

Plurality with Elimination

A series of runoff elections, or eliminations, where the candidate with the least amount of first-place votes is removed from the ballot each round. A winner is declared when a candidate has a majority of the first-place votes.

Pairwise Comparison

Every candidate is compared head-to-head with the other candidates. In each pair of comparisons, the candidate with the greater number of higher rankings is given a point. The candidate with the most points after all head-to-head comparisons are made is the winner.

Skill Check Answer

1. Sydney, with 53 votes that gave her a fifth place ranking.

13.1 Exercises

✓ CONCEPT CHECK

1. The _____ method of choosing a winner in an election is a single vote with a winner-take-all approach. The winner need not have a majority of the votes to win; she just needs the most votes.
2. The _____ method for elections pits each candidate against all of the others individually in order to choose a winner.
3. Setting up an election so that voters are required to rank all the candidates requires use of a _____.
4. In order to give weight to each of the different rankings for candidates, the _____ attributes each ranking a certain number of points, and the candidate with the most points wins the election.
5. In a _____ election, the winner must receive more than 50% of the votes.

💡 PRACTICE

Create a preference table to show the results for each set of preference ballots.

6. Candidates in the election are: Maria, Daniel, Eisa.

Maria, Eisa, Daniel: 29

Eisa, Daniel, Maria: 43

Eisa, Maria, Daniel: 33

Daniel, Maria, Eisa: 17

Maria, Daniel, Eisa: 3

Daniel, Eisa, Maria: 21

7. The candidates are: Lars, Noah, Stephen, Oliver, Jack.
 Stephen, Oliver, Jack, Lars, Noah: 212
 Noah, Oliver, Jack, Lars, Stephen: 133
 Oliver, Stephen, Jack, Noah, Lars: 543
 Jack, Stephen, Lars, Noah, Oliver: 24
 Oliver, Noah, Lars, Jack, Stephen: 179
 Noah, Lars, Stephen, Jack, Oliver: 8
 Stephen, Lars, Oliver, Noah, Jack: 201
 Stephen, Jack, Lars, Noah, Oliver: 11
8. The candidates are: Jones (J), Brown (B), Wang (W), Cohen (C), Diaz (D).
 JBDCW, CWJDB, WBDJC, BDCWJ, JBDCW, JBDCW, CWJDB,
 CWJDB, WBDJC, JBDCW, BWJDC, WBDJC, CWJDB, BWJDC,
 BWJDC, BJWCD, WBDJC, BWJDC, JBDCW, BJWCD, WBDJC,
 BWJDC, JBDCW, JBDCW, DBWCJ
9. Candidates in the election are: Nguyen (N), Dey (D), Smith (S), Mori (M),
 Abbadì (A).
 MADNS, NDSMA, DSMAN, SMAND, SMAND, NDSMA, MANDS,
 DSMAN, SAMND, NDSMA, SMAND, NDSMA, MANDS, SAMND,
 NDSMA, MANDS, MANS D, SDMAN, MADNS, MANDS, MANDS,
 SMAND, NDSMA, DSMAN, SAMDN, MADNS, SAMDN, SMAND,
 SMAND, NDSMA

Use the given preference table to answer each question.

10. Answer the following questions about the given preference table.

Preference Table for Candidates					
	Rankings				
1st	A	C	C	E	E
2nd	B	E	E	D	A
3rd	C	B	A	B	C
4th	E	D	B	C	D
5th	D	A	D	A	B
Total Votes	153	129	63	404	111

- How many possible unique rankings are there of the candidates in the election?
- How many people voted in the election?
- How many voters place Candidate C in first place?
- Which candidate wins using the plurality method?
- How many votes would a candidate need to have a majority?
- Do any of the candidates have a majority of first-place votes?

11. A wedding planning website asked visitors to rank the following six all-time favorite wedding songs. The song choices are *Nothing Compares to You* by Sinead O’Connor, *Love Me Tender* by Elvis Presley, *Close to You* by Maxi Priest, *When You Say Nothing at All* by Ronan Keating, *Just the Way You Are* by Billy Joel, and *You Made Me Love You* by Al Jolson. Use the preference table to answer the questions.

Rankings				
1st	Presley	Priest	O’Connor	Keating
2nd	Jolson	O’Connor	Presley	Joel
3rd	Keating	Joel	Priest	Presley
4th	Priest	Jolson	Keating	O’Connor
5th	Joel	Presley	Joel	Jolson
6th	O’Connor	Keating	Jolson	Priest
Total Votes	1320	2010	3167	2697

- How many possible unique rankings are there of the six wedding songs?
- How many people participated in the website questionnaire?
- Which song was ranked number one by the most voters?
- How many votes are required for a song to have a majority of first-place votes?
- Do any of the songs have a majority of first-place votes?

Complete each preference table.

12. The student programming board is deciding which band to bring to campus for a student concert. The possibilities are Matisyahu, Lady A, and The Black Keys. Complete the following preference table so that The Black Keys win the election using the majority rule.

