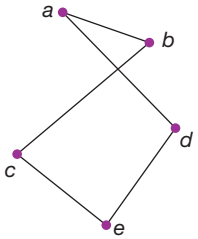
7. 

Graph *S*

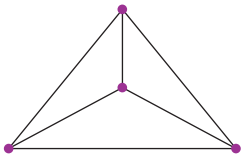
8. 

9. 

Determine the number of faces in each planar graph.

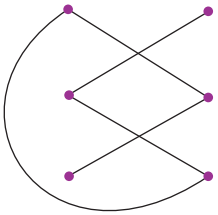
10. 

Graph *N*

11. 

Graph *P*

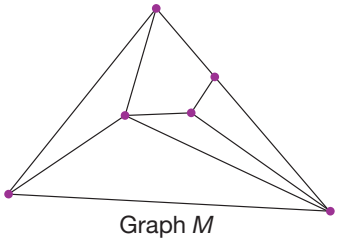
12.



Graph *E*

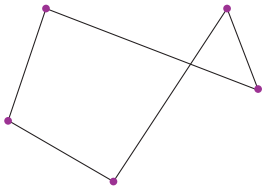
Verify Euler's formula for each graph.

13.



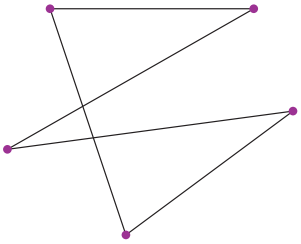
Graph *M*

14.



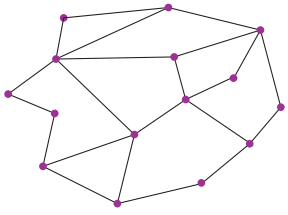
Graph *Z*

15.



Graph *F*

16.



Graph *C*

Use Euler's formula and the corollary of Euler's formula.

17. If a graph G has 9 vertices, what is the greatest number of edges that graph G can have and be a planar graph?

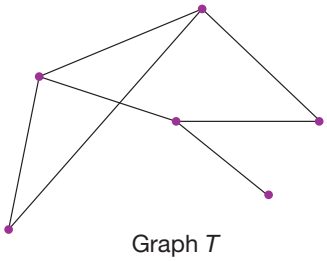
18. If a graph G has 16 vertices, what is the greatest number of edges that graph G can have and be a planar graph?

19. If G is a connected planar graph with 8 vertices and 10 edges, then how many faces does G have?

20. If G is a connected planar graph with 10 edges and 6 faces, then how many vertices does G have?

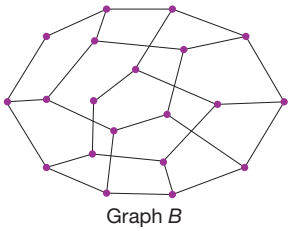
Determine if each graph is planar.

21.



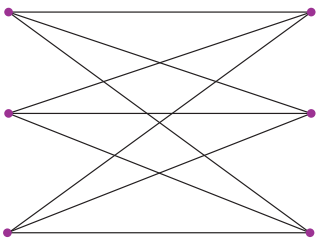
Graph *T*

22.



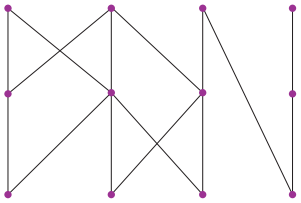
Graph *B*

23.



Graph *J*

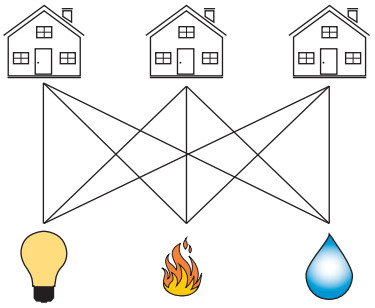
24.



Graph *D*

APPLICATIONS

25. A new housing development wants to connect three houses to their three utilities (gas, water and electric) without their cables/pipes crossing. Can it be done?



WRITING & THINKING

26. Show that a planar graph must have a vertex of degree at most 5.

27. Find the chromatic number of K_3, K_5, and K_n.

28. Find the chromatic number of $K_{5,5}$ and $K_{k,k}$.

29. Show that any planar graph can also be drawn on a sphere.

30. What happens if you draw the graph of a cube on a sphere and then flatten the faces so that they are the same size?

14.4 PROJECT

PLANAR POLYHEDRONS

In this section, you learned about planar graphs and how their numbers of v vertices, e edges, and f faces are related by Euler's formula: $v + f - e = 2$. You may have also learned about vertices, edges, and faces in geometry as part of the study of convex polyhedrons. A convex polyhedron is a solid made up of flat polygonal faces joined at their edges and vertices with the additional property that any line segment joining any two points on the surface of the polyhedron stays on or inside the polyhedron.