

10.2 Exercises

✓ CONCEPT CHECK

- When there are several outcomes at each stage of an experiment, as we saw with the start watch example in the first section, you can _____ together the number of possible outcomes at each stage of the experiment in order to obtain the total number of outcomes for the experiment.
- An experiment where repetition of outcomes is permitted is considered to be an experiment _____.
- The factorial of a number is the _____ of all the positive integers less than or equal to that number.
- When a selection of objects from a group is being made and order is not important, this is a _____. If order is important, it is a _____.
- True or False: $0! = 1$.

💡 PRACTICE

Evaluate each factorial expression.

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|-------------------------------|-------------------------------|------------------------|
| 6. $5!$ | 7. $9!$ | 8. $\frac{8!}{6!}$ |
| 9. $1!$ | 10. $0!$ | 11. $\frac{10!}{2!4!}$ |
| 12. $\frac{6!}{2!3!}$ | 13. $\frac{7!}{4(9-6)!}$ | |
| 14. $\frac{12!}{10!(12-10)!}$ | 15. $\frac{17!}{12!(15-10)!}$ | |

Evaluate each permutation or combination.

- | | | | |
|---|---|-------------------------------|-------------------------------|
| 16. ${}_8P_2$ | 17. ${}_7P_4$ | 18. ${}_5P_1$ | 19. ${}_4P_4$ |
| 20. ${}_3C_2$ | 21. ${}_{30}C_1$ | 22. $\frac{{}_3C_2}{{}_3P_2}$ | 23. $\frac{{}_5P_3}{{}_5C_3}$ |
| 24. $\frac{{}_5C_3}{{}_5C_2}$ | 25. $\frac{{}_5P_3}{{}_5P_2}$ | 26. $\frac{{}_6P_4}{{}_6C_2}$ | 27. $\frac{{}_7C_3}{{}_7P_3}$ |
| 28. ${}_7C_4 + {}_7C_3 + {}_7C_2 + {}_7C_1$ | 29. ${}_7P_4 + {}_7P_3 + {}_7P_2 + {}_7P_1$ | | |

 APPLICATIONS

30. How many three-digit area codes can be made from the digits 0 through 9? Assume that the digits may repeat and that area codes beginning with 0 are allowed.
31. How many three-digit area codes can be made from the digits 0 through 9 if the first digit is not allowed to be a 0 and the digits are allowed to repeat?
32. How many six-character password codes are possible if you are allowed both lowercase letters and the digits 0 through 9, and repeating characters are allowed?
33. When ordering a new e-reader, you have several choices to make. You can choose from five price ranges, decide between Wi-Fi and 3G, choose to have a one-year or three-year warranty, and pick a black, white, or silver casing. How many possible e-readers are there for you to choose from?
34. The Youngs are planning their next family night. They always have dinner out somewhere and then do something fun together. There are two adults and four boys in the family. Each family member is allowed two meal suggestions, and each boy is allowed three activity suggestions. Assuming no family members choose the same thing, how many different family night possibilities are there?
35. If there are seven children lining up for recess, how many different ways can they line up?
36. The college's soccer team will play 11 games next fall. Each game can result in one of three outcomes: a win, a loss, or a tie. Find the total possible number of outcomes for the season record.
37. Harper is deciding on her schedule for next semester. She must take each of the following classes: English 102, College Algebra, History 102, and Biology 101. If there are 16 sections of English 102, 14 sections of College Algebra, 7 sections of History 102, and 12 sections of Biology 101, how many different possible schedules are there for Harper to choose from? Assume there are no time conflicts between the different classes.
38. If there were no restrictions on the use of the number zero, in theory, how many seven-digit telephone numbers are possible?
39. How many four-digit even whole numbers exist?
40. In the new ice creamery, several choices need to be made before tasting “a little bit of heaven,” as the advertisement suggests. First, there are three cup sizes to choose from, then 39 different ice cream flavors to decide from, and finally several mix-ins to choose from: 10 candy bars, 8 fruits, 8 nuts, and 10 cookies or cakes. If you want a medium cup with one flavor of ice cream and two mix-ins, one candy bar, and one nut, how many possible choices are there for you to have your “taste of heaven”?

