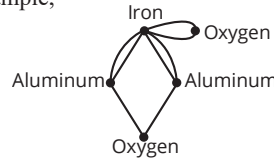


Skill Check Answers

1. Any of the following: c, b, a, e ; c, b, a, d ; c, b, a, b ; c, b, c, b ; c, d, a, e ; c, d, a, b ; c, d, a, d ; c, d, c, d 2. Adjacent vertices: x and y ; u and w ; v and x ; Incident edges: e_1 and e_2 ; e_3 and e_4 ; e_3 and e_5 ; e_4 and e_5
 3. Answers will vary. For example,



4. $B = \{b, e, j\}$
 5. Answers will vary.
 For example, a, b, d, h, g, e ; g, e, a, b, f, h ; b, f, h, d, e, a ; d, e, g, h, f, b .

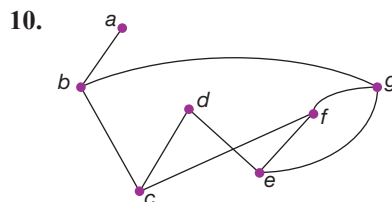
14.1 Exercises

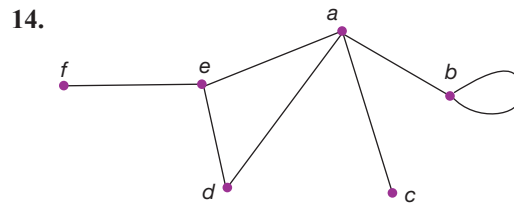
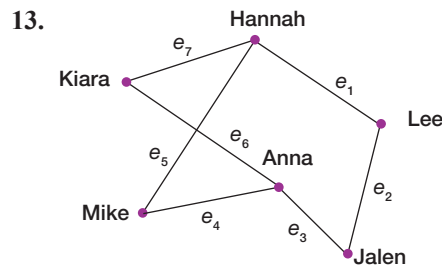
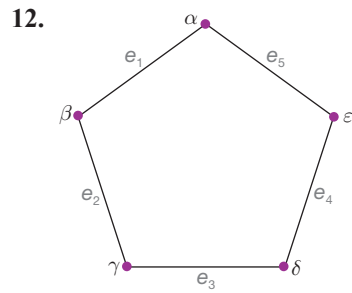
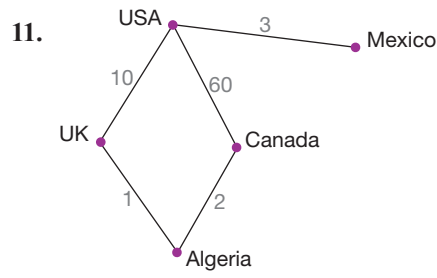
✓ CONCEPT CHECK

- The _____ of a vertex is the number of edges that are incident to that vertex.
- A/an _____ consists of a set of vertices and a set of edges that connect pairs of vertices.
- A/an _____ associates a pair of vertices in a graph.
- The end of an edge in a graph is called a/an _____.
- A/an _____ is an edge whose ends are the same vertex.
- Two edges are said to be _____ if they share a vertex.
- The _____ of a graph is the minimum number of colors needed in a vertex coloring of a graph.
- Two vertices are said to be _____ if they share an edge.
- A sequence of vertices such that each vertex is adjacent to the next in the list, the last vertex is adjacent to the first, and there are no repeated edges or vertices, except for the starting vertex, is called a/an _____.

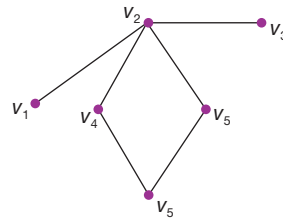
💡 PRACTICE

List the vertices and edges of each graph.



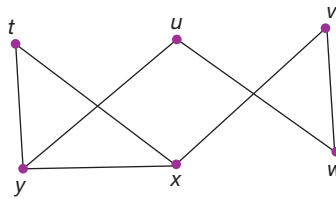


15. Use the graph to answer the following.



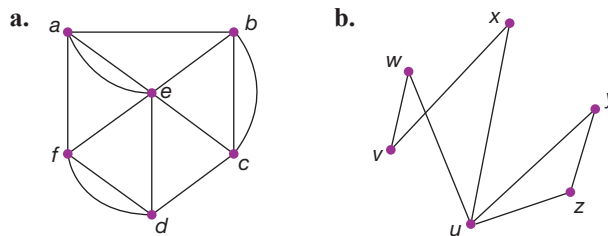
- Identify a walk of length two that starts at vertex v_1 and ends at vertex v_3 .
- Is the walk in part a. closed?
- Identify a closed walk.

