It seems like that might take a long time, though, and could get tedious. If we could find a pattern, we might find a more efficient way to solve the problem. Notice that in order to get from day 1 to day 14, we would be increasing by two 13 times. So the number of crunches done on day 14 would be equal to the number of crunches done on day one, 3, plus 2·13.

$$\begin{aligned} 3+2(13) &= 3+26 \\ &= 29 \end{aligned}$$

1.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. 56,317 can be written as 56,000 + 300 + 17 in expanded notation.

2. 42,360 can be written as forty-two thousand, three hundred sixty.

3. The word "and" is not used when reading or writing whole numbers.

Practice

4. Write 1892 in expanded notation.

5. Write 683,100 in words.

6. Write four hundred thousand, seven hundred thirty-six in standard notation.

Applications

Solve.

7. The largest lake in the United States is Lake Superior. It takes up an area of 82,103 square kilometers. Write 82,103 in words.

8. The largest collection of Joker playing cards consists of eight thousand, five hundred twenty cards amassed by Tony De Santis after inheriting a two thousand piece collection from the magician Fernando Riccardi. Write eight thousand, five hundred twenty in standard form.

Writing & Thinking

9. How are natural numbers and whole numbers different and how are they the same?

10. When are hyphens used to write numbers in English words?

Looking Ahead

To estimate fuel costs in the following example, the gas price is rounded to the nearest whole number and the quantity of gas used is rounded to the nearest 5 gallons. You did not review rounding to the nearest 5, but this is similar to rounding to the nearest ten. If the ones place contains a 0, 1, or 2, the digit in the tens place will remain unchanged and the ones digit will be replaced with 0. If the ones place contains a 3, 4, 5, 6, or 7, the digit in the tens place will remain unchanged and the ones digit will be replaced with 5. If the ones place contains an 8 or 9, the digit in the tens place will increase by 1 and the ones digit will be replaced with 0.

Example Preview

Katie rents a car while spending her vacation traveling in Russia. When she returns the car, she has driven 1350 miles and used about 54 gallons of gas. If gas costs an average of $3.969 per gallon, estimate how much she spent on fuel.

Solution

The problem asks for an estimate of the total amount of money Katie spent on fuel.

We know how many gallons of gas Katie used, and we know the average cost of gas per gallon. The total cost of gas can be found by multiplying these two amounts.

Since we are estimating, we can round the price of gas to the nearest dollar.

$$\$3.969 \approx 4$$

We could round to the nearest dime, or even to the nearest cent, but that would not make our calculations much easier, so there's not much benefit to rounding that way.

Similarly, we can round the number of gallons to the nearest ten, or even the nearest multiple of 5. Since the actual number of gallons is pretty close to 55 we'll use that number.

$$\$4 \cdot 55 = \$220$$

So Katie spent about $220 on fuel while in Russia.

1.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. Rounding means finding a number close to the given number, using a specified place of accuracy.

2. When rounded to the ten thousands place, 435,613 becomes 400,000.

3. To estimate the answer for a division problem, begin by rounding both the divisor and dividend.

4. If estimated, $4250 \div 51$ is $4000 \div 50 = 80$.

Practice

Estimate each answer; then find the actual answer.

5.
$$\begin{array}{r} 83 \\ 62 \\ +\ 78 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 63{,}504 \\ -\ 42{,}700 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 420 \\ \times\ 104 \\ \hline \end{array}$$

8. $11\overline{)99}$

Applications

Solve.

9. College cost for a private four-year college in the 2008–2009 academic year are as follows:

Tuition & Fees	$25,243
Room & Board	$8996
Books & Supplies	$1077

Estimate the total cost to attend for a year using rounded numbers to the nearest thousand. Then calculate the actual cost.

10. Brendon is running a sand volleyball tournament soon and must purchase some new equipment. He needs three new nets, which cost $159 each. He also needs five new sets of boundary lines, which cost $86 each. Estimate the total cost of the new equipment. Then calculate the actual cost.

Writing & Thinking

11. In your own words, define estimation.

12. Compare and contrast rounding and estimating.

Now we can use the estimates for the cost of one ream of paper and one cartridge of ink to estimate the total amount spent on each item. To estimate the cost of 6 reams of paper, we can multiply \$4 by 6. To estimate the cost of 8 cartridges of ink, we can multiply \$30 by 8. Then, to estimate the total cost of all the supplies, we find the sum of these two products.

$$\begin{aligned} 6\cdot\$4 + 8\cdot\$30 &= \$24 + \$240 \\ &= \$264 \end{aligned}$$

Following the order of operations, multiplication must be done before additon.

So Frances will need approximately \$264 to buy the office supplies.

1.R.3 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. Nine squared is equal to eighteen.

2. $2^7 = 128$

3. 7^0 is undefined.

4. According to the order of operations, multiplication is always performed before division.

Practice

For each exponential expression **a.** identify the base, **b.** identify the exponent, and **c.** evaluate the exponential expression.

5. 2^3

6. 4^0

Simplify.

7. $18 \div 2 - 1 - 3 \cdot 2$

8. $30 \div 2 - 11 + 2(5-1)^3$

Applications

Solve.

9. Neville bought 15 boxes of trading cards. Each box has 10 packs of trading cards. Each pack of trading cards contains 20 cards. He adds 132 cards that he already owns to the newly purchased cards. Then, Neville evenly distributes all of the cards to 6 of his friends. How many trading cards would each person get?

a. If you simplify the expression $15 \cdot 10 \cdot 20 + 132 \div 6$ using the order of operations, will you get the correct answer? If not, explain what is wrong with the expression.

b. What is the answer? If necessary, write the corrected expression to get the correct results when following the order of operations.

10. Robert is purchasing shirts for his weekend soccer team. The shirts he wants to buy are normally \$25 each but are on sale for \$10 off. His team has a total of 11 players. How much will he spend to buy the shirts?

a. If you simplify the expression $\$25 - \$10 \cdot 11$ using the order of operations, will you get the correct answer? If not, explain what is wrong with the expression.

b. What is the answer? If necessary, write the corrected expression to get the correct results when following the order of operations.

Writing & Thinking

11. Give one example where addition should be completed before multiplication.

1.R.4 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. Averages are found by performing addition and then division.

2. The sum of 312 and 4 is 1248.

3. The word "quotient" indicates multiplication.

4. After reading a problem carefully, the next step might be to make a diagram or draw a figure.

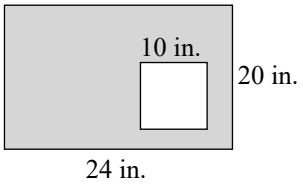
Applications

Solve.

5. Steven is calculating how many calories are in his lunch. He has a hamburger that has 354 calories, a medium fry that has 365 calories, and a chocolate milk shake that has 384 calories. How many total calories is his meal?

6. For a class in statistics, Anthony bought a new graphing calculator for $95, special graphing paper for $8, a USB flash drive for $10, a textbook for $105, and a workbook for $37. How much did he spend for this class?

7. A square that is 10 inches on a side is placed inside a rectangle that has a width of 20 inches and a length of 24 inches. What is the area of the region inside the rectangle that surrounds the square? (Find the area of the shaded region in the figure.)



