10.4 Exercises

CONCEPT CHECK

1. If A and B are events that have some outcomes in common, then the probability that A or B will happen is calculated by adding the individual probability of each and then _____ the probability that both events occur simultaneously.

2. If two events have __ outcomes in common, they are called mutually exclusive events.

3. The probability of events A and B happening is equal to the probability of A happening times the probability of B happening IF the two events are _____.

4. _____ probability is the probability of one event happening given that another event has already happened.

5. True or False: If the result of one event influences the probability of a second event, the two events are independent.

PRACTICE

Determine whether each situation contains mutually exclusive events.

6. Let event A consist of numbers that are prime. Let event B consist of numbers that are multiples of 5.

7. Let event A consist of the multiples of 3. Let event B consist of the factors of 10.

8. Let event A consist of randomly selecting a member of a concert audience who can play the guitar. Let event B consist of randomly selecting a member of a concert audience who cannot play the guitar.

9. Let event A consist of rolling a die and getting a 3. Let event consist of rolling a die and getting a 6.

Determine whether each situation contains independent events.

10. The color of car driven by three randomly chosen classmates.

11. A password must be six characters long with no repeated characters. Are the choices of consecutive characters independent?

12. There are 15 board members, of which seven are men and eight are women. Two randomly chosen members will serve on the United Way campaign committee. If you wish to find the probability that both members chosen are the same sex, do you treat these selections as independent events?

13. Are receiving a bill in Monday's mail and receiving a letter from your grandparents in Monday's mail independent events?

14. Naomi and Amelia both put two business cards into the basket at a coffee shop. The shop owner selects three cards from the basket. Are the two events that Naomi's card is chosen and Amelia's card is chosen independent?

APPLICATIONS

Calculate the probability of each set of events that are not mutually exclusive. Round your answer to the nearest millionth, if necessary.

15. A pair of dice is rolled. What is the probability that the sum of the numbers is an even number or a multiple of 3?

16. A bag of eleven marbles contains five marbles with red on them, three with green on them, seven with black on them, and four with black and red on them. What is the probability that a randomly chosen marble has either black or red on it?

17. What is the probability that a card selected from a deck will be either an ace or a spade?

18. The following is a table showing the results of a poll taken on campus.

Will You Vote in the Upcoming Election?

	Male	Female
Yes	16	24
No	19	11
Not decided	21	22

a. What is the probability that a randomly selected student from this poll would be a male who has not decided whether he will vote in the upcoming election?

b. What is the probability that a randomly selected student from this poll is female or will not vote in the upcoming election?

c. What is the probability that a randomly selected student from this poll has decided to vote in the upcoming election?

19. Out of a class of 30 students, there are 16 students who study Latin, 21 who study German, and 7 who study both. What is the probability that a randomly selected student from the class will study only Latin?

20. Of the 11 instructors in the English department, four are new to the department and three are female. However, there is only one who fits all of the descriptions. Find the probability that if you randomly choose a course taught by these instructors, you get either a new instructor or a female instructor.

21. The following is a table representing the students who are on the Student Government Board.

Students on the Student Government Board

	On-Campus Housing	Off-Campus Housing
Freshman	3	1
Sophomore	3	2
Junior	2	3
Senior	0	3
Graduate Student	0	2

Find the probability that a randomly chosen member of the Student Government Board is either a sophomore or lives in on-campus housing.

22. A box of a dozen chocolates contains 8 chocolates that contain nougat, 6 that contain caramel, and 2 that contain both caramel and nougat. Determine the probability that a randomly selected chocolate will contain only nougat.

23. A pair of dice is rolled. What is the probability that the sum of the numbers is divisible by 5 or a prime number?

24. A bag of meeple game pieces contains 10 meeples with yellow on them, 20 meeples with blue on them, 12 meeples with green on them, and 6 meeples that are striped yellow and blue. What is the probability that a randomly chosen meeple has either yellow or blue on it?

25. What is the probability that a card randomly selected from a standard deck will be either a face card or a diamond?

Calculate the probability of each set of mutually exclusive events. Round your answer to the nearest millionth when necessary.

26. Suppose that the probability of obtaining zero defective items in a sample of 50 items off the assembly line is 0.34 while the probability of obtaining 1 defective item in the sample is 0.46. What is the probability of the following?

a. Obtaining no more than one defective item in a sample.

b. Obtaining more than one defective item in a sample.

27. A pair of dice is rolled. What is the probability that the sum of the numbers is either 7 or 11?

28. A single letter from the word MISSISSIPPI is chosen. What is the probability of choosing an S or an I?

29. What is the probability that a card selected from a deck will be either an ace or a queen?

30. A reporter for an international newspaper is given an assignment that is randomly chosen from the following destinations worldwide: 13 continental United States assignments, 7 South American assignments, 21 European Union assignments, and 5 Asian assignments. Find the probability that he gets an assignment in Asia or South America.