16. Use the graph to answer the following.

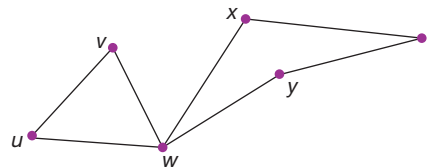


- a. Identify a walk of length 3 that starts at vertex u and ends at vertex v .
- b. Identify a walk of length 5 that starts at vertex t and ends at vertex t .
- c. Are the walks in parts a. and b. closed?

17. An **Euler trail** in a graph G is a walk that contains every edge in G precisely once. Each of the following graphs contains an Euler trail, identify it.

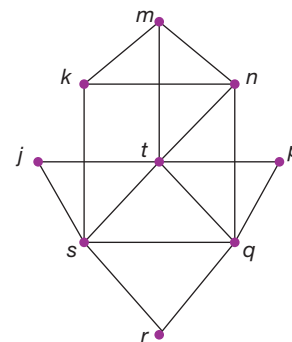


18. Identify two paths in the following graph. What is the length of each path?

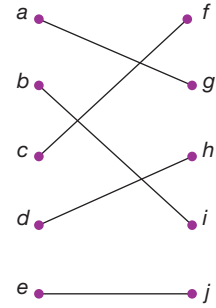
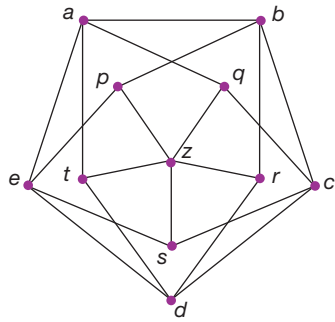


19. The length of a path is the number of edges in the path. Identify a path of the specified length in the graph.

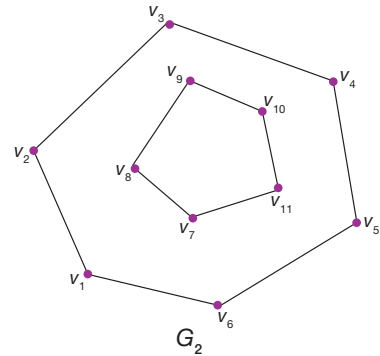
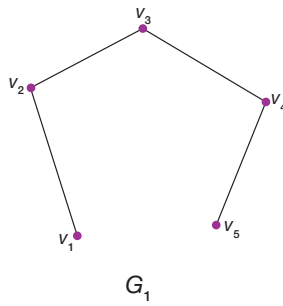
- a. Path of length 2
- b. Path of length 3
- c. Path of length 6



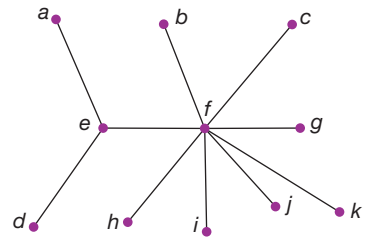
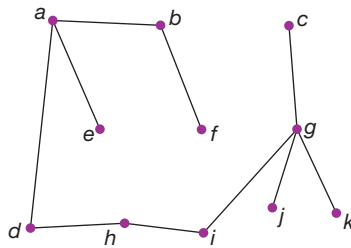
20. Which of the given graphs is connected?



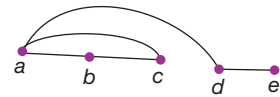
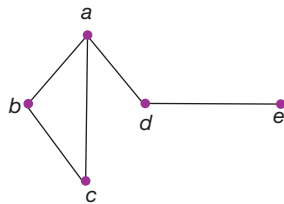
21. Which of the given graphs is connected?



22. Are the two given graphs the same graph? Explain.



23. Are the two given graphs the same? Explain.

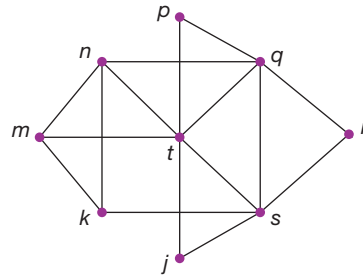


24. Draw a graph that has five vertices r, s, t, u, v , and the edges rt, ru, rv, st, su , and sv .

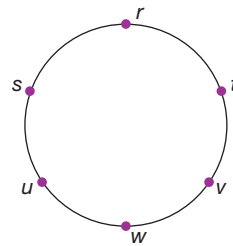
25. Draw a graph with vertices u, v, w, x, y, z , and edges $ux, uv, uy, vx, vy, vw, wy, wz, wx$, with no crossings.

