The following table summarizes each of the voting methods discussed previously versus the fairness criteria used in democratic elections.

Table 13.2.16: Voting Methods and Fairness Criteria

	Condorcet Criterion	Majority Criterion	Monotonicity Criterion	Irrelevant Alternatives Criterion	Dictator Criterion
Majority Rule Decision	Never violates	Never violates	Never violates	Never violates	May violate
Plurality	May violate	Never violates	Never violates	May violate	Never violates
Borda Count	May violate	May violate	Never violates	May violate	Never violates
Plurality with Elimination	May violate	Never violates	May violate	May violate	Never violates
Pairwise Comparison	Never violates	Never violates	Never violates	May violate	Never violates

As our table shows, it would appear that using a majority rule as an election method might in fact be the perfect voting system because it never violates any of the four fairness criteria in a democracy. However, the one "fly in the ointment," so to say, is exactly what we saw in the previous section—using the majority rule doesn't always produce a winner. This is why Arrow requires that valid voting systems always produce a distinct winner.

13.2 Exercises

✔ CONCEPT CHECK

1. The _____ states that if a candidate wins the head-to-head comparison—as in the pairwise comparison method—against every other candidate, then that candidate should also win the overall election in a fair voting system.

2. If a candidate receives a majority of votes in an election but does not win, the _____ has been violated.

3. The _____ requires that if Candidate X wins an election, then X would also win a second election if each voter were allowed to reorder the candidates in such a way that X only increases in ranking while the order of the other candidates remained the same.

4. If the winner of an election is changed when one or more candidates drops out of the election, the _____ has been violated.

5. The _____ is the only voting method that can violate the majority criterion.

APPLICATIONS

Use the given preference table to answer each question.

6. A group of students were asked to listen to auditions by four bands for the homecoming dance and then rank the bands. The following preference table summarizes their selections.

Preference Table for Homecoming Bands

	Rankings				
1st	Band B	Band A	Band C	Band D	Band A
2nd	Band D	Band D	Band B	Band A	Band C
3rd	Band C	Band B	Band A	Band B	Band B
4th	Band A	Band C	Band D	Band C	Band D
Total Votes	**23**	**15**	**16**	**11**	**5**

a. Which band is the plurality winner?

b. Which candidate wins the election using a pairwise method of comparison?

c. Does this election, using the plurality method, satisfy the Condorcet criterion? Explain your answer.

7. Susan, Courtney, and Jade ran for Freshman SGA representative. The following preference table shows the results of the race.

Preference Table for SGA Representatives

	Rankings		
1st	Susan	Courtney	Susan
2nd	Courtney	Susan	Jade
3rd	Jade	Jade	Courtney
Total Votes	**32**	**55**	**16**

a. Which candidate is the winner using the Borda count method?

b. Which candidate is the winner using the pairwise method of comparison?

c. Does the Borda count method satisfy the Condorcet criterion in this situation? Why or why not?

8. Work colleagues are trying to decide on which restaurant to use for their end-of-year party. Since they don't all agree on a favorite, they decide to rank the four possibilities and decide on a restaurant that way. The results are shown in the preference table.

Preference Table for Favorite Restaurants

	Rankings				
1st	Tanvi's	O'Reilly's Pub	Ralphie's	Tanvi's	Salty's
2nd	Ralphie's	Salty's	Tanvi's	O'Reilly's Pub	Ralphie's
3rd	Salty's	Tanvi's	Salty's	Salty's	O'Reilly's Pub
4th	O'Reilly's Pub	Ralphie's	O'Reilly's Pub	Ralphie's	Tanvi's
Total Votes	**8**	**3**	**4**	**11**	**9**

a. Which restaurant is the winner using the plurality with elimination method?

b. Which restaurant is the winner using the pairwise method of comparison?

c. Does the plurality with elimination method satisfy the Condorcet criterion if the colleagues chose a restaurant using plurality with elimination? Why or why not?

9. Jack is scheduling classes for next semester and has three possible classes for a particular time slot. He surveys students to see which class has the most interest out of Electricity and Magnetism, Heat and Optics, and Mechanics.

