1.3 Exercises

CONCEPT CHECK

1. True or False: When understanding a problem, it can be helpful to restate the problem using different phrasing.

2. True or False: When devising a plan to solve a problem, it can be helpful to look for patterns in the given data.

3. True or False: When carrying out the plan to solve a problem, if the plan you devised doesn't work, it's not worth creating a new plan and trying again.

4. True or False: It is useful to look back over your problem-solving process to help ensure that the conclusion you reached makes sense for the situation.

APPLICATIONS

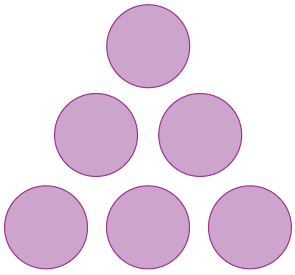
5. A child snuck out of the house one evening and bought a bag of candy. His older sister caught him sneaking back into the house and said that if he handed over one-fourth of the candy, she wouldn't tell their parents. So he gave her the candy. His younger sister saw him sneaking back to his room and said if he gave her twenty percent of the candy, she wouldn't tell their parents, so he gave her some candy. His younger brother found him hiding the candy in the room they shared and said if he gave him half of the candy, he wouldn't tell their parents. So he gave him some candy. After the series of negotiations, the child had 6 pieces of candy to hide. How many pieces of candy were in the bag when he returned home?

6. An employee baked a fresh batch of cookies and brought them to work the next day. She told a coworker that she'd give him one-fourth of the cookies plus three if he went to a meeting in her place. He accepted. She then told another coworker that she'd give her one-third of the remaining cookies plus two if she created a PowerPoint presentation for a meeting later in the week. She accepted. The employee finally told another coworker that she'd give him three-fourths of the remaining cookies if he researched a new product line for her. He accepted. After these work negotiations, she had three cookies left. How many cookies did she take to work with her that day?

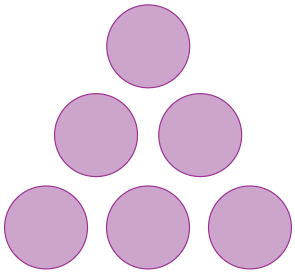
7. Complete the following magic square using the numbers 1 through 16, where the sum of every row, column, and diagonal is the same.

16	2		
5	11		8
	7	6	
4	14		1

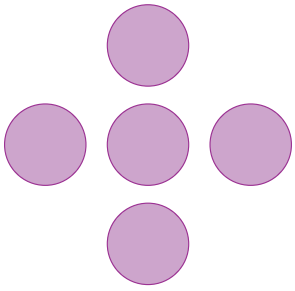
8. Arrange the numbers 1, 2, 3, 4, 5, and 6 in the given circles so that the sum of the numbers of each side is equal to 9.



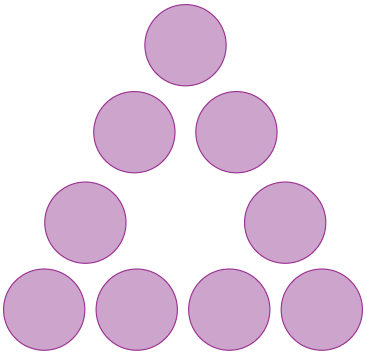
9. Arrange the numbers 1, 2, 3, 4, 5, and 6 in the given circles so that the sum of the numbers of each side is equal to 10.



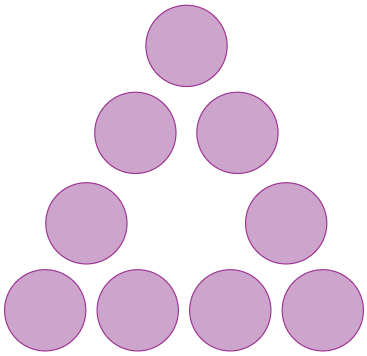
10. Place the digits 1, 2, 3, 4, and 5 in the circles so that the sums across (horizontally) and down (vertically) are equal.



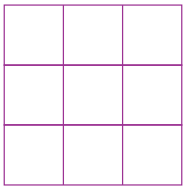
11. Arrange the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 in the given circles so that the sum of the numbers of each side is equal to 19.



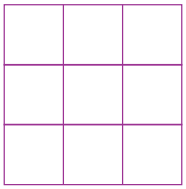
12. Arrange the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 in the given circles so that the sum of the numbers of each side is equal to 23.



13. Place the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 in the magic square so that the sum of the numbers in each row, column, and diagonal is equal to 15.



14. Use the digits 0, 1, 2, 3, 4, 5, 6, 7, and 8 to create another magic square so that the sum of the numbers in each row, column and diagonal is equal to 12.