26. Draw a graph that has 7 vertices a, b, c, d, e, f, g , and edges $ab, ac, ae, bc, cd, de, ef, fg$, and gg .

27. Draw a graph with vertices $\alpha, \beta, \pi, \sigma, \omega$, and edges $\alpha\beta, \beta\pi, \pi\sigma, \sigma\omega, \alpha\omega, \beta\omega$, and $\alpha\sigma$, with no edges crossing.
28. Find a vertex coloring of the graph. What is the chromatic number of that graph?

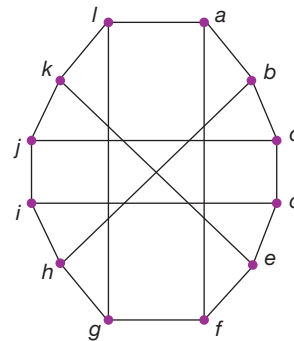


29. Identify a cycle in the following graph and determine its length.



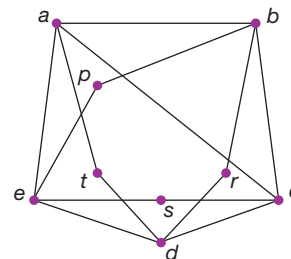
30. Identify a cycle of the required length in the graph.

- a. Cycle of length 4
- b. Cycle of length 6
- c. Cycle of length 8



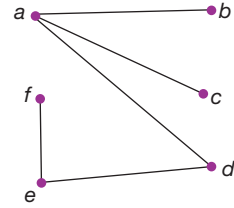
31. Identify the following features in the graph. In some cases, more than one answer is correct.

- a. An edge
- b. Two adjacent vertices
- c. Two incident edges
- d. Two vertices that are not adjacent
- e. A vertex of degree 4
- f. A cycle of length 5

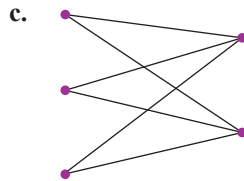
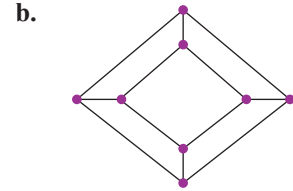
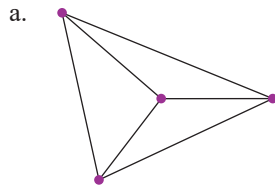


32. Identify the following features in the given graph. In some cases, more than one answer is correct.

- a. Two adjacent vertices
- b. Two incident edges
- c. Two vertices that are not adjacent
- d. A vertex of degree 2

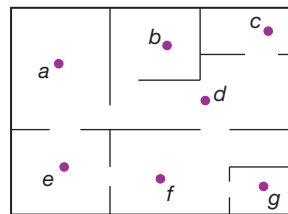


33. A **Hamilton cycle** in a graph G is a cycle that contains every vertex in G . Decide if the following graphs contain Hamilton cycles.

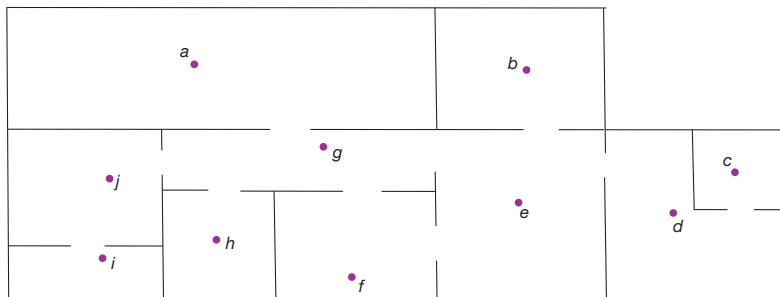


APPLICATIONS

34. The graph shows a blueprint of rooms in a museum. Find a way to place guards in the rooms so that every room either has a guard or shares a doorway with a room that has a guard and the minimum number of guards is used.



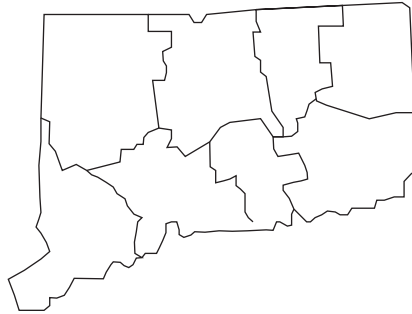
35. The graph shows a blueprint of rooms in a house. Find a way to place security cameras so that every room either has a camera or shares a doorway with a room that has a camera, and the minimum number of cameras is used.



36. Use a vertex coloring to assign colors to the counties of Vermont so that adjacent counties have different colors. What is the minimum number of colors required? Find a vertex coloring using the minimum number of colors.



37. Use a vertex coloring to assign colors to the counties of Connecticut so that adjacent counties have different colors. What is the minimum number of colors required? Find a vertex coloring using the minimum number of colors.



 **WRITING & THINKING**

38. Draw two different connected graphs, each with four vertices and four edges.