Rankings			
1st	The Black Keys	Lady A	Matisyahu
2nd	Lady A	Matisyahu	The Black Keys
3rd	Matisyahu	The Black Keys	Lady A
Total Votes	?	9	2

13. A hospital needs a new doctor. The hiring committee wants to get feedback from the current staff. They have the nursing staff and resident doctors rank the candidates in order of their preference. Complete the following preference table so that Perez is the staff's top choice using the plurality method.

Preference Table for New Doctor				
Rankings				
1st	Ross	Porter	Walker	Perez
2nd	Perez	Walker	Ross	Porter
3rd	Walker	Perez	Perez	Ross
4th	Porter	Ross	Porter	Walker
Total Votes	?	21	22	36

14. Phonetex is trying to decide which new feature to include in an advertising campaign: 12 megapixel camera, longer battery life, voice-activated commands, or reception quality. The company surveyed its current customers and asked them to rank the four features in order of preference. Complete the following preference table so that voice-activated commands is the favorite among current customers using the plurality with elimination method.

Preference Table for New Features				
Rankings				
1st	Camera	Battery Life	Voice-Activated	Voice-Activated
2nd	Reception Quality	Reception Quality	Reception Quality	Camera
3rd	Battery Life	Voice-Activated	Camera	Reception Quality
4th	Voice-Activated	Camera	Battery Life	Battery Life
Total Votes	44	11	?	56

15. Rhone County is electing a new county executive. The candidates are McMillian, Witt, and Longsdon. Complete the following preference table so that McMillian wins the election using the Borda count method.

Preference Table for Candidates			
Rankings			
1st	?	Witt	McMillian
2nd	?	McMillian	Longsdon
3rd	McMillian	Longsdon	Witt
Total Votes	601	632	650

16. *College Living Magazine* is compiling a list of the top five features of a college or university that students used when choosing an institution for higher education. They asked students to rank the importance of the following features of colleges and universities: size of the school, strength of the degree program, location, and number of student activities. Complete the following preference table so that location is the top influencer among students if the pairwise method of comparison is used.

Preference Table for Most Important Features					
		Rankings			
1st	Size	Location	Activities	Size	Strength
2nd	Activities	Strength	Size	Location	Activities
3rd	Strength	Size	Location	Strength	Location
4th	Location	Activities	Strength	Activities	Size
Total Votes	100	?	55	105	89

Calculate the number of pairwise comparisons that must be made in each election.

17. In the history club on campus, five students are running for the office of president.
18. Running for president of the National Association of College Students are the following students: L. Mayer, R. Schartz, M. Gillmore, L. Bilbo, R. Rosevear, K. Colletta, T. Yawn, S. Oswald, and K. Smith.

APPLICATIONS

19. The following table shows the number of votes for first, second, and third places that the top five college football players received in 2011 for the Heisman Trophy—College Football’s highest award. Determine the number of total points that each player would receive using the Borda count method if a first place vote receives 3 points, a second place receives 2 points, and third place receives 1 point.

Preference Table for 2011 Heisman Trophy Candidates				
Player	School	1st	2nd	3rd
Robert Griffin III	Baylor	405	168	136
Andrew Luck	Stanford	247	250	166
Trent Richardson	Alabama	138	207	150
Montee Ball	Wisconsin	22	83	116
Tyrann Mathieu	Louisiana State	34	63	99

Source: Heisman Trophy. "2011 - 77th Award Robert Griffin III Baylor University." <https://www.heisman.com/heisman-winners/robert-griffin-iii/>

20. In professional baseball, the Baseball Writers' Association of America hands out two Most Valuable Player (MVP) awards each year. One for the American League and one for the National League. The following table shows the voting breakdown for the top 10 players in the National League. Determine the number of total points that each player received using the Borda count method, given that a first place vote receives 14 points, a second place receives 9 points, third place receives 8 points, and on down to 1 point for tenth place.

National League MVP 2011											
Player	Team	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Ryan Braun	Milwaukee Brewers	20	12								
Matt Kemp	Los Angeles Dodgers	10	16	6							
Prince Fielder	Milwaukee Brewers	1	4	11	9	1	3		2		1
Justin Upton	Arizona Diamondbacks	1		8	11	6	3	1	1		1
Albert Pujols	St. Louis Cardinals			1	6	11	6	4		2	
Joey Votto	Cincinnati Reds			4	3	2	8	3	3	4	1
Lance Berkman	St. Louis Cardinals			1	2	6	3	7	2	4	3
Troy Tulowitzki	Colorado Rockies						3	4	8	5	4
Roy Halladay	Philadelphia Phillies			1		1	1	6	2		3
Ryan Howard	Philadelphia Phillies				1	3	1	1		1	3

Source: Baseball Writers' Association of America. "2011 NL MVP: Ryan Braun Slugs His Way to Award." <https://bbwaa.com/11-nl-mvp/>

21. In an attempt to choose a list of the "Best Places to Live in America," readers of a magazine were asked to rank eight towns. The following preference table summarizes the results of the survey.