8. The Lee family spent the following amounts for groceries: $338 in June; $307 in July; $318 in August. What was the average amount they spent for groceries in these three months?

Writing & Thinking

9. Make up three word problems that include key words to indicate operations such as addition, subtraction, multiplication and division. Underline the key words.

10. Give an example where you might use average (other than in a class).

4. Look Back

The primary objective of looking back is to check the answer to make sure it is the solution. Two numbers, 21 and 28, have a product of 588 and a sum of 49.

$$21(28) = 588$$
$$21 + 28 = 49$$

1.R.5 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The order in which the values are given is particularly important when working with subtraction and division problems.

2. "More than" and "increased by" are key phrases specifying the operation of subtraction.

3. Division is indicated by the phrase "five less than a number."

4. Key phrases for parentheses can be used to limit ambiguity in English phrases.

Practice

Write the algebraic expressions described by the English phrases. Choose your own variable.

5. six added to a number

6. twenty decreased by the product of four and a number

7. eighteen less than the quotient of a number and two

Translate each pair of English phrases into algebraic expressions. Notice the differences between the algebraic expressions and the corresponding English phrases.

8. a. six less than a number

b. six less a number

9. a. six less than four times a number

b. six less four times a number

Write the algebraic expression described by the English phrase using the given variables.

10. the cost of purchasing a fishing rod and reel if the rod costs x dollars and the reel costs \$8 more than twice the cost of the rod

Translate each algebraic expression into an equivalent English phrase. (There may be more than one correct translation.)

11. $-9x$

12. $\dfrac{9}{x+3}$

Writing & Thinking

13. Explain why translating addition and multiplication problems from English into algebra may be easier than changing subtraction or division problems. (Consider the properties previously studied.)

14. Explain the difference between $5(n+3)$ and $5n+3$ when converting from algebra to English.

We know from the problem that there are 74 total feet. We can now set the expression we developed equal to 74 and solve for x.

$$\begin{aligned} 2x + 52 &= 74 \\ 2x &= 22 \\ x &= 11 \end{aligned}$$

Solve this linear equation of the form $ax + b = c$.

Since x represents the total number of pigs, we need only subtract 11 from the total number of heads, 26, to determine the number of chickens. Using this information, we see that there are 11 pigs and 15 chickens.

4. Look Back

The final step in the problem solving process is to make sure our answer is feasible. In order to be feasible, 11 pigs and 15 chickens must have 26 heads and 74 feet.

To check for heads, we notice that $11 + 15 = 26$. To check that the number of feet is correct, we see that $4 \cdot 11 + 2 \cdot 15 = 74$. Therefore, our solution is feasible and accurate.

1.R.6 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If an equation of the form $ax + b = c$ uses decimal or fractional coefficients, the addition and multiplication principles of equality cannot be used.

2. The first step in solving $2x + 3 = 9$ is to add 3 to both sides.

3. To solve an equation that has been simplified to $4x = 12$, you need to multiply both sides by $\frac{1}{4}$, or divide both sides by 4.

4. When solving a linear equation with decimal coefficients, one approach is to multiply both sides in such a way to give integer coefficients before solving.

Practice

Solve each equation.

5. $3x + 11 = 2$

6. $-5x + 2.9 = 3.5$

7. $\frac{2}{5} - \frac{1}{2}x = \frac{7}{4}$

8. $\frac{y}{3} - \frac{2}{3} = 7$

Applications

Solve.

9. The tickets for a concert featuring the new hit band, Flying Sailor, sold out in 2.5 hours. If there were 35,000 tickets sold, solve the equation $35,000 - 2.5x = 0$ to find the number of tickets sold per hour.

10. All snacks (candy, popcorn, and soda) cost \$3.50 each at the local movie theater. Admission tickets cost \$7.50 each. After a long week, Carlos treats himself to a night at the movies. His movie night budget is \$25 and he spends all his movie money. Solve the equation $\$3.50x + \$7.50 = \$25.00$ to determine how many snacks Carlos can buy.

11. In probability, the probability of all possible outcomes of an event must add to 1. Suppose there are 6 possible outcomes of an event. One of the outcomes has a probability of 0.25. The other five outcomes all have the same probability. Solve the equation $5p + 0.25 = 1$ to find the probability of each of the other five outcomes.

Writing & Thinking

12. Find the error(s) made in solving each equation and give the correct solution.

a.

$$\frac{1}{3}x + 4 = 9$$

$$\mathbf{3} \cdot \frac{1}{3}x + 4 = \mathbf{3} \cdot 9$$

$$x + 4 = 27$$

$$x + 4 - \mathbf{4} = 27 - \mathbf{4}$$

$$x = 23$$

b.

$$5x + 3 = 11$$

$$(5x - \mathbf{3}) + (3 - \mathbf{3}) = 11 - \mathbf{3}$$

$$2x + 0 = 8$$

$$\frac{2x}{\mathbf{2}} = \frac{8}{\mathbf{2}}$$

$$x = 4$$

2.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If -8 lies to the right of a number on a number line, then -8 is less than that number.

2. All whole numbers have an opposite number.

3. All whole numbers are also integers.

4. The absolute value of a positive number is a positive number.

Practice

Graph each set of integers on a number line.

5. $\{-3, -1, 1\}$

6. $\{-5, -4, -3, -2, 0, 1\}$

Fill in each blank with the appropriate symbol that will make the statement true: <, >, or =.

7. $4 \_\_\_ 6$

8. -20 ___ -19

9. Simplify $|-4|$

10. List the possible values of x for $|x| = 5$

Applications

Represent each quantity with a signed integer.

11. The Alvin is a manned deep-ocean research submersible that has explored the wreck of the Titanic. The operating depth of the Alvin is 4500 meters below sea level.

12. Mount Everest is considered to be the highest mountain on Earth. Its peak reaches to a height of approximately 8844 meters.

Writing and Thinking

13. Explain, in your own words, how an expression such as $-y$ might represent a positive number.

14. Compare and contrast absolute value with opposites.

Looking Ahead

The vertical bars in the following example represent finding the number of items in a set. Finding the number of items which meet either of two criteria involves adding the number of items which meet one criterion with the number of items which meet the other criterion.

Example Preview

Find the number of playing cards in a standard deck of 52 cards that are either black cards or queens.

Solution

Let set A be the set of all black cards and set B be the set of all queens. Then the number of cards that are either black cards or queens can be written $|A \cup B|$, and we find the solution by applying the inclusion-exclusion principle.

$$|A \cup B| = |A| + |B| - |A \cap B|$$

There are 26 black cards in the deck and there are 4 queens. However, there are 2 queens that are also black cards.

Therefore, $|A \cup B| = 26 + 4 - 2 = 28$. Add and subtract real numbers.

So, there are 28 cards that are either black cards or queens.

2.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. When adding integers with unlike signs, the answer will be negative.

2. The sum of two positive numbers can equal zero.

3. The additive inverse of negative seven is -7.

4. When a number substituted for a variable makes a statement true, that number is said to be an equation.

Practice

Find the additive inverse (opposite) of each integer.

5. 15

6. –40

Add.

7. $-3+(-5)$

8. $12+14+(-16)$

Add. Be sure to find the absolute value first.

9. $|-7|+(-7)$

Determine whether the given integer is a solution to the equation by substituting for the variable and then adding.