41. As part of a fundraising campaign, a city's parks and recreation department is selling bicycle license plates featuring the blueprint of a future park. Each license plate has unique and consists of 2 capital letters followed by 2 numbers (from the digits 0 through 9).
- Determine the number of possible unique license plates if letters and numbers are allowed to be duplicated.
 - In an effort to decrease the number of license plates available, a decision was made to not allow any duplicates of letters or numbers on the license plates. Determine how many unique license plates are possible under this new rule.
42. A 5th grade class is choosing books to read for upcoming book reports. Each student has to complete two book reports. They must select two books from a selection of 20 books, then they must pick two book report formats from the following options: written report, poster, or shoebox diorama.
- If the teacher does not specify that the books and report types must be unique, how many possible combinations are there for a student to choose from?
 - If the teacher specifies that the books and the report types must be unique, how many possible combinations are there for a student to choose from?
43. A bakery has 10 different cupcake flavors and sells boxes that hold 4 cupcakes. How many different ways are there to fill a box with 4 cupcakes?
44. Suppose a bakery has 10 different cupcake flavors and sells variety boxes of 4 cupcakes. The variety boxes have the restriction that the same flavor cannot be added to the box back-to-back. (For example, if a chocolate cupcake with vanilla frosting is the first cupcake placed into the box, then the next cupcake placed into the box cannot be a chocolate cupcake with vanilla frosting.) How many different variety boxes of cupcakes can be created?
45. To determine the theme for a short story, an author places 15 different plot ideas into a box and draws out 3, placing each idea in order on a table. The author then writes a story that progresses through those 3 ideas. Determine how many possible short story topics can be created using this method.
46. A personal trainer creates a challenge for his clients where each client must choose one of five exercises to perform each day for three weeks (21 days). How many different ways can a client complete the month-long challenge?

Determine whether to use a permutation or combination to answer each question, and then determine the total number of outcomes.

47. There are 10 board members on the Community Arts Council. In how many ways can a president and treasurer be chosen? Assume that no member can hold both positions at the same time.
48. In how many ways can a committee of five people be chosen from a pool of 120 employees?

49. If there are 84 runners in a race, in how many ways can 1st, 2nd, and 3rd place ribbons be given out?
50. Elliot has to submit three photographs for the school art show. This semester he has taken 29 photographs that he thinks are show-worthy. In how many ways can he choose the photographs to submit?
51. Avery was born on 10/15/1995. How many eight-digit codes could she make using the digits in her birthday?
52. There are 18 tenured faculty in the biology department on campus. The department needs one tenured faculty member to facilitate undergraduate research, one member to supervise graduate advising, and one to coordinate grant proposals. In how many ways can these tasks be assigned, if a member may be appointed to only one duty?
53. A service organization on campus needs a group of six students from their organization's membership of 123 students to serve as program attendants at graduation. In how many ways can the attendants be chosen?
54. In how many ways can the letters in the word STATISTICS be arranged?
55. In how many ways can the letters in the word TENNESSEE be arranged?
56. A bookstore has a buy three get one free sale on used books. If the current selection of used books contains 18 unique books, how many different groups of four books can be chosen to qualify for the sale?
57. If a talent show has 12 contestants, in how many ways can the prizes for first, second, and third place be awarded?
58. A company is considering using a new productivity management system and wants to test out the latest system on a group of four employees. If the company has 50 employees, how many ways can the test group be selected?
59. A designer needs to select four colors to use to create a poster for an upcoming event. After some brainstorming, she comes up with eight different colors to use, all of which go together. How many different ways can she choose four colors from the eight possible colors?
60. An ice cream store can create 25 different flavors onsite but their display freezer limits the store to only selling 10 flavors per day. How many different flavor combinations can be created per day?
61. A movie theater's newest promotion is an early-bird classics week where a different classic movie is played each morning for a week (seven days). If the theater has the rights to show twenty different classic movies, how many different ways can seven movies be shown during the week?
62. The Renfield family is staying at a resort for their annual vacation. As part of the resort package, the family can choose three of twenty different excursions to go on from options such as zipline tours, guided hikes, and sundown bat watching. How many ways can the family choose which excursions to go on?

63. An instructor is creating a 10-question quiz where 6 questions are true and 4 are false. How many different ways can the questions be arranged in the quiz?
64. Naval signs are made by arranging colored flags in a vertical line. Using 6 flags, how many distinct symbols can be made if you have 2 red flags, 3 green flags, and 1 blue flag?
65. Allison purchases a variety pack of mochi ice cream that contains the following flavors: 2 salted caramel, 2 mango, 4 vanilla, and 4 chocolate. If she plans to eat one mochi per day, how many flavor arrangements are possible?

 **WRITING & THINKING**

66. Without calculating the permutations, decide which of the following words would produce the greatest number of four-letter arrangements.
- PASS
 - TEST
 - FAIR
 - FREE
67. Determine how many ways a group of three students from your class could be selected.

10.2 PROJECT

PASSWORDS AND SECURITY: DO HACKERS TEST ALL POSSIBILITIES?

Mark Zuckerberg, the founder and CEO of Facebook, is known to have kept “dadada” as his Twitter and Instagram passwords. Simple passwords of this type are a treat for hackers trying to gain access to an account. These passwords can be guessed in a short amount of time by what is called a dictionary attack. In this case, an intruder tests common words and their combinations until the correct password is found.

What about more secure passwords? Is it possible to simply test all possibilities for a password and gain access to someone’s account? In this activity, you will estimate the time it would take to correctly guess a social media account password.

Assume that a social media platform requires your password to have exactly 8 characters selected from any of the 26 letters of the alphabet, the numbers 0 through 9, and any of the characters #, \$, %, or &. In order to simplify our computations, let’s assume repetitions are allowed.

- Determine the number of distinct passwords that can be created using the requirements described above. The number is very large, so you should use a calculator or computer for your computation.