Notice that once the pre-professional group and its one representative is added in, the apportionment actually assigns the new branch two representatives, while the academic branch loses one. Because the addition of a new subgroup changed the apportionment of another subgroup, the new states paradox has occurred.

Hamilton's method of apportionment managed to be used for quite a long time in the United States House of Representatives before these paradoxes forced it to be replaced with some of the other methods we've looked at. However, although they avoided the paradoxes, these new methods created problems of their own. Occasionally, they violate what is known as the quota rule. The **quota rule** states that any fair apportionment method should assign every subgroup either its lower quota or its upper quota. Because the methods of apportionment introduced after Hamilton's all use a modified divisor, sometimes the modified quotas are different from the lower or upper quotas for that subpopulation.

THE QUOTA RULE

The **quota rule** states that any fair apportionment method should assign every subgroup either its lower quota or its upper quota.

Does a method exist that fairly apportions and does not violate any of the paradoxes or the quota rule? The short answer is no, although our examples in the text seem to suggest that it is possible because they adhere to the quota rule. However, in 1983, two mathematicians Michel Balinski and Peyton Young proved that there cannot be a perfect apportionment method. They proved that any apportionment method that does not violate the quota rule must produce paradoxes and any method that does not produce paradoxes must violate the quota rule. Although no perfect method for apportionment exists, we can make an informed decision about the most appropriate apportionment method, understanding the potential for flaws for paradoxes while striving to adhere to the quota rule.

Skill Check Answers

1. 25, 125 **2.** 6.4925; 3.8806; 6.6418 **3.** Math 4; History 8; Computer Science 6
4. Div. Pop 135 = 4; Div. Pop 98 = 3; Div. Pop 132 = 4
5. County 1 = 48; County 2 = 23; County 3 = 5; County 4 = 15; County 5 = 9

13.3 Exercises

CONCEPT CHECK

1. A _____ represents the number of items that will be apportioned to each subgroup.

2. The Huntington-Hill method of apportionment uses the _____ to determine if the standard quota should be rounded up or down.

3. The _____ occurs when an increase in the number of available items causes a group to lose an item—even though populations remain the same.

4. The _____ states that subgroup A can lose an item to subgroup B even when the rate of growth of the population of subgroup A is greater than in subgroup B.

5. The _____ happens when the addition of a new subgroup, with a corresponding increase in the number of available items, can cause a change in the apportionment of items among the other subgroups.

PRACTICE

Use the given table to solve each problem.

Student Enrollment in the University of California System

Campus	Enrollment	Full Time Equivalent (FTE)
Berkeley	33,558	14,161
Davis	32,290	20,883
Irvine	28,000	12,558
Los Angeles	37,221	28,292
Merced	2700	799
Riverside	20,956	4689
San Diego	25,938	18,274
San Francisco	18,140	4174
Santa Barbara	21,016	6081
Santa Cruz	15,012	4597

6. The University of California system consists of 10 campuses and has a budget of $2.6 billion in state funding. How much money in state funding would each campus receive if the state divided the money equally among the campuses (round to nearest dollar)?

7. If the state of California wanted to allocate the money equally based on total student enrollment, how much would be appropriated per student, rounded to the nearest cent?

8. Based on the result from Exercise 2, how much money would the campus of Los Angeles and Merced receive based on the number of students enrolled on each campus?

9. If the state apportions the funds based on FTE, how much money will the campuses of Davis and Santa Cruz receive (round to nearest dollar)?

10. As part of a "green" initiative, the state is wanting to apportion 500 new electric vehicles to their university system campuses. The state decides to apportion these vehicles based on the number of students at each university.
    a. Find the SD.
    b. Find the SQ for the San Francisco and Irvine campuses.

11. Supposed the state decides to apportion the 500 electric vehicles based on FTE.

a. Find the SD.

b. Find the SQ for the Riverside and Santa Barbara campuses.

12. Use the Hamilton method to apportion the 500 electric vehicles to all 10 campuses based on the number of students.

13. Use the Hamilton method to apportion the 500 electric vehicles to all ten campuses based on FTE. Compare your results with the apportionments from Exercise 7 and determine if the apportionments are different when the apportionment basis is different.

The given table shows the number of students enrolled in history, liberal arts math, and English during the fall and spring semesters. Use it to solve each problem.

