11.4 Exercises

CONCEPT CHECK

1. A _____ variable is a variable that can only take on certain values.

2. A _____ variable is a variable that can take on any value in a given interval.

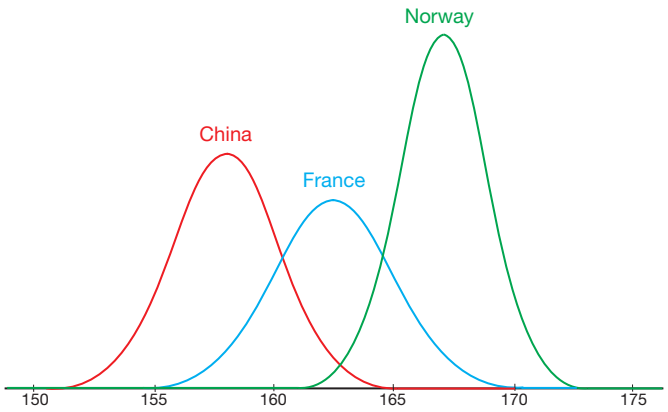
3. A standard score indicates how many _____ a data value lies away from the mean in a normal distribution.

4. True or False: According to the Empirical Rule, 99.7% of all data points in a data set lie within 2 standard deviations of the mean.

5. True or False: To find the area under a normal curve that lies to the right of a given z-value, look up the negative of the z-value in the Standard Normal Distribution table.

PRACTICE

6. Below are three normal distributions representing the height of adults in China, France, and Norway. Use the graph to answer the following questions.

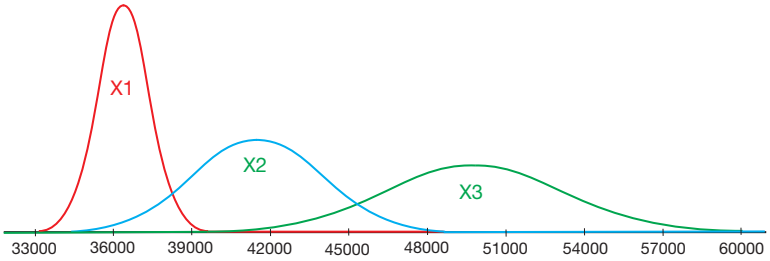


a. Determine which distribution has the largest mean and which has the smallest.

b. Determine which distribution has the largest standard deviation and which has the smallest.

c. What can you conclude about the heights of people in China and Norway?

7. Below are three normal distributions representing the prices of BMW X1, X2, and X3 models. Use the graph to answer the following questions.



a. Determine which distribution has the largest mean and which has the smallest.

b. Determine which distribution has the largest standard deviation and which has the smallest.

c. What can you conclude about the prices of BMW X1 and X2 models?

Calculate the standard score for each given value. Round your answer to the nearest hundredth.

8. $\mu = 57, \sigma = 11$

a. $x_1 = 63$

b. $x_2 = 38$

c. $x_3 = 58$

9. $\bar{x} = 1123, s = 241$

a. $x_1 = 1284$

b. $x_2 = 900$

c. $x_3 = 1364$

d. $x_4 = 1123$

10. $\bar{x} = 3.19, s = 0.06$

a. $x_1 = 3.13$

b. $x_2 = 3.22$

c. $x_3 = 3.00$

11. $\mu = 178.15, \sigma = 49.3$

a. $x_1 = 73.9$

b. $x_2 = 267.3$

c. $x_3 = 199.5$

Use the *z*-score formula to complete each table.

12. Find the missing value in each row of the table. Round answers to the same number of decimal places given in the table.

	z	x	μ	σ
a.		82.1	74.0	6.3
b.	1.05	162.3		8.9
c.	3.04		34.5	5.02
d.	–2.73	379	634	

13. Find the missing value in each row of the table. Round answers to the same number of decimal places given in the table.

	z	x	μ	σ
a.		4.33	6.10	2.04
b.	–2.39	–57		139.8
c.	0.58		118	21.2
d.	2.78	68	43	

Find the percentage of data points that lie below each z-score.