Preference Table for Classes

	Rankings			
1st	H & O	Mechanics	H & O	E & M
2nd	E & M	H & O	E & M	Mechanics
3rd	Mechanics	E & M	Mechanics	H & O
Total Votes	**81**	**132**	**36**	**13**

a. Which class has the most interest using the majority rule?

b. Which class has the most interest using the Borda count method?

c. Does the Borda count method satisfy the majority criterion if Jack uses this method to fill the available time slot? Explain your answer.

10. The history department is adopting a new textbook for the Freshman Introduction class. Instructors in the history department were asked to rank the top four textbooks in order of usability for the class. The preference table shows the outcome of their rankings.

Preference Table for History Textbooks

	Rankings			
1st	*Excursions Through Time*	*Thinking Historically*	*A Look in Time*	*Through the Ages*
2nd	*Thinking Historically*	*A Look in Time*	*Thinking Historically*	*A Look in Time*
3rd	*A Look in Time*	*Excursions Through Time*	*Through the Ages*	*Thinking Historically*
4th	*Through the Ages*	*Through the Ages*	*Excursions Through Time*	*Excursions Through Time*
Total Votes	**7**	**3**	**7**	**4**

a. Use the plurality with elimination method to choose a new textbook for adoption.

b. After some discussion in the department, the seven instructors whose rankings appear in the first column decide to change their votes so that the textbooks titled *A Look in Time* and *Thinking Historically* swap rankings. Create a new preference table to show this change and then use the table to decide on a textbook using the plurality with elimination method once again.

c. Is the monotonicity criterion satisfied? Explain your answer.

11. Charles Beauregard High School sponsors a senior class trip each year. This year, the students can choose between trips to Charleston, SC; Jacksonville, FL; New Orleans, LA; Chicago, IL; or Atlanta, GA. The students were asked to rank their choices in order to choose a destination.

Preference Table for Senior Class Trip

	Rankings			
1st	New Orleans, LA	Charleston, SC	Atlanta, GA	Chicago, IL
2nd	Chicago, IL	New Orleans, LA	New Orleans, LA	Charleston, SC
3rd	Atlanta, GA	Atlanta, GA	Chicago, IL	Atlanta, GA
4th	Charleston, SC	Jacksonville, FL	Charleston, SC	New Orleans, LA
5th	Jacksonville, FL	Chicago, IL	Jacksonville, FL	Jacksonville, FL
Total Votes	**375**	**348**	**289**	**115**

a. Which city would be the senior trip destination using the plurality with elimination method?

b. After consideration of travel time, the senior class sponsors decided to eliminate Chicago, IL, from the options. Which city is now the winning destination using plurality with elimination?

c. Does the plurality with elimination method satisfy the irrelevant alternatives criterion? Explain your answer.

12. A nonprofit organization is looking to use social networking sites to increase interest. They poll donors to decide where to focus their resources. The donors are asked to rank the following sites by how often they use them.

Preference Table for Social Networking Sites

	Rankings			
1st	Facebook	Pinterest	Facebook	Twitter
2nd	Twitter	Facebook	Twitter	Facebook
3rd	Google+	Twitter	LinkedIn	LinkedIn
4th	LinkedIn	LinkedIn	Pinterest	Google+
5th	Pinterest	Google+	Google+	Pinterest
Total Votes	**23**	**40**	**44**	**35**

a. Which site is given the higher ranking using the pairwise method of comparison?

b. If the nonprofit decides to eliminate Pinterest and Google+ from the list and adjust the votes accordingly, which site would then be the top place for the nonprofit to focus on using the pairwise method?

c. Does the pairwise method satisfy the irrelevant alternatives criterion for this preference table? Why or why not?

13. The men of the fraternity Chi Rho on campus are electing a new president. The candidates are M. Jones, H. Kennedy, and T. Parchment. The summary of the rankings of the candidates from the members is given in the chart.

Preference Table for Chi Rho Presidential Candidates

	Rankings			
1st	H. Kennedy	M. Jones	H. Kennedy	T. Parchment
2nd	M. Jones	T. Parchment	T. Parchment	H. Kennedy
3rd	T. Parchment	H. Kennedy	M. Jones	M. Jones
Total Votes	**8**	**14**	**10**	**11**

a. Name the fraternity's next president using the Borda count method.

b. Because of unexpected family issues, M. Jones had to withdraw from the university on short notice. Create a new preference table to show this change and then use the table to determine the next president using the Borda count method.

c. Is the irrelevant alternatives criterion satisfied with this change? Explain your answer.

