Chapter 13 Exercises

Create a preference table to show the results for each set of preference ballots.

1. The candidates are: Smith (S), Patel (P), Harvey (H), Knight (K), Jordan (J).

Preference Voting Results

HKPJS	PJKSH	HKPJS	SHJPK	SHKPJ
SHJPK	KJPHS	PHSJK	PJSHK	PJSHK
JPHKS	SHJKP	KPJSH	KPJSH	SHJPK
PJKSH	SHJPK	PJSHK	HKPJS	SHJPK
SHKPJ	HKPJS	PJKSH	KJPHS	KJPHS
PJKSH	KJPHS	HKPJS	KJPHS	SHJPK
KJPHS	PJSHK	SHKPJ	SHJKP	KJPHS

2. Candidates in the election are: Lawson, Eric, Misty.

Misty, Eric, Lawson: 27

Lawson, Misty, Eric: 41

Misty, Lawson, Eric: 31

Eric, Misty, Lawson: 15

Eric, Lawson, Misty: 5

Lawson, Eric, Misty: 19

Calculate the number of pairwise comparisons that must be made in each election.

3. Seven students are running for senior class president.

4. Running for local councilman: J. Pitts, K. McMillian, J. Wallace, D. McLaughlin, K. Smith, and T. Knight.

Use the given preference table to answer each question.

5. Answer the following questions about the given preference table.

Preference Table for Candidates

	Rankings				
1st	A	E	C	C	A
2nd	D	B	A	A	E
3rd	B	C	B	E	C
4th	C	A	D	B	D
5th	E	D	E	D	B
Total Votes	**221**	**212**	**109**	**84**	**167**

a. How many possible unique rankings are there of the candidates in the election?

b. How many people voted in the election?

c. How many voters place Candidate A in first place?

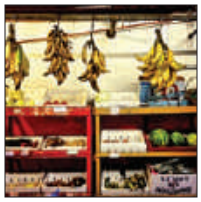
d. Which candidate wins the using plurality method?

e. How many votes would a candidate need to have a majority?

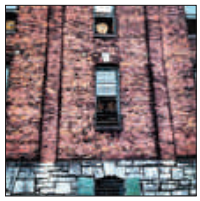
f. Do any of the candidates have a majority of first-place votes?

6. A photography club held a contest where the pictures had to be taken solely with a cell phone. The club members were asked to rank the following entries in order of preference. The preference table summarizes the results of the photo contest.

1.



2.



3.

4.

5.

Preference Table for Best Photo

	Rankings			
1st	3.	3.	2.	5.
2nd	1.	5.	4.	4.
3rd	2.	4.	1.	3.
4th	4.	1.	3.	2.
5th	5.	2.	5.	1.
Total Votes	**54**	**83**	**65**	**92**

a. Determine the photo winner using the Borda count method.

b. Determine the winner using the plurality with elimination method.

c. Determine the winner with the pairwise method of comparison.

d. Suppose that Picture 4 was thrown out for violating the rules. Pictures ranked below Picture 4 simply move up a ranking. Which picture would be declared the winner using the Borda count method with the new preference table? Is this a different winner than in part **a.**?

7. In the Campus Sing-Off, each contestant is ranked by the general student body via the Internet, and by all student organizations sponsoring a contestant. The top 10 entries receive votes in each ballot. The votes are distributed in the following manner.

1st place = 12 points 2nd place = 10 points 3rd place = 8 points
4th place = 7 points 5th place = 6 points 6th place = 5 points
7th place = 4 points 8th place = 3 points 9th place = 2 points
10th place = 1 point

The following table shows how the student organization ranked each contestant in the first semifinal round. The contestants are listed by their last names and sponsor organizations.