14. $z = -0.19$ **15.** $z = 1.46$ **16.** $z = 3.07$

17. $z = -2.22$ **18.** $z = 0$

Find the percentage of data points that lie above each z-score.

19. $z = 1.03$ **20.** $z = -1.87$ **21.** $z = -3.10$

22. $z = 2.84$ **23.** $z = 0$

Find the percentage of data points that lie between each pair of z-scores.

24. $z_1 = -1.00$, $z_2 = 1.00$ **25.** $z_1 = -2.40$, $z_2 = 1.73$ **26.** $z_1 = 2.00$, $z_2 = 3.00$

27. $z_1 = -3.01$, $z_2 = -0.56$ **28.** $z_1 = 0$, $z_2 = 2.61$

Find the percentage of data points that lie below z_1 and above z_2.

29. $z_1 = -1.10$, $z_2 = 1.10$ **30.** $z_1 = -2.84$, $z_2 = 2.84$ **31.** $z_1 = -1.75$, $z_2 = 0.53$

32. $z_1 = 1.09$, $z_2 = 2.88$ **33.** $z_1 = -0.01$, $z_2 = 0.02$

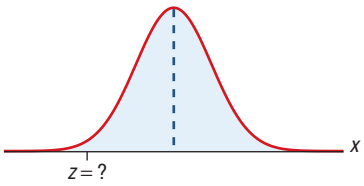
APPLICATIONS

34. Scores on a test have a mean of 73 and a standard deviation of 11. Steve has a score of 68. Convert Steve's score to a z-score.

35. The annual rainfall in a town has a mean of 47.22 inches and a standard deviation of 10 inches. Last year there was 51 inches of rain. How many standard deviations from the mean is that?

36. Mason's weekly poker winnings have a mean of \$144 and a standard deviation of \$51. Last week he won \$165. How many standard deviations from the mean is that?

37. The mean score for a set of data is marked by the dotted line on the following graph. Which value is a likely z-score for the indicated value? Choose from **a.** –2.1, **b.** 0, or **c.** 2.7.



38. The average IQ score for adults is 100 with a standard deviation of 15. Assume that the distribution of IQ scores is approximately normal.

a. Find the percentage of adults who have an IQ score less than 90.

b. Find the percentage of adults who have an IQ score which exceeds the mean by at least 15 points.

c. Find the percentage of adults who have an IQ score between 100 and 120.

d. Find the percentage of adults who have an IQ score less than 55 or more than 145.

39. Assume the weights of offensive linemen in the NFL follow a normal distribution with a mean of 300 pounds and a standard deviation of 12.3 pounds.

a. Find the percentage of linemen in the NFL who weigh more than 320 pounds.

b. Find the percentage of linemen in the NFL who weigh between 275 and 325 pounds.

c. Find the percentage of NFL linemen who weigh at least 260 pounds.

d. Find the percentage of NFL linemen who weigh at most 315 pounds.

40. Although there is some controversy around the precise average body temperature of adults, new data suggest that the mean is 98.2° with a standard deviation of 0.6° and has a bell-shaped distribution.

a. According to this distribution, approximately what percentage of body temperatures are between 97° and 99.4°?

b. Approximately what percentage of temperatures are greater than 98.8°?

c. Approximately what percentage of temperatures are no more than 98.2°?

41. In 2011, high school seniors had the following mean and standard deviation on the mathematics portion of the SAT exam: $\mu = 514$ and $\sigma = 117$.

a. Approximately what percentage of scores were greater than 397 but less than 631?

b. What two scores have approximately 95% of the data between them?

d. Describe where the best and worst 0.3% of scores lie.

42. The speeds of vehicles on a certain part of a highway are normally distributed with a mean of 60 mph and a standard deviation of 6 mph. Use the Empirical Rule to answer the following questions.

a. Approximately what percentage of vehicles travel at a speed between 54 mph and 66 mph?

b. Approximately what percentage of vehicles travel at a speed greater than 72 mph?

c. Approximately what percentage of vehicles travel at a speed not greater than 78 mph?

