

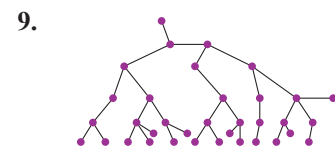
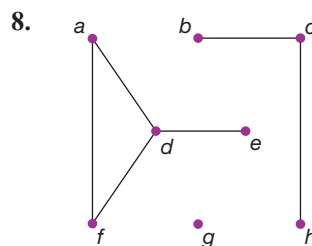
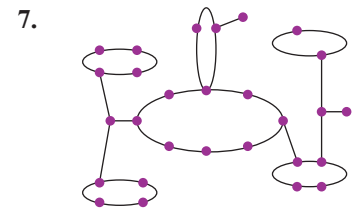
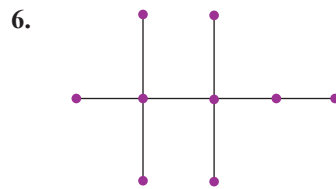
# 14.2 Exercises

## ✓ CONCEPT CHECK

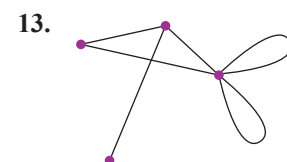
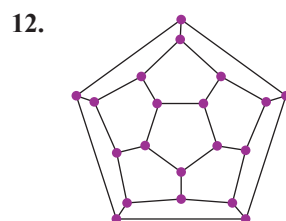
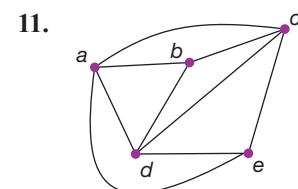
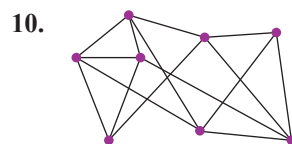
1. A tree is a \_\_\_\_\_ graph with no cycles.
2. If graph  $T$  is a tree on  $v$  vertices, then graph  $T$  has  $v - 1$  \_\_\_\_\_.
3. A \_\_\_\_\_ is a subgraph of a connected graph, which is itself connected and contains all the vertices of the original graph but has no cycles.
4. To find a spanning tree, start by identifying any \_\_\_\_\_ in the graph and delete enough edges to break the \_\_\_\_\_ while keeping the graph connected.
5. A vertex of degree \_\_\_\_\_ in a tree is called a leaf.

## 💡 PRACTICE

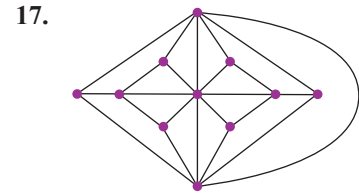
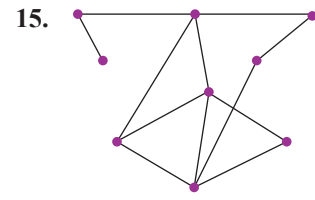
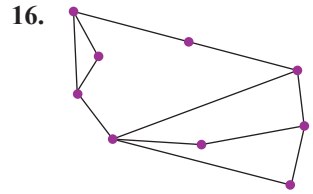
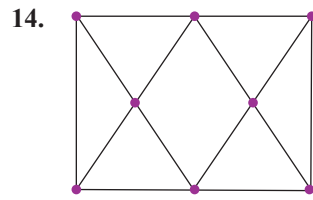
Determine if each graph is a tree.



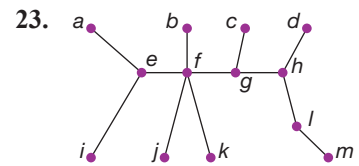
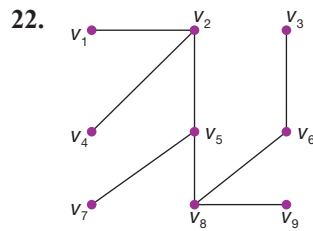
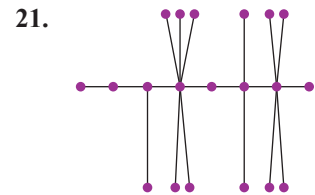
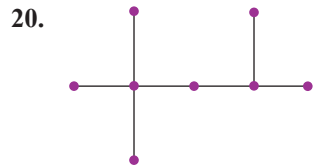
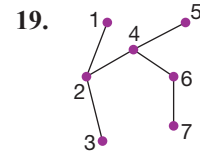
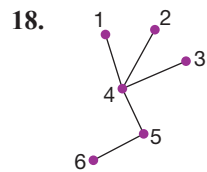
Determine the number of edges needed to form a tree in each graph.



Find a spanning tree in each graph.



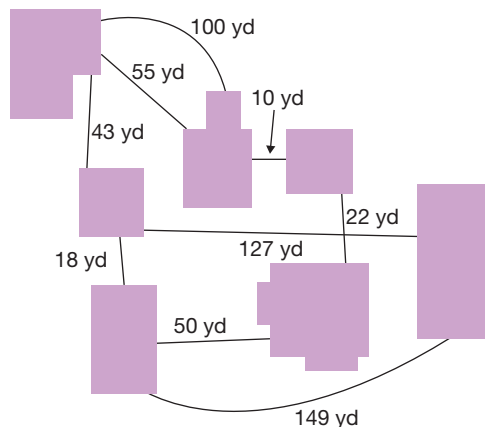
Find the number of leaves in each tree.



24. Let  $T$  be a tree with internal vertices of degree 4, 5, 6, 2, and 3. How many leaves does  $T$  have?

### APPLICATIONS

25. The sidewalks at a university are laid out according to the following graph. Each corresponding edge is labeled with the length of that section of sidewalk. If all of the sidewalks are covered with snow, which sections of sidewalks should be cleared to connect all sections of the university while clearing as little snow as possible?



26. A traveling salesman has to visit a number of destinations by a route that is as short as possible. Let  $G$  be a graph in which the vertices are the destinations, the edges represent roads that join the destinations, and the weights represent the lengths of the roads.
- If the minimum spanning tree has a weight of 100 miles, show that the salesman must drive at least 100 miles to visit all of the destinations.
  - Show that he needs to drive no more than 200 miles to complete his route.
27. Eurojet Hovercrafts has five hubs connecting European destinations on both sides of the English Channel. If the routes form a tree, with the five hubs having routes to 4, 5, 7, 3, and 6 ports respectively, how many cities are served by Eurojet Hovercrafts?

### WRITING & THINKING

28. What is the chromatic number of a tree? (**Hint:** Recall from Section 14.1 that the chromatic number of a graph is the number of colors used in a minimum vertex coloring.)

## 14.2 PROJECT

### ALGEBRA TREES

In this project, you will explore the use of trees to represent algebraic expressions and other mathematical formulas. Writing expressions in this way can be useful when trying to create a computer program that performs complicated calculations. For example, if we want to represent the expression  $x + y$  we could use the following tree.