Voting Results for First Semifinal																		
Harris; SGA					1st						3rd							
Icin; Social Work Club	6th		6th	6th			7th	6th	2nd	7th	8th	3rd	2nd	10th	7th	9th	10th	5th
Green; ΑΛΠ	2nd	6th			3rd	1st	8th	3rd		8th	4th	1st	7th	6th		10th	6th	8th
Lawson; FCA	9th													7th				
Albert; GSA	1st	8th	2nd	7th		7th	1st	2nd	6th	6th	2nd	2nd	4th	9th	2nd	1st	1st	7th
Roman; ΓΒΘ	4th	7th	3rd		6th		9th	7th		3rd	5th	5th	10th	3rd	8th	6th	2nd	1st
Switzer; Galois Club		9th		4th	8th	9th				10th	10th	9th			3rd	8th	3rd	
Belton; Hispanic Culture Center			7th		9th	10th			9th									10th
Finley; History Club		4th		5th	10th		10th	10th		9th			3rd		1st			
Issen; NAEA				10th		6th			8th			10th	8th	5th	6th	4th		9th
San Marie; NBS	7th		9th		2nd												8th	
Centus; NTSS	5th	1st	1st	8th	5th	4th		8th	10th	2nd				4th			4th	3rd
Dennis; ΩΨΦ		3rd	10th	3rd		8th	2nd		3rd		7th	7th		8th	10th		5th	
Russan; ΦΑ	3rd	5th	4th	1st		5th	3rd	1st	1st	1st	9th	4th	1st		4th	2nd	9th	4th
Hunter; ΦΜΑ					4th	3rd	5th	5th	7th			6th	6th			7th		
Austin; ΣΓΡ		10th					6th	9th										
Molda; Student Design Group	8th		5th	9th	7th	2nd	4th		5th	5th	6th	8th	5th	1st	9th	5th	7th	2nd
Irene; Voices of Praise	10th	2nd	8th	2nd				4th	4th	4th	1st		9th	2nd	5th	3rd		6th

a. Determine the number of total points each contestant received in the first semifinal round from the student organizations.

b. Which top 10 contestants did the organizations think should move on to the second semifinal?

8. Online voting for the best student music video was available for two weeks. Students were asked to rank the videos in order of preference. The following table shows the results of the student votes.

Preference Table for Best Student Music Video

	Rankings					
1st	Video 2	Video 1	Video 2	Video 3	Video 3	Video 4
2nd	Video 1	Video 2	Video 1	Video 4	Video 2	Video 1
3rd	Video 4	Video 3	Video 3	Video 2	Video 4	Video 2
4th	Video 3	Video 4	Video 4	Video 1	Video 1	Video 3
Total Votes	**143**	**212**	**99**	**43**	**121**	**151**

a. Determine the winning video by using the plurality method.

b. Determine the winning video by using the pairwise method of comparison.

c. Determine the winning by using the plurality with elimination method.

d. If you were determining a winning video, which video would you declare the winner based on the online votes? How would you defend your choice based on the answers to **a.–c.**?

9. In the 1992 United States Presidential Election, three candidates shared most of the November 3 popular vote, although there was a small portion of voters for other candidates. The following table displays the breakdown of the popular vote along with the electoral vote for the election.

1992 US Presidential Election Results

Presidential Candidate	Popular Vote		Electoral Vote
	Count	Percentage	
George H. Bush	39,104,545	37.45%	168
Bill Clinton	44,909,889	43.01%	370
Ross Perot	19,742,267	18.91%	0
Other	669,958	0.63%	0
Totals	**104,426,659**	**100%**	**538**

Source: Federal Election Commission. "Federal Elections 92. Election Results for the US President, the US Senate, and the US House of Representatives." June 1993. http://www.fec.gov/pubrec/fe1992/federalelections92.pdf

a. Who won the popular vote by plurality? Was it a majority?

b. Who won the electoral vote by plurality? Was it a majority?

c. Suppose Perot had dropped out of the election. Is it possible that Clinton would have lost the presidency? Explain your answer.

10. The student government is deciding which author to bring to campus next year for the annual Books to Tables talk. The possibilities are Marci Shimoff, coauthor of *Chicken Soup for the Woman's Soul*, Peter Legge, author of *How to Soar with the Eagles*, and Tania Aebi, author of *Maiden Voyage*. Complete the following preference table so that Tania Aebi wins the election using the majority rule.

Preference Table for Author for Books to Tables Talk

	Rankings		
1st	Tania Aebi	Peter Legge	Marci Shimoff
2nd	Peter Legge	Marci Shimoff	Tania Aebi
3rd	Marci Shimoff	Tania Aebi	Peter Legge
Total Votes	**?**	**5**	**4**

11. The Campus Housing Welcome Back committee is deciding what movie to show free to students the first week of school. The movie options are *Django Unchained*, *Zero Dark Thirty*, *Silver Linings Playbook*, and *Life of Pi*. Students were asked to rank the movies by preference. The following preference table summarizes their selections.