10. $x+5=7; \quad x=-2$

Applications

Solve.

11. A submarine dives to a depth of 250 feet below the surface. It rises 75 feet before diving an additional 100 feet. What is the final depth of the submarine?

12. The temperature at 2 a.m. was −17 °C. By 2 p.m. the temperature increased a total of 15 °C. What was the temperature at 2 p.m.?

Writing & Thinking

Choose the response that correctly completes each statement. In each problem, give two examples that illustrate your reasoning.

13. If x and y are integers, then $x + y$ is (never, sometimes, always) equal to 0.

14. Explain how the sum of the absolute values of two integers might be 0. (Is this possible?)

Looking Ahead

When finding the number of elements in a set, you want to make sure not to double count any elements. To ensure this, when finding the number of items that meet at least one of two criteria, you will subtract the number of items which meet both criteria.

Example Preview

A survey of 400 students yielded the following information: 252 were seniors, 215 were commuters, and 145 of the seniors were commuters. How many of the 400 surveyed students were seniors or were commuters?

Solution

To determine how many students were seniors or were commuters, we need to find the union of the two sets. Note that the students that were seniors and were also commuters were counted in both categories. Since some students are included in both sets, we need to use the inclusion-exclusion principle.

If we let set A consist of the set of all seniors and set B consist of the set of all commuters, then $|A| = 252$, $|B| = 215$, and we are looking for $|A \cup B|$. We were told that 145 of the seniors were commuters, which means that $|A \cap B| = 145$. Substituting these values into the formula for the inclusion-exclusion principle gives the following.

$$\begin{aligned} |A \cup B| &= |A| + |B| - |A \cap B| \\ &= 252 + 215 - 145 \qquad \text{Add and subtract real numbers.} \\ &= 322 \end{aligned}$$

So, there are a total of 322 students who were seniors or were commuters.

2.R.3 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. Moving to the right on the number line is equivalent to moving in a positive direction.

2. Like addition, subtraction is both commutative and associative.

3. The expression "15 − 7" can be thought of as "fifteen plus negative seven."

Practice

Perform the indicated operations.

4. $17 - 22$

5. $-17 - 30$

6. $-3-(-3)+(-2)$

Applications

Solve.

7. Beginning with a temperature of 8° above zero, the temperature was measured hourly for 4 hours. It rose 3°, dropped 7°, and rose 1°. What was the final temperature recorded?

8. In a 5-day week, the NASDAQ stock market posted a gain of 145 points, a loss of 100 points, a loss of 82 points, a gain of 50 points, and a gain of 25 points. If the NASDAQ started the week at 6300 points, what was the value of the market at the end of the week?

Writing & Thinking

9. Under what conditions can the difference between two negative numbers be a positive number?

10. Give two examples to illustrate why subtraction is not commutative.

Looking Ahead

You will learn how to calculate the number of proper subsets which can be formed from a given set. The order of operations is used when determining how many proper subsets a set has. The exponent should be ev aluated before subtracting.

Example Preview

K contains the top organized religions in the world in 2015 according to Infoplease. If K = {Christianity, Islam, Hinduism, Buddhism, Sikhism, Judaism, Bahaism, Confucianism, Jainism, Shintoism}, how many proper subsets does K contain?

Solution

If the cardinal number of a set is n, then there are $2^n - 1$ proper subsets contained in the set.

Begin by counting the number of religions in K. You will see that it contains 10 religions. Using the formula for the number of proper subsets, we have the following.

$$\text{Number of proper subsets} = 2^{10} - 1 = 1023$$

2.R.4 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The product of zero and an integer is undefined.

2. If a negative integer is divided by a positive integer, the result will be a negative number.

3. When zero is divided by any nonzero integer, the result is zero.

4. If there are no grouping symbols, multiplication should always be performed before division.

Practice

Multiply.

5. $0(-5)$

6. $(-5)(3)(-4)$

Divide.

7. $\frac{-12}{3}$

8. $\frac{35}{0}$

Simplify the expression using the order of operations.

9. $-6^2 + 7(12) - 3^2$

Applications

Solve.

10. Alicia bought shares of two companies on the stock market. She paid \$9000 for 90 shares in one company and \$6600 for 110 shares in another company. What was the average price per share for the 200 shares?

11. In a speech class the students graded each other on a particular assignment. On this speech, three students scored 60, three scored 70, five scored 80, five scored 82, and four scored 85. What was the average score on this speech?

Writing & Thinking

12. If you multiply an odd number of negative numbers together, do you think that the product will be positive or negative? Explain your reasoning.

13. Explain, in your own words, why the following expression cannot be evaluated:
$(24 - 2^4) + 6(3 - 5) \div (3^2 - 9)$.

Sugar: $\dfrac{1\frac{3}{4}}{2} = \dfrac{x}{3}$

$$1\frac{3}{4} \cdot \frac{\overset{3}{\cancel{6}}}{\cancel{2}} = \frac{x}{\cancel{3}} \cdot \frac{\overset{2}{\cancel{6}}}{\cancel{1}}$$

$$\frac{7}{4} \cdot 3 = 2x$$

$$\frac{21}{4} = 2x$$

$$x = \frac{21}{8} \text{ or } 2\frac{5}{8}$$

Salt: $\dfrac{\frac{1}{4}}{2} = \dfrac{y}{3}$

$$\frac{1}{4} \cdot \frac{\overset{3}{\cancel{6}}}{\cancel{2}} = \frac{y}{\cancel{3}} \cdot \frac{\overset{2}{\cancel{6}}}{\cancel{1}}$$

$$\frac{3}{4} = 2y$$

$$y = \frac{3}{8}$$

Peanuts: $\dfrac{1\frac{1}{3}}{2} = \dfrac{z}{3}$

$$1\frac{1}{3} \cdot \frac{\overset{3}{\cancel{6}}}{\cancel{2}} = \frac{z}{\cancel{3}} \cdot \frac{\overset{2}{\cancel{6}}}{\cancel{1}}$$

$$\frac{4}{3} \cdot 3 = 2z$$

$$4 = 2z$$

$$z = 2$$

So the amounts needed for 3 dozen cookies are $2\frac{5}{8}$ cups of sugar, $\frac{3}{8}$ teaspoon of salt, and 2 cups of peanuts.

4.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. In $\frac{11}{13}$, the denominator is 11.

2. $\frac{0}{6} = 0$

3. $\frac{17}{0}$ is undefined.

Practice

For the figure, a) write the fraction for the number of days remaining in June (not crossed out) and b) write the fraction for the number of days that have been crossed out for June.

4.

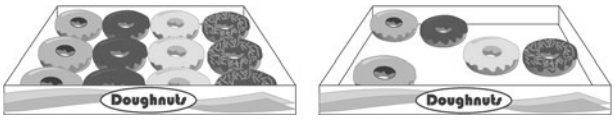
June

S	M	T	W	T	F	S
~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~	~~7~~
~~8~~	~~9~~	~~10~~	~~11~~	~~12~~	~~13~~	~~14~~
~~15~~	~~16~~	~~17~~	~~18~~	~~19~~	~~20~~	~~21~~
~~22~~	~~23~~	24	25	26	27	28
29	30					

5. Graph $\frac{3}{5}$ on a number line.

Write the remaining amount as **a.** a mixed number and **b.** an improper fraction.

