# 10.6 Exercises

## CONCEPT CHECK

1. A random variable's numeric value is determined by the outcome of a _____ _____.

2. Expected value is a _____ average.

3. True or False: In the scenario where you win \$1 if a fair six-sided die lands on a 1, \$2 if it lands on a 2, and so on, the expected value must be an integer 1 through 6.

4. True or False: The expected value of a game is the average of the maximum and minimum outcomes.

## PRACTICE

Calculate the expected value of each scenario. Round your answer to the nearest hundredth, if necessary.

5.

| $x_i$ | $P(x_i)$ |
|---|---|
| 1 | 0.21 |
| 2 | 0.58 |
| 3 | 0.06 |
| 4 | 0.15 |
| 5 | 0.0 |

6.

| $x_i$ | $P(x_i)$ |
|---|---|
| –\$1.50 | 0.3 |
| \$0.00 | 0.5 |
| \$2.75 | 0.1 |
| \$5.00 | 0.1 |

7.

| $x_i$ | $P(x_i)$ |
|---|---|
| 25 | $\frac{1}{3}$ |
| 15 | $\frac{2}{5}$ |
| 10 | $\frac{1}{15}$ |
| 5 | $\frac{1}{5}$ |

**8.**

| $x_i$ | $P(x_i)$ |
|---|---|
| 1 | 0.12 |
| 2 | 0.09 |
| 3 | 0.33 |
| 4 | 0.14 |
| 5 | 0.04 |
| 6 | 0.28 |

**9.**

| $x_i$ | $P(x_i)$ |
|---|---|
| \$2.25 | 0.1 |
| –\$1.43 | 0.2 |
| \$4.67 | 0.3 |
| \$3.11 | 0.3 |
| \$0.56 | 0.1 |

**10.**

| $x_i$ | $P(x_i)$ |
|---|---|
| 10 | $\frac{4}{21}$ |
| 5 | $\frac{1}{7}$ |
| 35 | $\frac{1}{6}$ |
| 15 | $\frac{3}{10}$ |
| 20 | $\frac{1}{5}$ |

**11.** The number of even numbers showing when a pair of standard six-sided dice are rolled.

| $x_i$ | $P(x_i)$ |
|---|---|
| 0 | $\frac{9}{36}=\frac{1}{4}=0.25$ |
| 1 | $\frac{18}{36}=\frac{1}{2}=0.5$ |
| 2 | $\frac{9}{36}=\frac{1}{4}=0.25$ |

**12.** The number of odd numbers showing when three standard six-sided dice are rolled.

| $x_i$ | $P(x_i)$ |
|---|---|
| 0 | 0.125 |
| 1 | 0.375 |
| 2 | 0.375 |
| 3 | 0.125 |

## APPLICATIONS

**13.** Alex wanted to learn how to play guitar, so his father bought him an inexpensive model for training. The number of strings $x_i$ that will require a replacement within a year is distributed as follows.

| $x_i$ | $P(x_i)$ |
|---|---|
| 0 | 0.262 |
| 1 | 0.393 |
| 2 | 0.246 |
| 3 | 0.082 |
| 4 | 0.015 |
| 5 | 0.002 |
| 6 | 0 |

Find the average number of strings to be replaced in a year.

**14.** A shooter fires at 5 targets. The number of hits $x_i$ is distributed as follows.

| $x_i$ | $P(x_i)$ |
|---|---|
| 0 | 0.01 |
| 1 | 0.08 |
| 2 | 0.23 |
| 3 | 0.34 |
| 4 | 0.26 |
| 5 | 0.08 |

Find the average number of targets the shooter will hit.

**15.** An insurance policy for a car costs \$1500 per year. According to the past data, the probability of having a minor accident during a year is 0.2, and the average amount of insurance claim is \$5000. The probability of having a serious accident is 0.01, and the average claim is \$30,000.

**a.** Find the average amount of the insurance claim.

**b.** Find the average income of an insurance company from the sale of one policy.

**16.** A horse in a circus is trained to jump over obstacles. It will jump over one obstacle with a probability of 60%, over two obstacles with a probability of 30%, and over three obstacles with a probability of 10%. Find the expected number of obstacles that the horse will jump over.

**17.** A student bought a second-hand laptop. During the first year of its usage, he will have to spend \$100 on repairs with the probability of 0.45 and \$200 with the probability of 0.35. Find the average amount the student will have to spend on repairs in the first year.

**18.** Suppose that there is a 60% chance that Michael will go to the gym twice a week, a 35% chance that he will go once, and the remaining 5% that he will not go to the gym.

**a.** What is the expected value for the number of times Michael goes to the gym in a week?

**b.** Assuming there are 4 weeks in a month, what is the average number of times Michael goes to the gym in a month?

**19.** A coffee shop analyzed the sales for the latest month and got the following data. 55% of customers spent about \$5, 25% spent about \$9, and the remaining 20% spent about \$11.

**a.** What is the expected amount that a customer will spend?

**b.** If the average number of customers per day is 80, what is the average daily revenue of the coffee shop?

**20.** A device consists of four independently operating nodes. The probability that exactly one node will fail in an experiment is 0.3. The probability of two nodes failing at once is 0.05. The probability of three nodes failing simultaneously is 0.03. The probability of all four nodes failing at the same time is negligible.