Preference Table for Movie Showing

Movie	Rankings						
1st	*Zero*	*Zero*	*Django*	*Silver*	*Life*	*Django*	*Django*
2nd	*Django*	*Life*	*Life*	*Zero*	*Django*	*Silver*	*Zero*
3rd	*Life*	*Silver*	*Zero*	*Django*	*Zero*	*Zero*	*Silver*
4th	*Silver*	*Django*	*Silver*	*Life*	*Silver*	*Life*	*Life*
Total Votes	**30**	**23**	**15**	**16**	**11**	**15**	**27**

a. Which movie is the plurality winner?

b. Which movie wins using a pairwise method of comparison?

c. Does using the plurality method satisfy the Condorcet criterion? Explain your answer.

12. An online poll asked readers to rank the top three law schools in the country. The following preference table shows the results after two days of online voting.

Preference Table for Top Law Schools

	Rankings			
1st	Stanford	Yale	Stanford	Harvard
2nd	Harvard	Stanford	Harvard	Yale
3rd	Yale	Harvard	Yale	Stanford
Total Votes	**84**	**132**	**39**	**13**

a. Which law school would win using the majority rule if voting closed today?

b. Which law school would win using the Borda count method if voting closed today?

c. Does the Borda count method satisfy the majority criterion for these poll results? Explain your answer.

13. The Association of Marketing Professionals is choosing a city for its next conference. The committee making the decision asks the association's members to rank the top five cities they would like to visit. The following preference table gives the results of the member survey. Would the plurality with elimination method satisfy the irrelevant alternatives criterion if the committee decided to eliminate Jacksonville, FL, from the options after the vote? Explain your answer.

Preference Table for Conference Location

	Rankings			
1st	Boston, MA	Nashville, TN	San Diego, CA	New Orleans, LA
2nd	New Orleans, LA	Boston, MA	Boston, MA	Nashville, TN
3rd	San Diego, CA	San Diego, CA	New Orleans, LA	San Diego, CA
4th	Nashville, TN	Jacksonville, FL	Nashville, TN	Boston, MA
5th	Jacksonville, FL	New Orleans, LA	Jacksonville, FL	Jacksonville, FL
Total Votes	**375**	**348**	**289**	**115**

14. The Chamber of Commerce is electing a new president. The candidates are N. Pitts, A. Palmer, and J. Layne. The summary of the rankings of the candidates from the members is given in the table. Is the irrelevant alternatives criterion satisfied using the Borda count method if N. Pitts has to withdraw from the ballot after votes are cast because he moved? Explain your answer.

Preference Table for Chamber of Commerce President

	Rankings			
1st	A. Palmer	N. Pitts	A. Palmer	J. Layne
2nd	N. Pitts	J. Layne	J. Layne	A. Palmer
3rd	J. Layne	A. Palmer	N. Pitts	N. Pitts
Total Votes	**12**	**15**	**13**	**16**

15. *Living Day-to-Day* magazine wants to name the number-one feature homebuyers want in a house. They ask their readers to rank eight home features in order of importance. The eight home features are open-concept homes, smaller homes, outdoor living spacings, neutral decor, modern kitchens, linen closest & smart storage options, energy-efficient fixtures & appliances, and two-car garage with organization. The following preference table displays the results of the rankings. Determine if the Condorcet criterion is satisfied if the magazine uses the plurality method to name the top feature.