6. Isabella brought 2 boxes of doughnuts to a meeting. The figure shows the remaining amount of doughnuts.



7. Graph $3\frac{1}{4}$ on a number line.

8. Change $1\frac{3}{5}$ to an improper fraction.

Applications

Solve.

9. In a class of 35 students, 6 students received As on a mathematics exam. What fraction of students received an A? What fraction of students did not receive an A?

10. A certain brand of plain bagels has 146 calories per bagel. 115 calories come from the carbohydrates in the bagel. What fraction of the calories is from carbohydrates?

Writing & Thinking

11. In your own words, list the parts of a fraction and briefly describe the purpose of each part.

12. Show and explain, using diagrams and words, why $2\frac{3}{5} = \frac{13}{5}$.

$$\frac{\$4.05}{150 \text{ index cards}} = \frac{\$4.05 \div 150}{150 \text{ index cards} \div 150}$$
$$\approx \frac{\$0.03}{1 \text{ index card}} \quad \text{Round to the nearest hundredth,}$$

or approximately \$0.03 per index card.

For the second option, we have

$$\frac{\$117}{30 \text{ index cards}} = \frac{\$1.17 \div 30}{30 \text{ index cards} \div 30}$$
$$\approx \frac{\$0.04}{1 \text{ index card}} \quad \text{Round to the nearest hundredth,}$$

or approximately \$0.04 per index card.

Since \$0.03 is the lower of the two unit prices, the better buy is \$4.05 for 150 index cards.

4.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. Two hundred thousand, four hundred six and twelve hundredths can be written as 200,406.12.

2. 92.586 is greater than 92.6.

3. On a number line, any number to the left of another number is larger than that other number.

4. When a decimal number is rounded, all numbers to the right of the place of accuracy become zeros in the final answer.

Practice

5. Write $2\frac{57}{100}$ in decimal notation.

6. Write 20.7 in words.

7. Write six and twenty-eight thousandths in decimal notation.

8. Arrange 0.2, 0.26, and 0.17 in order from smallest to largest. Then, graph the numbers on a number line.

9. Round 3.00652 to the nearest ten-thousandth.

a. The digit in the ten-thousandths position is ___.

b. The next digit to the right is ___.

c. Since ___ is less than 5, leave ___ as it is and replace ___ with 0.

d. So 3.00652 rounds to _____ to the nearest ten-thousandth.

Applications

In each exercise, write the decimal numbers that are not whole numbers in words.

10. The tallest unicycle ever ridden was 114.8 feet tall, and was ridden by Sam Abrahams (with a safety wire suspended from an overhead crane) for a distance of 28 feet in Pontiac, Michigan, on January 29, 2004.

11. One quart of water weighs approximately 2.0825 pounds.

Writing & Thinking

12. Discuss situations where you think it is particularly appropriate (or necessary) to write numbers in English word form.

13. With **a.** and **b.** as examples, explain in your own words how you can tell quickly when one decimal number is larger (or smaller) than another decimal number.

a. The decimal number 2.765274 is larger than the decimal number 2.763895.

b. The decimal number 17.345678 is larger than the decimal number 17.345578.

For the second option, we have

$$\frac{\$117}{30 \text{ index cards}} = \frac{\$1.17 \div 30}{30 \text{ index cards} \div 30}$$
$$\approx \frac{\$0.04}{1 \text{ index card}} \qquad \text{Round to the nearest hundredth,}$$

or approximately \$0.04 per index card.

Since \$0.03 is the lower of the two unit prices, the better buy is \$4.05 for 150 index cards.

4.R.3 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The decimal points should be aligned vertically when multiplying decimal numbers.

2. When multiplying decimal numbers, the answer should have the same number of decimal places as the total number of decimal places in the numbers being multiplied.

3. Multiplying by 100 requires that the decimal point be moved 100 places to the right.

4. Moving the decimal point in a divisor requires that the decimal point also be moved in the dividend.

Practice

Multiply.

5. Multiply $(5.6)(-0.02)$

6. Multiply $10(-45.6)$

7. Divide $-1.62 \div 9$

8. Divide $\frac{167}{10}$

Applications

Solve.

9. To buy a car, you can pay $2036.50 in cash, or you can put down $400 and make 18 monthly payments of $104.30. How much would you save by paying cash?

10. A professor has graded a test of five students, and their scores were 76.4, 100, 84.7, 10.2, and 68.3. What is the average of these five scores?

Writing & Thinking

11. In your own words, discuss the similarities and differences between multiplication with whole numbers and multiplication with decimal numbers.

Looking Ahead

You will work with sales tax, which is typically stated as a percent. To make calculations involving sales tax, it is easiest to change the tax rate to a decimal, as is shown in the following example.

Example Preview

A new blu-ray player costs $136.99 in the store. What would your total cost be if the sales tax is 7.5%? Round your answer to the nearest cent, if necessary.

Solution

To find the total cost of the blu-ray player, we first need to find the sales tax and then add that value to the original price.

We begin by changing the tax rate to a decimal and then multiply by the original price to get the sales tax. Note that we round the answer to the nearest cent.

$$\text{sales tax} = 0.075 \cdot \$136.99$$

$$\approx \$10.27$$

Now we add the sales tax to the original price to get the total cost.

$$\text{total cost} = \$136.99 + \$10.27 = \$147.26$$

So the total cost for the blu-ray player is $147.26.

4.R.4 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. It is not possible to have a percent greater than 100%.

2. A decimal number that is between 0.01 and 0.10 is between 10% and 100%.

3. To change from a percent to a decimal, simply omit the percent sign.

4. Fractions that have denominators other than 100 cannot be changed to a percent.

Practice

5. Write $\frac{20}{100}$ as a percent.

6. Write 1.12 as a percent.

7. Write 60% as a decimal number.

8. Write $\frac{4}{5}$ as a percent.

9. Write 150% as a fraction or mixed number and reduce, if possible.

Applications

Solve.

10. A savings account is offering an interest rate of 0.04 for the first year after opening the account. Change 0.04 to a percent.

11. Suppose that sales tax is figured at 7.25%. Change 7.25% to a decimal.

Writing & Thinking

12. Describe a situation where more than 100% is possible. Describe a situation where it is impossible to have more than 100%.

13. Justify why mixed numbers are a larger percentage than proper fractions alone. (Consider the value of 100%.)

4.R.5 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The exponent in the number 1.4×10^4 indicates that the decimal point should be moved 4 places to the right.

2. The exponent in the number 2.5×10^{-3} indicates that the decimal point should be moved 3 places to the right.

3. The number 3.53×10^5 is less than 8.72×10^{-4}.

4. The number 4000 written in scientific notation is 0.4×10^4.

Practice

Write the following numbers in scientific notation.

5. 86,000

6. 0.0000000002368

Write the following numbers in decimal form.

7. 4.2×10^{-2}

8. 3.067×10^{10}

First write each of the numbers in scientific notation. Then perform the indicated operations and leave your answer in scientific notation.

9. $0.0003 \cdot 0.0000025$

10. $23{,}400{,}000{,}000 \cdot 5{,}500{,}000{,}000$

Applications

Solve.