31. The following table shows the breakdown of opinions for both faculty and students in a recent survey about the new restructuring of the campus to be a walking campus.

Survey Results on Restructuring Campus to a Walking Campus

	Favor	Oppose	Neutral	Total
Faculty	12	4	3	19
Student	33	57	28	118
Total	45	61	31	137

a. Find the probability that a randomly selected person is either a faculty member in favor of the change or a student who has an opinion either for or against.

b. Find the probability that a randomly selected person is either neutral or in favor of the restructuring.

32. The probability of the stoplight being green at the intersection of Meeting Street and Main Street is 0.55, while the probability of it being yellow is 0.15. Find the probability that the light is red when you get to the intersection of Meeting Street and Main Street. Assume that the light will be working and will be a solid color: red, yellow, or green.

33. In a box of pens and pencils, the probability of randomly choosing a sharpened pencil is 0.54 and the probability of randomly choosing a pen from the box is 0.39. Find the probability of randomly selecting either an unsharpened pencil or a pen from the box.

34. A couple is deciding where to spend their honeymoon. They've narrowed down their options to 4 locations in the United States, 3 locations in Mexico, and 3 locations in the United Kingdom. What is the probability of the couple choosing a honeymoon location in Mexico?

35. A community extracurricular soccer league uses volunteers for the coaching positions. This year, only 7 of the 16 volunteers have previous coaching experience. What is the probability that a team will have a coach with no previous coaching experience?

Calculate the probability of each set of independent events. Round your answer to the nearest millionth, if necessary.

36. Suppose the probability that my pet will be alive in five years is 0.65 and the probability that my cousin's pet will be alive in five years is 0.48. Find the probability that both of these pets will be alive in five years assuming that they are independent events.

37. Two dice are thrown. Find the probability of getting an even number on the first die and an odd number on the second die.

38. The following table shows the student demographics for a sociology class.

Sociology 101 Student Demographics

	Male	Female
Freshman	3	11
Sophomore	4	9
Junior	0	3
Senior	1	0

a. Find the probability that a randomly selected student from the class is a male.

b. Find the probability that if two students are randomly selected, without replacement, the first is a female junior and the second is a male sophomore.

39. Find the probability of choosing a heart and then an ace from a standard deck of cards with replacement.

40. On any given day at the beach, there is a 49% chance of precipitation. What is the probability that you will get precipitation for three days in a row on your beach vacation? Assume that the weather on a particular day at the beach is independent of the weather the day before.

Calculate the probability of each set of dependent events. Round your answer to the nearest millionth, if necessary.

41. A bag of marbles contains 7 marbles with green on them, 10 marbles with pink on them, 6 marbles with blue on them, and 3 marbles with both pink and blue on them. Two marbles are selected from the bag in a row without replacement. Determine the probability that a marble with blue and pink on it was selected first and a marble with blue on it was selected second.

42. A group of candidates are sitting in a lobby waiting to be interviewed. Six of the candidates have previous experience related to the job, eight of the candidates have a bachelor's degree, and three of the candidates have both previous experiences and a bachelor's degree. Determine the probability that the next candidate called has a bachelor's degree and previous experience followed by a candidate that only has a bachelor's degree.

43. A group of friends are deciding which two movies to watch for movie night. They narrowed their options down to 6 comedies, 5 romances, 4 action films, and 3 romantic comedies. What is the probability that the first movie they select is a comedy and the second movie they select is a romantic comedy? (Assume they will not watch the same movie twice in a row.)

44. Suppose you are trying out different types of mouthwash and the store you visit has 10 mouthwashes that contain alcohol, 7 mouthwashes that contain fluoride, 3 mouthwashes that contain neither alcohol nor fluoride, and 5 mouthwashes that contain both alcohol and fluoride. If you choose two different mouthwashes at random, what is the probability that the first mouthwash you select contains both alcohol and fluoride and the second mouthwash you select contains fluoride?

Calculate each conditional probability. Round your answer to the nearest millionth, if necessary.

45. A swim team consists of four boys and three girls. A relay team of four swimmers is chosen at random from the team members. What is the probability that there are two boys on the relay team given that there are two girls on the relay team?

46. Emma is playing Monopoly, a game played with two dice. What is the probability that the sum of the two dice she rolls is less than 4 given that she rolls an odd number?

47. Hunter bets his friend that he can draw two aces in a row from a standard deck of cards. What is the probability that Hunter draws a second ace given that his first card was an ace?