Preference Table for Homebuyers' Number One Feature

Feature	Rankings					
1st	Outdoor	Smaller	Open	Kitchen	Smaller	Outdoor
2nd	Open	Outdoor	Smaller	Outdoor	Efficient	Kitchen
3rd	Kitchen	Kitchen	Outdoor	Efficient	Garage	Storage
4th	Neutral	Efficient	Neutral	Open	Open	Efficient
5th	Efficient	Storage	Kitchen	Storage	Outdoor	Garage
6th	Garage	Garage	Storage	Garage	Kitchen	Neutral
7th	Storage	Neutral	Efficient	Smaller	Storage	Open
8th	Smaller	Open	Garage	Neutral	Neutral	Smaller
Total Votes	**421**	**234**	**331**	**472**	**253**	**119**

16. Driving Divas asks its readers to rank the following car features in order of "must haves" in a new car. The car features are remote keyless entry, OnStar system, antilock brakes (ABS), electronic stability/skid-control system, telescoping steering wheel/adjustable pedals, and rear-seat DVD player. Complete the preference table so that rear-seat DVD player is the winner using the Borda count method, but the Condorcet criterion is violated in a head-to-head matchup with antilock brakes.

Preference Table for Must-Have Car Feature

Feature	Rankings			
1st	DVD	Keyless	Adjustable	Stability
2nd	Adjustable	?	DVD	ABS
3rd	ABS	?	OnStar	OnStar
4th	Keyless	Stability	ABS	Keyless
5th	OnStar	OnStar	Stability	Adjustable
6th	Stability	Adjustable	Keyless	DVD
Total Votes	**35**	**44**	**28**	**20**

17. In his next newspaper column, Tom plans to publish his results from a study on the top six qualities that make a great leader. The following preference table summarizes the rankings for the traits. Complete the preference table so that "sense of humor" is the top trait if the site uses the Borda count method to count the votes, and the majority criterion is not violated.

Preference Table for Top Leadership Qualities

Quality	Rankings			
1st	?	Honesty	Honesty	Communication
2nd	?	Sense of Humor	Communication	Sense of Humor
3rd	Confidence	Ability to Delegate	Sense of Humor	Honesty
4th	Commitment	Commitment	Confidence	Ability to Delegate
5th	Ability to Delegate	Confidence	Ability to Delegate	Commitment
6th	Communication	Communication	Commitment	Confidence
Total Votes	**159**	**66**	**42**	**50**

Use the given table to solve each problem.

Population per County

County	Population
A	8578
B	9878
C	10450
D	7565
E	4563
F	6347

18. Suppose a state decides to apportion 250 new highway patrol officers on the basis of number of residents in a county.

a. Find the standard divisor, SD.

b. Find the standard quota, SQ, for county A and county B. Round to three decimal places.

19. Use the Hamilton method to apportion the 250 patrol officers based on the population of the county.

Use the given table to solve each problem.

Student Enrollment in the University of Texas System

Campus	Enrollment	Full Time Equivalent (FTE)
Arlington	33,439	20,594
Austin	51,112	46,402
Brownsville	13,836	9398
Dallas	18,684	12,537
El Paso	18,160	16,271
San Antonio	30,968	23,198
Tyler	5064	4810
Permian Basin	16,266	2696
Pan American	21,016	15,494

20. The University of Texas system consists of nine campuses and has a budget of $13.1 billion. How much money in funding would each campus receive if the system divided the money equally among the campuses (round to nearest dollar)?

21. If the state of Texas wanted to allocate the money equally based on total student enrollment, how much would be appropriated per student?

22. Based on the result from Exercise 21, how much money would the campuses of Arlington and Pan American receive based on the student enrollment on each campus.

23. If the state apportions the funds based on FTE, how much money will the campuses of Austin and Dallas receive (round to nearest dollar)?

24. The University of Texas system wants to apportion 825 new electric vehicles to their campuses. The state decides to apportion these vehicles based on the student enrollment at each university.

a. Find the SD. Round to three decimal places.

b. Find the SQ for the Brownsville and Tyler campuses. Round to three decimal places.

25. Suppose the state decides to apportion the 825 electric vehicles based on FTE.

a. Find the SD. Round to three decimal places.

b. Find the SQ for the San Antonio and Permian Basin campuses. Round to three decimal places.

26. Use the Jefferson method to apportion the 825 electric vehicles to all 10 campuses based on the number of students.

27. Use the Hamilton method to apportion the 825 electric vehicles to all 10 campuses based on FTE. Compare your results with the apportionments from Exercise 26 and determine if the apportionments are different when the apportionment basis is different.

Solve each problem.