11. One light-year is approximately 9.46×10^{15} meters. The distance to a certain star is 4.3 light-years. How many meters is this?

12. The mass of an atom of gold is approximately 3.25×10^{-22} grams. What would be the mass of 2000 atoms of gold? Express your answer in scientific notation.

Looking Ahead

Checking an answer or evaluating a function for a specific value are common occurrences in algebra that involve real numbers.

Example Preview

Evaluate the function $f(x) = x^2 + 7x - 3$ for $x = -5$.

Solution

Substitute -5 for x in the function and simplify.

$$\begin{aligned} f(x) &= x^2 + 7x - 3 \\ f(-5) &= (-5)^2 + 7(-5) - 3 \\ &= 25 - 35 - 3 \\ &= -13 \end{aligned}$$

5.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The sum of a positive number and a negative number is always positive.

2. The sum of two positive numbers can equal zero.

3. The expression "15 – 7 " can be thought of as "fifteen plus negative seven."

4. If two numbers have the same sign, both the product and the quotient of the two numbers will be negative.

5. The mean of a set of numbers is always positive.

Practice

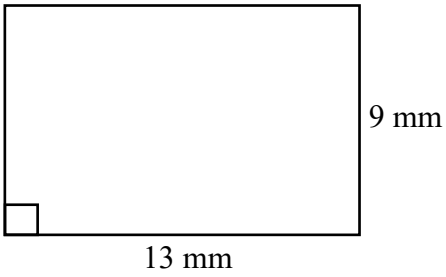
6. Compute the value of the following sum.

$-16.6 + 3.7$

7. Compute the value of the following sum.

$-60 + (-79) + 42$

8. Find the perimeter of the rectangle.



9. Find the product.

$3(-5)(-4)$

10. Find the product.

$0(-33)$

11. Find the quotient.

$\frac{45}{3}$

12. Find the quotient.

$\frac{46}{0}$

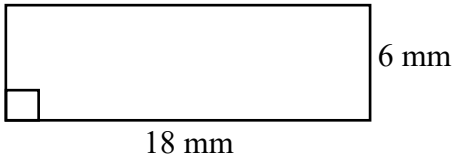
Applications

Solve.

13. A pilot flew a plane from an altitude of 10000 feet to an altitude of 3600 feet. What was the change in altitude?

14. In April, Mr. Burton opened a checking account and made deposits of $968, $1387, $986, and $369. He also wrote checks for $193, $480, $52, $468, and $545. What was his balance at the end of the month?

15. Find the area of the rectangle:



Writing & Thinking

16. If you multiply an odd number of negative numbers together, do you think that the product will be positive or negative? Explain your reasoning.

Looking Ahead

Simplifying algebraic expressions is a common step when solving systems of linear equations.

Example Preview

Solve the system of linear equations algebraically.

$$\begin{aligned} y &= 2x + 4 \\ x + 2y &= 18 \end{aligned}$$

Step 1: Solve one of the equations for one of the variables.

Since the first equation is already solved for y, we can move on to the next step.

Step 2: Substitute the expression found in Step 1 into the other equation and solve for the remaining variable.

Use the expression $2x + 4$ as a substitution for y in the second equation.

$$\begin{aligned} x + 2y &= 18 \\ x + 2(2x + 4) &= 18 \\ x + 4x + 8 &= 18 \\ 5x + 8 &= 18 \\ 5x &= 10 \\ x &= 2 \end{aligned}$$

Step 3: Substitute the value found in Step 2 into the original equation from Step 1 and solve for the remaining variable.

Using the original equation from Step 1, find the y-coordinate by substituting $x = 2$ into the equation.

$$\begin{aligned} y &= 2x + 4 \\ &= 2(2) + 4 \\ &= 4 + 6 = 10 \end{aligned}$$

Thus, our point of intersection for the two lines is (2, 10).

5.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. A variable that does not appear to have an exponent has an exponent of 1.

2. In the term $-9x$, nine is being subtracted from x.

3. In the term "$12a$," 12 is the constant.

4. Like terms have the same coefficients.

Practice

Identify the like terms in the list of terms.

5. $-5, 3, 7x, 8, 9x, 3y$

Simplify each expression by combining like terms.

6. $2a + 14a - 25a$

7. $5x^2 - 3x^2 + 2x$

8. $3(n+1) + n$

Writing & Thinking

9. Discuss like and unlike terms and give an example of each.

10. Explain the difference between -13^2 and $(-13)^2$.

Solution

This situation can be modeled by the equation $\$60 = \$7.50x$, where x is the number of 4-packs of cupcakes purchased. To determine how many 4-packs of cupcakes he can buy, we solve for x.

$$\begin{aligned} \$60 &= \$7.50x \\ \frac{\$60}{\$7.50} &= x \\ 8 &= x \end{aligned}$$

Therefore, Gareth can buy 8 packs of cupcakes.

5.R.3 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. When an algebraic expression is added to both sides of an equation, the new equation has the same solutions as the original equation.

2. The process of finding the solution set to an equation is called simplifying the equation.

3. A linear equation in x is also called a first-degree equation in x.

4. Equations with the same solutions are said to be equivalent equations.

Practice

Determine whether the given number is a solution to the given equation by substituting and then evaluating.

5. $y + (-5) = -3; y = 2$

6. $-1 - |y| = -8; y = -7$

Solve the linear equation using equivalent equations to isolate the variable. Express your solution as an integer, as a simplified fraction, or as a decimal number.

7. $w + 11 = 4$

8. $9.4 - 7.3 = 7.8a - 6.8a - 7.5$

9. $-6 = 2u$

10. $-\frac{6}{5}y + \frac{2}{5}y = \frac{2}{3} - \frac{1}{3}$

Applications

Solve.

11. An office supply company offers a number of different packages to help save their customers money. One of the packages includes dry erase markers, 22 boxes of paper clips, and 22 boxes of printer ink. If the total number of boxes in the package is 67, use the formula $x + 2(22) = 67$ to find the number of boxes of dry erase markers that come in the package.

12. Janice lives 154 miles away from her cousin. The distance between Janice's house and her cousin's house is 9 times farther than the distance between Janice's house and her best friend's house. Solve the equation $9x = 154$ to find the number of miles between Janice's house and her best friend's house.

Express your answer as an integer, as a simplified fraction, or as a decimal number rounded to two places.

Writing & Thinking

13. a. Is the expression $6 + 3 = 9$ an equation? Explain.

b. Is $x = 4$ a solution to the equation $5 + x = 10$? Explain.

$$\$500 = \$250 + \$15x$$
$$\$250 = \$15x$$
$$\frac{\$250}{\$15} = x$$
$$16.667 \approx x$$

Since we can't invite a fraction of a person, we'll need to round. Rounding up will result in an amount higher than \$500, so we round down to 16. Therefore, Alex can invite 16 people (including herself).

5.R.4 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The first step in solving $2x + 3 = 9$ is to add to both sides.

2. To solve an equation that has been simplified to $4x = 12$, you need to multiply both sides by $\frac{1}{4}$, or divide both sides by 4.

3. If an equation has no solution, it is called an identity.

4. The most general form of a linear equation is $ax + b = cx + d$.

Practice

Solve the following equation. Express your answer as an integer, simplified fraction, or decimal rounded to two decimal places.