Preference Table for Best Places to Live in America					
Rankings					
1st	Louisville, CO	Middleton, WI	Leesburg, VA	Leesburg, VA	Hanover, NH
2nd	Papillion, NE	Louisville, CO	Middleton, WI	Milton, MA	Leesburg, VA
3rd	Hanover, NH	Liberty, MO	Milton, MA	Louisville, CO	Milton, MA
4th	Milton, MA	Milton, MA	Liberty, MO	Papillion, NE	Solon, OH
5th	Liberty, MO	Papillion, NE	Hanover, NH	Solon, OH	Middleton, WI
6th	Solon, OH	Solon, OH	Papillion, NE	Hanover, NH	Papillion, NE
7th	Middleton, WI	Leesburg, VA	Solon, OH	Middleton, WI	Louisville, CO
8th	Leesburg, VA	Hanover, NH	Louisville, CO	Liberty, MO	Liberty, MO
Total Votes	345	231	211	202	16

- Determine a winner using the plurality method.
- Determine a winner using the plurality with elimination method.
- Which method do you think chooses the winner that best reflects the votes, or would you choose a different method? Explain your answer.

22. The following preference table summarizes the election results for city mayor of Clamptonville.

		Rankings			
1st	Costa	Little	Little	Torres	
2nd	Braugh	Allen	Costa	Braugh	
3rd	Little	Torres	Braugh	Allen	
4th	Torres	Braugh	Allen	Little	
5th	Allen	Costa	Torres	Costa	
Total Votes	1231	542	1001	654	

- Determine the winner for city mayor using the Borda count method.
 - Determine the winner for city mayor using the plurality with elimination method.
 - Determine the winner with the pairwise method of comparison.
 - Suppose that Braugh withdrew from the race before the votes were counted. Candidates ranked below him in any of the orders simply move up a ranking. Who would be declared the winner using the Borda count method with the new preference table? Is this a different winner than in part a.?
23. A university’s Department of Business is electing a new chairman. As part of the process, each candidate gives a brief question and answer session with the business students. The students are then asked to rank the candidates in order of preference. The following table shows the results of the student votes.

		Rankings					
1st	B	A	C	C	D	B	
2nd	A	B	D	B	A	A	
3rd	D	C	B	D	B	C	
4th	C	D	A	A	C	D	
Total Votes	43	12	13	21	51	9	

- Determine the student preference for chairman by using the pairwise method of comparison.
- Determine the student preference for chairman by using the plurality with elimination method.
- Determine the student preference for chairman by using the plurality method.
- If you were chosen to represent the students by announcing a preference for chairman, which candidate would you recommend? How would you defend your choice based on the answers to a.–c.?

24. The Tennessee Department of Transportation is considering a new bus route between its state capital, Nashville, and one of the other four largest cities: Knoxville, Memphis, Chattanooga, and Clarksville. Each of the 99 State Representatives was asked to rank the cities in order of preference for the new route. Their votes are summarized in the following table.

Preference Table for New Bus Route						
Rankings						
1st	Chattanooga	Memphis	Chattanooga	Knoxville	Clarksville	Memphis
2nd	Memphis	Knoxville	Clarksville	Memphis	Knoxville	Knoxville
3rd	Clarksville	Clarksville	Knoxville	Chattanooga	Memphis	Chattanooga
4th	Knoxville	Chattanooga	Memphis	Clarksville	Chattanooga	Clarksville
Total Votes	23	13	9	33	12	9

- Which city would receive the new bus route using the pairwise method of comparison?
 - Which city received the most first-place votes? Is this a majority?
 - Which city would receive the new bus route using the plurality with elimination method?
 - Which city would receive the new bus route using the Borda count method?
 - Explain which method you think is most “fair” in choosing the new bus route.
25. In the 1968 United States Presidential Election, three candidates shared most of the November 5 popular vote, although there was a small portion of votes for other candidates. The following table displays the breakdown of the popular vote along with the electoral vote for the election.

1968 US Presidential Election Results			
Presidential Candidate	Popular vote		Electoral Vote
	Count	Percentage	
Richard Milhous Nixon	31,783,783	43.40%	301
Hubert Horatio Humphrey	31,271,839	42.70%	191
George Corley Wallace	9,901,118	13.50%	46
Eugene McCarthy	25,634	0.00%	0
Other	217,624	0.30%	0
Total Votes	73,199,998		538

Sources: 1968 Presidential General Election Results, <http://www.uselectionatlas.org>.
Electoral Votes for President and Vice President, <http://www.archives.gov>.

- Who won the popular vote by plurality? Was it a majority?
- Who won the electoral vote by plurality? Was it a majority?
- Compare the outcomes of the popular vote between Nixon and Humphrey to that of the electoral vote for the two candidates.
- Suppose Wallace had dropped out of the election. Is it possible that Humphrey would have won the presidency? Explain your answer.