**a.** Find the expected number of failing nodes in the experiment.

**b.** If a similar test is performed simultaneously on three devices, what is the expected number of failing nodes?

**21.** A large entrepreneur plans to invest in a certain project. According to forecasts, the profit from the project will be \$20,000 with a probability of 0.5, \$30,000 with a probability of 0.2, or \$25,000 with a probability of 0.3. What is the expected profit on this investment?

**22.** Suppose Piper eats out twice a week 15% of the time, she eats out once a week 35% of the time, and she doesn't eat out anytime during the week 50% of the time. What is the expected value for the number of times Piper eats out during a week?

**23.** Suppose that you and a friend are playing cards and decide to make a bet. If you draw two aces in succession from a standard deck of cards without replacing the first card, you win \$50.00. Otherwise, you pay your friend \$10.00.

**a.** What is the expected value of your bet?

**b.** If the same bet was made 25 times, how much would you expect to win or lose?

**24.** A European roulette wheel has only one green slot instead of two. Using Example 2 from this section as a guide, calculate the expected winnings on a European roulette wheel if a player bets \$1.00 on red to play the game.

**25.** Jim likes to day-trade on the Internet. On a good day, he averages a $1100 gain. On a bad day, he averages a $900 loss. Suppose that he has good days 25% of the time, bad days 35% of the time, and the rest of the time he breaks even.

**a.** What is the expected value for one day of Jim's day-trading hobby?

**b.** If Jim day-trades every weekday for three weeks, how much money should he expect to win or lose?

**26.** A university in town is raffling off $20,000 for student scholarships. You can buy one ticket for $10, three tickets for $25, or five tickets for $40. Assume that the university sells 10,000 tickets.

**a.** Find the expected value for each of the three ticket options: purchasing just one ticket, purchasing three tickets, or purchasing five tickets.

**b.** Should you buy one, three, or five tickets in order to maximize the money you expect to have at the end of the raffle?

**27.** You need to borrow money from your sister. She's feeling quirky on the day you ask and says she wants you to flip a coin. Heads, you get $15, tails you get $5. Thinking this is weird, you ask your mother for money instead. She says she'll let you roll a die and she'll give you $2 times the number that appears on the die. Before agreeing to either of these unique offers from the "mathy" folk in your family, you decide to see which is the better offer by calculating the expected value for each method (realizing that you too fit the bill of a "mathy" member of your family). Which offer should you take? Explain your reasoning.

**28.** Assume that stock in Degree Compass, a predictive analytics company in higher education, returns the percentages shown in the table.

**Degree Compass Stock Returns**

| Annual Return Rate | Probability |
|---|---|
| 15% | 0.17 |
| 30% | 0.51 |
| 45% | 0.32 |

Calculate the expected value of the return rate for stock in Degree Compass.

**29.** During the NCAA basketball tournament season, affectionately called *March Madness*, part of one team's strategy is to always foul their opponent's tall forward. Because he is so tall, he makes 57% of shots he takes close to the basket. However, when he is fouled, his free-throw shooting percentage is only 51.5%. The shots he makes close to the basket are worth two points and each of the two free throw shots after being fouled are worth one point.

**a.** Calculate the expected value of the number of points the forward makes when he takes a shot close to the basket.

**b.** Calculate the expected value of the number of points the forward makes when he shoots two foul shots.

**c.** Based on these expected values, is fouling the tall forward a good strategy? Explain your answer.

**30.** On your next multiple-choice test, each question has four incorrect answers and one correct answer to choose from. Your professor tells you that each correct answer you make, you receive 1 point, but you lose $\frac{1}{4}$ point for each incorrect answer.

**a.** What is your expected gain or loss on a question if you have no idea of the correct answer and end up simply guessing?

**b.** What is your expected gain or loss if you guess on all 25 questions?

## 10.6 PROJECT

## EXPECTED VALUE AND INSURANCE PREMIUMS

When you purchase a homeowners insurance policy, you pay a certain amount of money (the premium) to the insurance company. If nothing happens to your home, the insurance company keeps the money. If you make a claim, the company will pay to fix your home, usually spending a lot more than what your premium was. Due to this set up, the insurance company makes money on some policies and loses money on others.

In this activity, we will investigate the way an insurance company computes insurance premiums.

Suppose that in a certain neighborhood, an insurance company has used historical data to determine that the probability of a house fire occurring at a home over the course of one year is 0.02%.

**1.** What is the probability that there will not be a fire at a house in the neighborhood over the course of one year?

Assume that the insurance company charges a $300 annual premium for fire insurance. If there is a fire, the insurance company will pay out $200,000 to the homeowner.

**2.** Fill in the table below. What happens when there is no fire? Explain why this is.

**Probability and Payout for Insurance Company**

| Event | Probability | Payout – Premium |
|---|---|---|
| Fire | | |
| No Fire | | |

**3.** Determine the expected value that the insurance company will pay out.

**4.** What does the expected value say about the average loss or gain for the insurance company on each policy they sell?

**5.** How much would the insurance company expect to earn on average if it sold 10,000 policies in a year?