14. For the end of year banquet, the program committee needs to decide on the type of cuisine. They have four choices: Italian, French, Mexican, or Chinese. The following preference table summarizes the rankings of the members surveyed. Determine if the Condorcet criterion is satisfied if the committee uses the Borda count method to choose a cuisine.

Preference Table for Banquet Cuisine

	Rankings				
1st	French	Mexican	Italian	Italian	Mexican
2nd	Italian	Italian	Mexican	Chinese	French
3rd	Mexican	Chinese	French	Mexican	Chinese
4th	Chinese	French	Chinese	French	Italian
Total Votes	**4**	**21**	**14**	**11**	**9**

15. In preparation for their Oscar party, a sorority decides to have an election of their own. In the Best Actor category are George Clooney, Johnny Depp, Colin Firth, and Zac Efron. The rankings from the sorority members are shown in the following preference table. Determine if the majority criterion will be satisfied if the sorority uses the Borda count method to choose the winner for Best Actor.

Preference Table for Best Actor

	Rankings				
1st	Johnny Depp	Zac Efron	Johnny Depp	George Clooney	Colin Firth
2nd	Colin Firth	Colin Firth	George Clooney	Colin Firth	Zac Efron
3rd	Zac Efron	George Clooney	Zac Efron	Johnny Depp	Johnny Depp
4th	George Clooney	Johnny Depp	Colin Firth	Zac Efron	George Clooney
Total Votes	**26**	**16**	**16**	**15**	**9**

16. *Pretty People* magazine wants to name the number one activity for a first date amongst singles. They enlist the help of an online dating site that asks members to rank five activities in order of their preference as part of their sign-up process. The five activities include: movie, nice dinner, picnic, sporting event, and concert. The following preference table displays the results of the rankings. Determine if the Condorcet criterion is satisfied if the magazine uses the plurality method to name the top activity.

Preference Table for First Date Activities

	Rankings					
1st	Nice Dinner	Picnic	Picnic	Sporting Event	Sporting Event	Movie
2nd	Picnic	Sporting Event	Sporting Event	Concert	Nice Dinner	Nice Dinner
3rd	Concert	Concert	Nice Dinner	Picnic	Movie	Concert
4th	Movie	Nice Dinner	Concert	Nice Dinner	Concert	Picnic
5th	Sporting Event	Movie	Movie	Movie	Picnic	Sporting Event
Total Votes	**40**	**34**	**21**	**22**	**19**	**20**

17. United Way asks its volunteers to rank the nonprofit organizations it serves in order to give out the annual award for Local Volunteer of the Year. This year's organizations include the Breast Cancer Foundation, Community Garden, Big Brother/Big Sister, YMCA, and Manna Café. Complete the preference table so that Community Garden is the winner using the Borda count method, but the Condorcet criterion is violated.

Preference Table for Local Volunteer of the Year Award

	Rankings		
1st	MC	CG	BB/BS
2nd	BB/BS	BCF	MC
3rd	CG	BB/BS	CG
4th	YMCA	YMCA	BCF
5th	BCF	MC	YMCA
Total Votes	**?**	**16**	**10**

18. An online dating site asks new users to rank certain traits in order of importance when they are matched with another user. The site uses the top choice as the first criterion for matching people together. The following preference table summarizes the rankings for appearance, personality, income, profession, and height. Complete the preference table so that personality is the top trait if the site uses the Borda count method to count the votes and the majority criterion is not violated.

Preference Table for Important Traits

	Rankings			
1st	?	Personality	Income	Appearance
2nd	?	Profession	Personality	Income
3rd	Profession	Appearance	Appearance	Personality
4th	Height	Height	Profession	Height
5th	Income	Income	Height	Profession
Total Votes	**45**	**56**	**33**	**48**

19. College dining is considering opening a national fast food franchise on campus. Their choices have been narrowed down to Subway, Taco Bell, McDonald's, KFC, or Pizza Hut. Students were asked to rank these establishments in order of preference, and the results are shown in the preference table. If college dining uses the plurality method to choose the students' top choice of Subway, complete the preference table so that the irrelevant alternatives criterion is violated when KFC is removed from the race.