15. A rectangle has an area of 72 square inches. Its length and width are whole numbers.

a. What are the possible dimensions of the rectangle?

b. Which of those dimensions yield a rectangle with the smallest perimeter?

16. Carlos is placing an order online for curbside pickup and notices a promotion that if he spends $75 on his order (before tax), he will receive a $20 gift card for a future purchase. He needs to purchase a pack of LED lightbulbs for $32.99, a bottle of laundry detergent for $15.99, a throw blanket for $24.99, and a travel coffee mug for $17.99. Can he purchase three of the four items with this online order and use the gift card to purchase the remaining item? If so, which three of items should he purchase?

17. Marcus is considering which presents to buy for his son's birthday. He wants to limit the overall cost to $100, including tax, so he is aiming for the gift total to be no more than $90 before taxes. His son has the following toys on his wish list: fire station playset ($19.69), dinosaur expedition playset ($22.99), toy soldiers set ($35.99), construction truck set ($31.99), miniature racetrack with cars ($9.99), and a medieval castle ($85.89). If Marcus wants to purchase as many gifts as possible and stay within the set budget, which toys should he buy?

18. Emmet has five bills due this month: water & sewer ($95.50), internet ($75.45), electric ($115.79), gas ($63.98), and phone ($112.79). He currently only has $350 to put towards bills. Each company imposes at $20 late fee if the bill is not paid on time. It is possible for Emmet to pay four of the bills in order to incur only one late fee? If so, which bills should he pay?

19. Sofía is planning a weekend trip and wants to limit her spending to $500, not including travel expenses. She'll stay for two nights at a hotel that costs $115 per night (including taxes and fees), and she estimates her meals will cost a total of $120. The activities in the area that Sofía is considering include an aquarium ($58), an art museum ($25), a zipline tour ($75), and a wine tasting tour ($65). Can Sofía pick three of the activities and stay within her budget? If so, which three activities?

20. Azra has four assignments that are due by the end of the week and only has 20 hours to spend working on them. Any late assignments will incur a 10% grade deduction. She estimates that her math assignment will take 4 hours, her literature assignment will take 7.5 hours, her sociology assignment will take 8 hours, and her business ethics assignment will take 5 hours. All courses are equally important to her. Can Azra complete three of the four assignments and only incur one late penalty? If so, which three assignments?

21. Find the sum of the whole numbers from 1 to 900.

22. Find the sum of the even numbers from 1 to 500.

23. Find the sum of the odd numbers from 1 to 700.

24. The Alpha Zeta Math Club is having a pizza party. The president of the club decides to have fun cutting the pizza and challenges the members to cut a pizza into 11 pieces with only 4 straight cuts. Show a way that this could be done.

25. There are eight teams in the intramural basketball league. Each of the teams will play each other five times. What is the total number of games played in the league?

26. A local bowling league consists of seven teams and each team plays each other team three times. What is the total number of games played in the league?

27. At a birthday party, a child was permitted to cut his own cake to serve to the guests. The cake was round, and the child cut it into 16 pieces using only 5 cuts. (Each cut was vertical through the top of the cake.) Draw a diagram of how this was possible.

28. A farmer looks over a field and sees 37 heads and 98 feet. Some are goats, some are chickens. How many of each are there?

29. Jessica needs to mail a USB drive to her friend. She uses a combination of 41-cent stamps and 8-cent stamps to pay $1.71 in postage. How many of each stamp did Jessica use?

30. While inspecting tanks at an aquarium, a keeper counts 148 legs and 16 heads in a tank that holds octopuses and crabs, which have 8 and 10 legs, respectively. How many of each creature are there?

31. Ruth is sending a care package to her granddaughter, Sarah, who is in college. The care package contains two of Sarah's favorite cookies: fudge swirls, which come in packs of 12, and extra chocolate chunk, which come in packs of 5. The care package contains a total of 107 cookies. How many packs of each cookie type were included?

32. The product of two whole numbers is 196 and their sum is 35. What are the two numbers?

33. The product of two whole numbers is 902 and their sum is 63. What are the two numbers?

34. Twice the difference of a number and 1 is 4 more than that number. Find the number.

35. A number is multiplied by 8, and that product is added to 3. The sum is equal to the product of 5 and 7. Find the number.

WRITING & THINKING

36. One of the most interesting and useful patterns in all of mathematics is called Pascal's triangle, as shown. It is named after the French mathematician Blaise Pascal (1623–1662) who showed that these numbers play an important role in the theory of probability.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

a. Copy the array and fill in the next three rows of numbers using the pattern(s) that you notice.

b. Compute the sum of the elements in each row of Pascal's triangle for the first 6 rows.

c. Look for a pattern in the sums of the first 6 rows. See if you can predict the sum of the elements for row 7, then check by adding. Do the same for rows 8 and 9. State the general rule for the sum of the elements in words.