Course Enrollment per Semester

Subject	Fall	Spring
History	1902	1922
Liberal Arts Math	14,200	14,200
English	3898	3938

14. If there are 197 full time teaching positions available for apportionment among the three departments based on course enrollment, answer each of the following questions.

a. Find the number of teaching positions that should be apportioned to each department in the fall using Jefferson's method.

b. Find the number of teaching positions that should be apportioned to each department in the spring using Jefferson's method.

c. Does Jefferson's method create an example of the population paradox?

d. Find the number of teaching positions that should be apportioned to each department in the fall using Hamilton's method.

e. Find the number of teaching positions that should be apportioned to each department in the spring using Hamilton's method.

f. Does Hamilton's method create an example of the population paradox?

g. Find the number of teaching positions that should be apportioned to each department in the fall using the Huntington-Hill method.

h. Find the number of teaching positions that should be apportioned to each department in the spring using the Huntington-Hill method.

i. Does the Huntington-Hill method create an example of the population paradox?

APPLICATIONS

15. An English teacher at a high school can teach six classes. There are 35 students enrolled in English I, 43 in English II, and 48 in English III.

a. Find the SD and SQ to determine how many sections of each course should be offered?

b. Use the Jefferson method to determine the apportionment of students to the courses to determine the number of sections needed per course. determine the number of sections needed per course.

16. Repeat Exercise 15 b. using the Webster method.

17. Repeat Exercise 15 b. using the Hamilton method.

18. Repeat Exercise 15 b. using the Huntington-Hill method.

19. A county is divided into four districts with the populations of: Northern: 5500, Southern: 6350, Eastern: 3470, and Western: 1950. There are 16 seats on the county board to be apportioned.

a. Use the Jefferson method to apportion the board seats.

b. Use the Huntington-Hill method to apportion the board seats.

c. Compare the apportionments for the Jefferson and Huntington-Hill methods and determine if there is a difference in how the seats are apportioned.

20. A biology department uses 25 graduate assistants in teaching its undergraduate courses. The enrollments for each of the courses that these students teach is as follows. How many graduate assistants should be assigned to each course using the Jefferson method?

Enrollment per Course

Course	Enrollment
Survey of Biology	450
Zoology	200
Cell Biology	175
Plant Biology	280

21. Use the Hamilton method to round each of the following numbers to a whole number while preserving the total.

$$12.65 + 3.48 + 2.57 + 4.39 + 1.91 = 25$$

22. Use Hamilton's method to round each of the following numbers to a whole number while preserving the total.

$$32.61 + 58.37 + 55.02 + 23.11 + 54.89 = 224$$

23. Suppose there are 76 faculty members in the sciences, 86 in the humanities, and 16 in the professional and trade schools. An 11-person faculty committee is to be formed.

a. Use Hamilton's method to determine the allocation of committee members based on department size.

b. Use Jefferson's method to determine the allocation of committee members based on department size.

c. Use the Huntington-Hill method to determine the allocation of committee members based on department size.

d. Use Webster's method to determine the allocation of committee members based on department size.

24. Suppose Learn-A-Lot University has enrollments on its three campuses as follows. There are 40 police officers to be distributed among these campuses based on enrollment. Use Hamilton's method to apportion the police officers.

Enrollment per Campus			
Campus	1	2	3
Enrollment	10,170	9150	680

25. If the number of police officers to be apportioned increases by 1 to 41 (see Exercise 24) determine the apportionment of officers using Hamilton's Method and show that the Alabama paradox occurs when the number of officers increases by 1.

26. Suppose a country has six states with populations as given in the table. There are 250 seats in the House of Representatives for this country. Use Webster's method to apportion the representatives.

Population by State	
State	**Population**
A	1646
B	6936
C	154
D	2091
E	685
F	988
Total	**12,500**

27. Suppose a college homecoming planning committee has 17 members. The makeup of the committed is to be based on the size of the classes: Freshman = 422, Sophomore = 356, Junior = 321, and Senior = 288.

a. Find the number of members from each class to be apportioned to the committee using Webster's method.

b. Find the number of members from each class to be apportioned to the committee using Jefferson's method.

c. Find the number of members from each class to be apportioned to the committee using the Huntington-Hill method.