5. $4x - 2 = -6$

6. $-10 = 4x - 14$

7. $\frac{3}{8}z - \frac{8}{3}z - \frac{11}{3} = \frac{11}{12}$

8. $8x - 1 = 5x + 11$

9. $-3 + (3n + 2) = -3(-3n + 9) + 2$

10. $0.5u - 1.5 = 2.7u + 2.9$

Determine if the following equation is a conditional equation, an identity, or a contradiction.

11. $6(x + 2) + 5x = -3(x - 4) + 14x$

12. $3(x + 4) = 8(4 - 3x) + 27x$

Applications

Solve.

13. The tickets for a figure skating performance sold out in hours. If there were 25,000 tickets sold, solve the equation $25{,}000 - 4x = 0$ to find the number of tickets sold per hour.

14. A rectangular shaped park is to have a perimeter of 900 yards. If the width must be 70 yards because of a building code, solve the equation $2l + 2(70) = 900$ to determine the length of the park.

Writing & Thinking

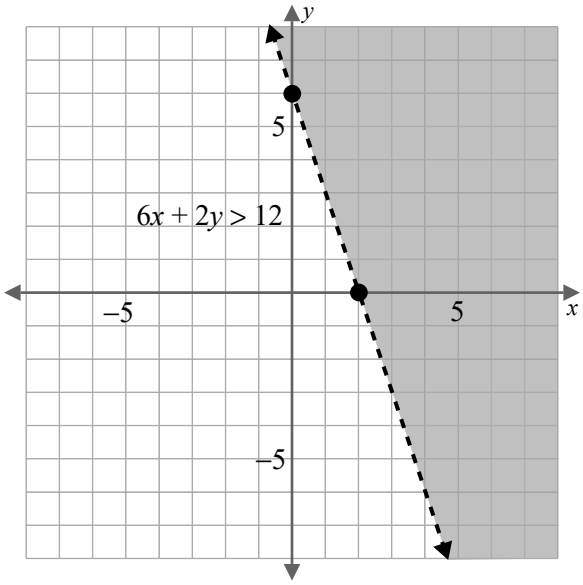
15. Answer each question.

a. Simplify the expression $3(x + 5) + 2(x - 7)$.

b. Solve the equation $3(x + 5) + 2(x - 7) = 31$.

c. How are the methods you used to answer questions a. and b. similar? How are they different?

Since the point (0,0) does not make the inequality true, we know that the point is not in the solution set. Thus, we shade the other half-plane.



5.R.5 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If only one endpoint is included in an interval, it is called a half-open interval.

2. When both sides of a linear inequality are multiplied by a negative constant, the sense of the inequality should stay the same.

3. To check the solution set of a linear inequality, every solution in the solution set must be checked in the original inequality.

4. The infinity symbol ∞ does not represent a specific number.

Practice

Graph each interval on a real number line and tell what type of interval it is.

5. $x \leq -3$

6. $-1.5 \leq x < 3.2$

Solve each inequality and graph the solution set. Write each solution set using interval notation.

7. $x + 1 > 5$

8. $-2x \geq 6$

9. $4x - 7 \geq 9$

10. $5x + 6 \geq 2x - 2$

Applications

Solve.

11. A statistics student has grades of 82, 95, 93, and 78 on four hour-long exams. He must average 90 or higher to receive an A for the course. What scores can he receive on the final exam and earn an A if:

a. The final is equivalent to a single hour-long exam (100 points maximum)?

b. The final is equivalent to two hourly exams (200 points maximum)?

12. Allison is going to the post office to buy 34¢ stamps and 3¢ adjustment stamps. Since the current postage rate is 49¢, she will need 5 times as many 3¢ adjustment stamps as 34¢ stamps. If she has $12.25 to spend, what is the largest number of 34¢ stamps she can buy?

Writing & Thinking

13. a. Write a list of three situations where inequalities might be used in daily life.

b. Illustrate theses situations with algebraic inequalities and appropriate numbers.

$$t = \frac{-6 \pm \sqrt{6^2 - 4(0.7)(-8500)}}{2(0.7)}$$
$$t = \frac{-6 \pm \sqrt{36 + 23{,}800}}{1.4}$$
$$t = \frac{-6 \pm \sqrt{23{,}836}}{1.4}$$
$$t \approx \frac{-6 \pm 154.389}{1.4}$$
$$t \approx \frac{-6 + 154.389}{1.4} \text{ or } \frac{-6 - 154.389}{1.4}$$

This means that $t \approx 105.992$ years or $t \approx -114.564$ years. Since we are looking for a future date, the only logical answer would be $t \approx 105.992$ years. But what month and year does this represent?

As the initial population was measured in 2012, and we are talking about 105 and some fraction of a year later, the year is going to be 2012 + 105 = 2117. Now multiply 0.992 of a year by the number of months in a year, 12, to find the month.

$$12 \cdot 0.992 = 11{,}904$$

The number of months is 11.904, which means that 11 months have passed as well as part of the 12th month, which is December. So the population of Pleasantburg will reach 19,500 in December of 2117.

5.R.6 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The quadratic formula will always work when solving quadratic equations.

2. If the discriminant is a perfect square, the quadratic equation is factorable.

3. When using the quadratic formula, if the discriminant is greater than zero, there are infinite solutions.

4. If the discriminant is less than zero, there is no real solution.

Practice

Find the discriminant and determine the nature of the solutions of each quadratic equation.

5. $x^2 + 6x - 8 = 0$

6. $x^2 - 8x + 16 = 0$

Solve each of the quadratic equations using the quadratic formula.

7. $x^2 + 4x - 4 = 0$

8. $x^2 - 2x + 7 = 0$

9. $3x^2 - 7x + 4 = 0$

Applications

Solve.

10. An orange is thrown down from the top of a building that is 300 feet tall with an initial velocity of 6 feet per second. The distance of the object from the ground can be calculated using the equation $d = 300 - 6t - 16t^2$, where t is the time in seconds after the orange is thrown.

a. On a balcony, a cup is sitting on a table located 100 feet from the ground. If the orange is thrown with the right aim to fall into the cup, how long will the orange fall? Round to the nearest hundredth. (**Hint:** The distance is 100 feet.)

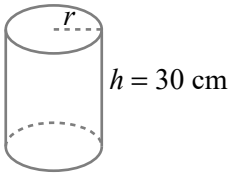
b. If the orange misses the cup and falls to the ground, how long will it take for the orange to splatter on the sidewalk? (**Hint:** What is the height of the orange when it hits the ground?)

c. Approximately how much longer would it take for the orange to fall to the sidewalk than it would for the orange to fall into the cup?

Writing & Thinking

11. Find an equation of the form $Ax^4 + Bx^2 + C = 0$ that has the four roots ± 2 and ± 3. Explain how you arrived at this equation.

12. The surface area of a right circular cylinder can be found using the following formula: $S = 2\pi r^2 + 2\pi rh$, where r is the radius of the cylinder and h is the height. Estimate the radius of a circular cylinder of height 30 cm and surface area 300 cm^2. Explain how you used your knowledge of quadratic equations.



5.R.7 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The graph of every ordered pair that has a positive x-coordinate and a negative y-coordinate can be found in Quadrant IV.