48. The probability that a student passes Intermediate Algebra is 0.55. The probability that a student passes College Algebra given that they pass Intermediate Algebra is 0.70. What is the probability that a student passes both College Algebra and Intermediate Algebra?

49. On each point in racquetball, a player is allowed two serves. Suppose while playing racquetball, Tim gets his first serve in about 75% of the time. He gets his first serve in and wins the point about 50% of the time. What is the probability that he wins the point, given that he gets his first serve in?

50. Suppose 170 people were randomly selected and asked whether their job earnings during the past month were less than $4000 or greater than or equal to $4000. Their responses are given in the following table.

	Earnings		
Age	**<$4,000**	**≥$4,000**	**Unemployed**
18–27	14	7	9
28–37	16	17	3
38–47	4	31	0
48–57	6	28	2
>58	12	10	11

If one response was selected at random, find the probability that it indicated that the person earned more than $4,000.

51. Suppose 170 people were randomly selected and asked whether their job earnings during the past month were less than $4000 or greater than or equal to $4000. Their responses are given in the following table.

	Earnings		
Age	**<$4,000**	**≥$4,000**	**Unemployed**
18–27	14	7	9
28–37	16	17	3
38–47	4	31	0
48–57	6	28	2
>58	12	10	11

If one response was selected at random, find the probability that it indicated that the person earned at least $4,000 given that they were of age 38 to 47.

52. Suppose 170 people were randomly selected and asked whether their job earnings during the past month were less than $4000 or greater than or equal to $4000. Their responses are given in the following table.

	Earnings		
Age	**<$4,000**	**≥$4,000**	**Unemployed**
18–27	14	7	9
28–37	16	17	3
38–47	4	31	0
48–57	6	28	2
>58	12	10	11

If one response was selected at random, find the probability that it indicated that the person was of age 18 to 27 given that they were unemployed.

Use Bayes' Theorem to calculate each probability. Round your answer to the nearest millionth, if necessary.

53. One of the world's most common parasites is Toxoplasma gondii which causes a disease called Toxoplasmosis. It is possible that 50% of the world's population is infected by toxoplasmosis. Cats spread Toxoplasma, and about 26% of the world population own cats. If 42% of people infected by toxoplasmosis own a cat, what is the probability that a cat owner is infected by toxoplasmosis?

54. The company expects the probability of economic recovery to be 4% and the probability of the company's revenue growth due to economic recovery to be 78%. Regardless of whether the economy grows, the company's revenue will grow with a 32% probability. What is the probability that the economy has risen if the company's revenue has grown?

55. Suppose 34% of students at a certain school know French and 23% know German. The share of French-speaking students who also know German is 12%. What is the probability that a student speaks French given that he or she speaks German?

56. Major depressive disorder affects approximately 7.1% of the US population aged 18 and older. Moreover, depression is the cause approximately two-thirds of the reported suicides in the US every year. Given that 0.015% of Americans die by suicide each year, what is the probability of a person committing suicide if they have depression?

10.4 PROJECT

THE PROBABILITY OF SPAM FILTERING

According to the website statista.com, 28.5% of all email traffic in 2019 was made up of spam—those pesky, useless, and potentially dangerous messages that just clog our email inboxes. Most email servers these days can filter spam automatically. Spam messages often have certain suspicious phrases in the subject lines. For example, "You Have Been Selected" is one such phrase.

An incoming email is checked for key elements, such as this phrase, then the server decides whether to put the email in your mailbox or send it to the spam folder.

In this activity, you will estimate the probability that an email with a specific subject line is classified as spam.

Let $P(S)$ be the probability that an email you have received is spam and $P(S^c)$ be the probability that the email is not spam.

1. According to statista.com, what were the values of $P(S)$ and $P(S^c)$ in 2019?

Let's assume that 10% of all spam messages contain the word *selected* in the subject line. In order to simplify our notation, we will name the events as follows.

S = email is spam

S^c = email is not spam

W = subject line contains the word *selected*

W^c = subject line does not contain the word *selected*

2. Express the statement "10% of all spam messages contain the word *selected* in the subject line" as a conditional probability.

3. We also will assume that 0.5% of all nonspam messages also contain *selected* in the subject line. Express the previous statement as a conditional probability.

Since every message can be classified as either spam or not spam, the probability that any message has *selected* in the subject line is the following.

$$P(W) = P(W \mid S)P(S) + P(W \mid S^c)P(S^c)$$

4. Compute the value of $P(W)$.

5. Finally, determine the probability that an email is spam, knowing it has the word *selected* in the subject line. (**Hint:** Use Bayes' Theorem.)