Preference Table for New Franchise on Campus

	Rankings			
1st	Subway	Taco Bell	KFC	Subway
2nd	McDonald's	Subway	Taco Bell	Taco Bell
3rd	Taco Bell	Pizza Hut	Subway	Pizza Hut
4th	KFC	McDonald's	Pizza Hut	McDonald's
5th	Pizza Hut	KFC	McDonald's	KFC
Total Votes	**56**	**60**	**?**	**45**

20. Sigma Air is exploring the possibility of establishing a new hub in one of the following cities: Cheyenne, WY; Little Rock, AR; Amarillo, TX; and Portland, OR. They surveyed their Gold and Platinum members to ask their preference for the new hub. Complete the preference table of the rankings so that using the plurality with elimination method Amarillo is the winner, and the Condorcet criterion is satisfied.

Preference Table for New Hub Location

	Rankings		
1st	Amarillo, TX	Portland, OR	Cheyenne, WY
2nd	Portland, OR	Little Rock, AR	Amarillo, TX
3rd	Cheyenne, WY	?	Little Rock, AR
4th	Little Rock, AR	?	Portland, OR
Total Votes	**70**	**34**	**37**

21. The following preference table summarizes the outcome of an election.

Preference Table for Candidates

	Rankings			
1st	Luke	Luke	Lauren	Blake
2nd	Hannah	Blake	Blake	Luke
3rd	Lauren	Lauren	Hannah	Lauren
4th	Blake	Hannah	Luke	Hannah
Total Votes	**31**	**9**	**23**	**41**

a. Which candidate is the plurality winner?

b. Which candidate wins the election using a pairwise method of comparison?

c. Does this method satisfy the Condorcet criterion? Why or why not?

22. Every year, the Oviedo Little League gives out a Coach of the Year award. The board of directors selects three names to put on the ballot, and parents send in their votes. The results are shown in the following preference table.

Preference Table for Coach of the Year

	Rankings			
1st	Hall	McKee	Price	Hall
2nd	McKee	Price	McKee	Price
3rd	Price	Hall	Hall	McKee
Total Votes	**7**	**62**	**21**	**11**

a. Which coach would win using the majority rule?

b. Which coach would win using the Borda count method?

c. Does the Borda count method satisfy the majority criterion

23. A high school math teacher is trying to pick a name for her puppy. She's narrowed it down to four choices and has decided to let the senior class vote. Determine if the majority criterion will be satisfied if the teacher uses the Borda count method to choose the winner.

Preference Table for Puppy Name

	Rankings				
1st	Paladin	Knight	Knight	Paladin	Paladin
2nd	Ranger	Paladin	Whiskey	Ranger	Whiskey
3rd	Whiskey	Ranger	Paladin	Whiskey	Knight
4th	Knight	Whiskey	Ranger	Knight	Ranger
Total Votes	**26**	**73**	**68**	**51**	**25**

24. Suzie's family has a reunion every other year. On the last night of each reunion, family members can propose a location for the next reunion. Family members rank the top four proposals.

Preference Table for Reunion Location

	Rankings			
1st	Charleston	San Diego	Dallas	New Orleans
2nd	Dallas	Charleston	San Diego	Dallas
3rd	San Diego	New Orleans	Charleston	Charleston
4th	New Orleans	Dallas	New Orleans	San Diego
Total Votes	**6**	**5**	**4**	**3**

a. Use the plurality with elimination method to choose where the next reunion should be held.

b. After some discussion among the cousins, the 3 family members whose rankings appear in the rightmost column decide to change their votes so that Charleston is their first pick, but the rest of their ranking keeps the same order. Create a new preference table to show this change and then use the table to decide on a location using the plurality of elimination method once again.

c. Is the monotonicity criterion satisfied? Explain your answer.

13.2 PROJECT

WHAT'S FAIR? DEPENDS ON WHO YOU ASK

In Section 13.2, we discussed five different conditions for determining the fairness of an election. Now it's time to explore how voting procedures and ideas about fairness impact elections in the real world. In this project, we'll consider the fairness of ranked-choice voting, which is equivalent to the plurality with elimination method that is used with preference ballots.

As a recap, here are the fairness criteria.

- The **Condorcet criterion** states that if a candidate wins the head-to-head comparison against every other candidate, then that candidate should also win the overall election in a fair voting system.