2. To find the y-value that corresponds with $x = 2$, substitute 2 for x into the given equation and solve for y.

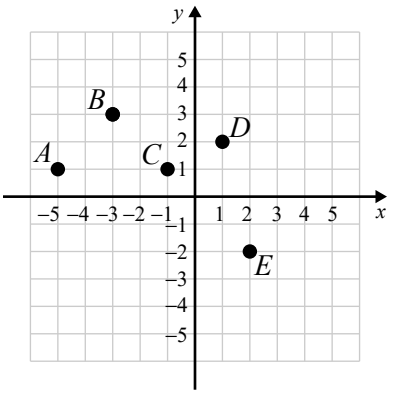
3. If $(-7, 3)$ is a solution of $y = 3x + 24$, then $(-7, 3)$ satisfies $y = 3x + 24$.

4. If point $A = (0, 4)$, then point A lies on the x-axis.

Practice

List the set of ordered pairs corresponding to the points on the graph.

5.



Plot each set of ordered pairs and label the points.

6. $\{A(4,-1), B(3,2), C(0,5), D(1,-1), E(1,4)\}$

Determine the missing coordinate in each of the ordered pairs so that the point will satisfy the equation given.

7. $x - 2y = 2$

a. $(0, \_\_)$

b. $(4, \_\_)$

c. $(\_\_, 0)$

d. $(\_\_, 3)$

Complete the table so that each ordered pair will satisfy the given equation. Plot the resulting sets of ordered pairs.

8. $y = 2x - 3$

x	*y*
0	
	–1
–2	
	3

Determine which, if any, of the ordered pairs satisfy the given equation.

9. $2x - 3y = 7$

a. $(1, 3)$

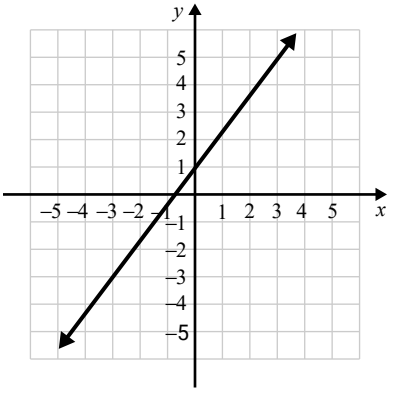
b. $\left(\frac{1}{2}, -2\right)$

c. $\left(\frac{7}{2}, 0\right)$

d. $(2, 1)$

The graph of a line is shown. List any three points on the line. (There is more than one correct answer.)

10.



Applications

Solve.

11. At one point in 2017, the exchange rate from US dollars to Euros was $E = 0.85D$ where E is Euros and D is dollars.

a. Make a table of ordered pairs for the values of D and E if D has the values \$100, \$200, \$300, \$400, and \$500.

b. Plot the points corresponding to the ordered pairs.

12. Given the equation $F = \frac{9}{5}C + 32$ where C is temperature in degrees Celsius and F is the corresponding temperature in degrees Fahrenheit:

a. Make a table of ordered pairs for the values of C and F if C has the values $-20°$, $-10°$, $-5°$, $0°$, $5°$, $10°$, and $15°$.

b. Plot the points corresponding to the ordered pairs.

5.R.8 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The y-intercept is the point where a line crosses the y-axis.

2. The terms ordered pair and point are used interchangeably.

3. A horizontal line does not have a y-intercept.

4. All x-intercepts correspond to an ordered pair of the form $(0, y)$.

Practice

Graph each linear equation by locating at least two ordered pairs that satisfy the given equation.

5. $x + y = 3$

6. $x = 1$

7. $y = -3$

Graph each linear equation by locating the x-intercept and the y-intercept.

8. $y = 4x - 10$

9. $3x - 7y = -21$

Applications

Solve.

10. The amount of potassium in a clear bottle of a popular sports drink declines over time when exposed to the UV lights found in most grocery stores. The amount of potassium in a container of this sports drink is given by the equation $y = -30x + 360,$ where y represents the mg of potassium remaining after x days on the shelf. Find both the x-intercept and y-intercept, and interpret the meaning of each in the context of this problem.

11. Mr. Adler has found that the grade each student gets in his Introductory Algebra course directly correlates with the amount of time spent doing homework, and is represented by the equation $y = 7x + 30,$ where y represents the numerical score the student receives on an exam (out of 100 points) after spending x hours per week doing homework. Find the y-intercept and interpret its meaning in this context.

Writing & Thinking

12. Explain, in your own words, why it is sufficient to find the x-intercept and y-intercept to graph a line (assuming that they are not the same point).

13. Explain, in your own words, how you can determine if an ordered pair is a solution to an equation.

5.R.9 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If the domain of a linear function is not explicitly stated, the implied domain is the set of all values of x that produce real values for y.

2. A relation is a function in which each domain element has exactly one corresponding range element.

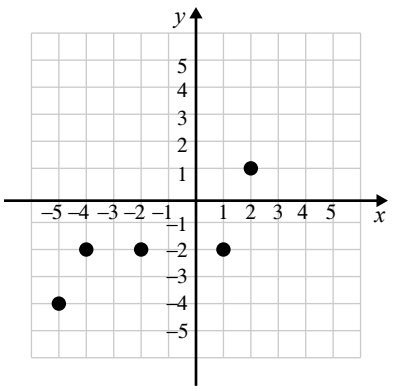
3. In a function, the range elements can have more than one corresponding domain element.

4. If $s = \{(1,-6),(3,5),(4,0),(1,2)\}$, then s is a function.

Practice

List the sets of ordered pairs that correspond to the points. State the domain and range and indicate if the relation is a function.

5.

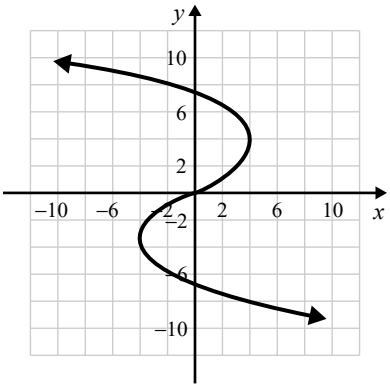


Graph the relation. State the domain and range and indicate which of the relation is a function.

6. $h = \{(1, -5), (2, -3), (-1, -3), (0, 2), (4, 3)\}$

Use the vertical line test to determine whether the graph represents a function. State the domain and range using interval notation.

7.



State the domain of the function.

8. $h(x) = \dfrac{7}{3x}$

Find the values of the function as indicated.

9. $F(x) = 6x^2 - 10$

a. $F(0)$

b. $F(-4)$

c. $F(4)$

Applications

Solve.

10. A nurse hangs a 1000-milliliter IV bag which is set to drip at 120 milliliters per hour. Create a model of this situation to represent the amount of IV solution left in the bag after x hours.

a. The y-intercept is the amount of IV solution in the bag initially (time = 0). What is the y-intercept?

b. The slope is equal to the rate that the IV solution is dispensed per hour. What is the slope? (**Hint:** Consider whether the amount of IV solution in the bag is increasing or decreasing and how this would affect the slope.)

c. Write an equation in slope-intercept form to model this situation.

d. Write the equation from Part **c.** using function notation.

e. State the domain and range of the function.

f. State any additional restrictions that should be made on the domain for it to make sense in the context of this problem.

g. How much IV solution is left in the bag after 5 hours?