43. A machine is set to pour a mean of 497 mL of spring water in each bottle with a standard deviation of 2 mL. Assume that the distribution of the amount of water is bell-shaped. Use the Empirical Rule to answer the following questions.

a. Approximately what percentage of bottles filled by the machine contain more than 491 mL and less than 503 mL of water?

b. Between what two amounts of water will 68% of all bottles contain?

c. How much water will the top 2.5% of bottles contain? How much water will the bottom 2.5% of bottles contain?

44. Ava scored a 92 on a test with a mean of 71 and a standard deviation of 15. Charlotte had a score of 688 on a test with a mean of 493 and a standard deviation of 150. Which score was better with respect to their test?

45. Avery started training to run a 5K. Her first race was a 5K for charity. She finished in 37.3 minutes. The average race time for the charity run was 36.42 with a standard deviation of 1.73 minutes. In her second race, Avery finished in 36.5 minutes. The race had a mean time of 33.02 minutes with a standard deviation of 2.45 minutes. In which race did Avery place higher in the list of finishers?

46. Michael works at a company where the mean annual salary is $37,000 with a standard deviation of $6800. Daniel works at a company where the mean annual salary is $39,000 with a standard deviation of $8200. If they both have a salary of $35,000, whose salary is better compared to the other people in their respective companies? Assume that the salaries are normally distributed.

47. The average height of students in Class A is 59.6 inches with a standard deviation of 2.7 inches. The average height of students in Class B is 57.2 inches with a standard deviation of 2.4 inches. Amanda is in Class A and is 62.2 inches tall. Sofia is in Class B and is 60.1 inches tall. Which girl is taller compared to the rest of their respective classes?

48. The Intelligence Quotient (IQ) score is one way to measure human intelligence. It is determined using a set of tests and then standardized to have a normal distribution with the mean of 100 and the standard deviation of 15. What percentage of people have an IQ score higher than 125?

49. The average weight of newborn boys in the US is 3530 grams with a standard deviation of 577 grams. Assuming that the distribution of weights is normal, what percentage of newborn boys weigh less than 3 kilograms?

50. Suppose you want to determine the voltage in a circuit, and your voltmeter allows you to find the voltage with a standard deviation of 0.65 volts. If the exact voltage is 95 volts, what is the probability that you obtain a value greater than or equal to 96 volts?

51. The systolic blood pressure of healthy adults is normally distributed with a mean of 116 mm Hg and a standard deviation of 23 mm Hg. However, if the blood pressure is higher than 130 mm Hg at rest, it can be a sign of hypertension. With this in mind, what percentage of healthy adults have blood pressure higher than 130 mm Hg?

52. Assume that the retirement age of people in a certain country is normally distributed with a mean of 62.4 years and a standard deviation of 2.7 years. What percentage of people in the country retire before the age of 60?

53. Assume that the daily milk yield of a cow has a normal distribution with a mean of 25 liters and a standard deviation of 7.5 liters. What is the probability that the cow will produce less than 20 liters of milk in a day?

WRITING & THINKING

54. What is the minimum z-score that a piece of data would need to have in order to be in the top 10% of a normally distributed set of data?

55. What is an "average" z-score? Explain your answer.

56. What z-score represents the 1st quartile? 2nd quartile? 3rd quartile?

11.4 PROJECT

IS THAT NORMAL?

In Section 11.4, you learned how to use a normal distribution to calculate z-scores and answer questions about probability. We found that variables such as height, body temperature, weight, and blood pressure are common examples of data that are normally distributed. Because of the relative ease of calculating probabilities using normal distributions, it's tempting to assume, when collecting and analyzing data, that the variable or characteristic in question has a normal distribution. However, if the data set doesn't have a normal distribution, any conclusions we draw using standard scores is suspect at best. Fortunately, there is a method available to check the data beforehand and determine whether the population from which the data are taken has a normal distribution. It is called a *normality test*.

Normality Test

Step 1: Sort the data values so they are arranged from smallest to largest, and assign a rank to each value with 1 corresponding to the smallest value and n corresponding to the largest value, where n is equal to the sample size.