28. A science teacher at a high school can teach five classes. There are 27 students enrolled in Physical Science, 24 in Biology, and 37 in Anatomy.

a. Find the SD and SQ of each course. Round to three decimal places.

b. Use the Jefferson method to determine the apportionment of students to the courses to determine the number of sections needed per course.

29. Repeat Exercise 28 b. using the Huntington-Hill method.

30. Repeat Exercise 28 b. using the Hamilton method.

31. An English department uses 19 graduate assistants in teaching its undergraduate courses. The enrollments for each of the courses these students teach is as follows. Using Webster's method, how many graduate assistants should be assigned to each course?

Students per Course

Course	Students
Comp I	675
Comp II	455
Survey of Lit	197
Poetry	385

32. A state consists of six counties: A, B, C, D, E, and F.

The senate for the state is to have 30 members apportioned to the counties based on the old populations using Webster's method. The new populations are also given.

Population by County

County	Old Population	New Population
A	8578	9244
B	9878	9166
C	10,450	10,580
D	7565	10,254
E	4563	6680
F	6347	5993

a. Apportion the members of the senate using the old populations.

b. Apportion the members of the senate using the new populations.

c. Apportion the members of the senate using the Huntington-Hill method and the old populations.

d. Apportion the members of the senate using the Huntington-Hill method and the new populations.

e. Does the population paradox occur when calculating the apportionments using the Huntington-Hill method and the old and new populations?

33. The city of Hillsboro wants to allocate $55 million to youth sports and activity programs. The current participation rates are listed in the table.

Participants per Activity

Activity	Number of Participants
Swimming	245
Library	2595
Baseball	1150
Softball	978
Theater	753
Computer Hobbies	477
Music	1658

a. Find how much the city should allocate to each activity based on the number of participants using the Jefferson method.

b. Find how much the city should allocate to each activity based on the number of participants using Webster's method.

c. Find how much the city should allocate to each activity based on the number of participants using the Huntington-Hill method.

d. Find how much the city should allocate to each activity based on the number of participants using the Hamilton method.

34. Three groups, A, B, and C, have the number of members shown in the table. If Jefferson's method is used to apportion the representatives of the groups on a panel, does the Alabama paradox occur if the number of representatives is increased from 40 to 41?

Members per Group

Group	Members
A	3340
B	4500
C	8875

35. Using the table in Exercise 34, if Webster's method is used to apportion the representatives of the groups on a panel, does the Alabama paradox occur if the number of representatives is increased from 40 to 41?

36. What is the quota for $[8, 5, 3, 2]$ in a simple majority?

37. What is the quota for $[8, 5, 3, 2]$ for a two-thirds majority?

38. Consider the weighted voting system $[25: 9, 7, 4, 3, 2, 2, 1]$.

a. What is the quota for this voting system?

b. What is the weight of P_5?

c. If only the last six voters vote for a motion, does the motion pass?

d. If P_1 and P_2 vote against a motion, does the motion pass?

e. What are the possible winning coalitions for this voting system?

39. Six partners in a law firm have the following voting system $[96: 34, 25, 18, 18, 12, 4]$.

a. How many voters are in the system?

b. What is the quota for the system?

c. What are all of the winning coalitions?

d. Are there any critical players?

e. Do any of the players have veto power?

f. Is there a dummy player?

g. Determine the Banzhaf Power Index for each player.

40. In the weighted voting system $[q: 10, 7, 4, 4, 2, 2]$, if every voter has veto power, what is q?

41. A voting system is represented by $[61: 35, 30, 24, 21]$.

a. List all sequential coalitions in which the voter with weight 35 is pivotal.

b. List all sequential coalitions in which the voter with weight 30 is pivotal.

c. Calculate the Shapley-Shubik Power Index for each player in the system.

d. Are any of the voters dictators?

e. Do any of the voters have veto power?

42. A professional football team is getting ready for the amateur player draft. The head coach, general manager, head scout, and team physician will use the voting system $[8: 5, 4, 3, 2]$ to make decisions regarding players to draft.

a. List all sequential coalitions in which P_1 is a pivotal player.

b. List all sequential coalitions in which P_2 is a pivotal player.

c. List all of the winning coalitions.

d. Calculate the Shapley-Shubik Power Index for each player.

e. Calculate the Banzhaf Power Index for each player.

f. Is there a player with veto power?