2. Estimating products can be done by rounding each number to the place of the _____

_____

3. In order to estimate with division, round both the _____

_____

Looking Ahead

Following the order of operations is necessary when simplifying formulas and expressions. The following example shows the importance of the order of operations when determine an individual's monthly net pay amount.

Example Preview

A recent graduate from a liberal arts college acquired a job as a sociologist. Her yearly salary is $34,500. Her employer withholds $5175 in state and federal income taxes and $2639.25 in FICA taxes throughout the year. Determine her monthly net pay amount.

Solution

The monthly net pay amount is the annual salary minus withholdings for all required taxes, divided over 12 months of the year. In this case, we need to subtract $5175 and $2639.25 from the yearly salary before dividing by 12.

$$\text{Monthly Net Pay}: \frac{\$34,500-(\$5175+\$2639.25)}{12}=\frac{\$26685.75}{12}\approx\$2223.81$$

Therefore, the sociologist can expect to collect a monthly paycheck of $2223.81.

6.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. An estimate of the sum 71.369 + 49.1 is 120.

2. One way to estimate the product of decimal numbers is to round the numbers to the rightmost nonzero digit before performing the multiplication.

3. An estimate of the quotient 16.469 ÷ 3.87 would be 4.

4. Experience and understanding are needed to decide whether or not a particular answer is reasonably close to an estimate.

5. According to the rules for order of operations, addition and subtraction should be performed before multiplication and division.

Practice

Estimate each answer, then find the actual answer.

6.
$$\begin{array}{r} 29.03 \\ +\ 3.79 \\ \hline \end{array}$$

7.
$$\begin{array}{r} 51.21 \\ -\ 25.13 \\ \hline \end{array}$$

8. $(6.3)(1.6)$

9. $3.1\overline{)6.36}$

10. Simplify $3.1(50-25.8)-12.9$

Applications

Solve.

11. Jim is packing three sculptures in a box for shipping. The weights of the sculptures are 5.63 pounds, 12.4 pounds, and 3 pounds. The shipping materials weigh 17.4 pounds.

a. Estimate the total weight.

b. Find the actual weight.

12. Peter Sagan rode 125.09 miles in 5.35 hours.

a. Estimate how fast he was riding per hour.

b. What was his average speed per hour (to the nearest hundredth)?

Now You Try It!

Use the space provided to work out the solution to the next example.

Example A Application: Determining Commission

Lynsay earns a salary of $1250 a month plus a commission of 5% on all electronics she sells at her job at the local computer store. What did she earn the month she sold $28,640 in electronics?

Solution

Looking Ahead

Converting between percentages and decimal numbers is an important skill to have for personal finance, as many values related to personal finance are reported as percentages. A key step when using formulas that involve percentages is to first convert the provided percentage as a decimal number.

Example Preview

An online lending company is offering simple-interest personal loans based on consumer credit scores. An individual with a credit score of 720 can get an interest rate of 9.99%. If the individual takes out a $3500 loan and doesn't pay it back for 4 years, how much interest would accrue over that time period?

Solution

In this case, the principal $P = \$3500$ and the time $t = 4$. The interest rate is given as a percentage and must first be converted to a decimal number. That is, $r = 9.99\% = 0.0999$. Substituting these values into the simple interest formula and simplifying gives us the following.

$$I = (\$3500)(0.0999)(4) = \$1398.60$$

Thus, the interest accrued on the loan over 4 years is $1398.60.

6.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement is true. (**Note:** There may be more than one acceptable change.)

1. To change a decimal number to a percent, move the decimal point two places to the left and add the sign.

2. When using the basic formula $R \cdot B = A$, the word "of" means to divide.

3. If an item is selling for a 35% discount, the customer will pay 65% of the original price.

4. A car was purchased in 1965 for \$3800. It sold for \$1200 in 2011. This is an example of depreciation.

Practice

5. Change the following decimal to a percent.

0.012

6. Change the following percent to a decimal.

11%

7. Change $1\frac{1}{4}\%$ to a fraction and reduce, if possible.

8. Change the following fraction to a percent.

$\frac{16}{25}$

Find the unknown quantity. Round your answer to two decimal places, if necessary.

9. 111% of 189 is _____

10. 50% of _____is 803.

11. _____% of 773 is 343.

Applications

Solve.

12. A sales clerk receives a monthly salary of $950 plus a commission of 7% on all sales over $3200. What did the clerk earn the month that he sold $13,500 in merchandise? Follow the problem-solving process and round your answer to the nearest cent, if necessary.

13. The population of white-tailed deer in a region was counted to be 321. The population in the same region the previous year was 300. Find the percent increase in the white-tailed deer population. Round your answer to the nearest hundredth, if necessary.

14. A few years ago, Sarah acquired a parcel of land valued at $13,800. Today, that same parcel of land has a value of $14,628. Find the percent increase in the property's value. Round your answer to the nearest hundredth, if necessary.

Writing & Thinking

15. A man weighed 200 pounds. He lost 20 pounds in 3 months. Then he gained back 20 pounds 2 months later.

a. What percent of his weight did he lose in the first months?

b. What percent of his weight did he gain back?

c. The loss and gain are the same, but the two percentages are different. Explain why.

Solution

Consumers aged 35 to 54 years old spent approximately $21,000 on food and $45,000 on housing in 2018.

6.R.3 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. Graphs should always be clearly labeled, easy to read, and have appropriate titles.

2. Circle graphs show trends over a period of time.

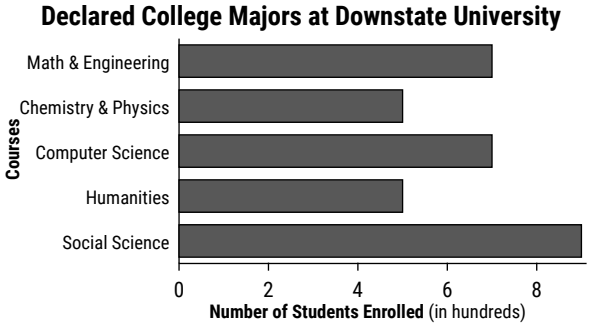
3. The frequency is the number of data items in a class.

4. Numbers that are halfway between the upper limit of one class and the lower limit of the next class are the class boundaries.

Applications

Answer the questions using the given graphs.

5. The following bar graph shows the number of students in five fields of study at a university.



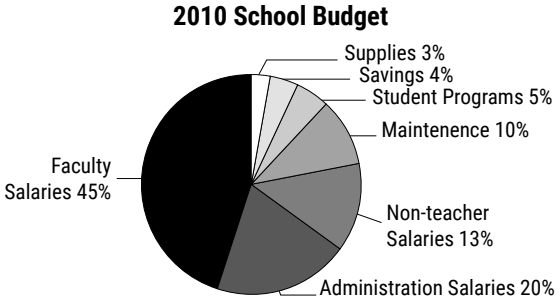
a. Which field(s) of study has the largest number of declared majors?

b. Which field(s) of study has the smallest number of declared majors?

c. How many declared majors are indicated in the entire graph?

d. What percent are computer science majors? Round your answer to the nearest tenth of a percent.

6. The following circle graph represents the various areas of spending for a school with a total budget of $34,500,000.



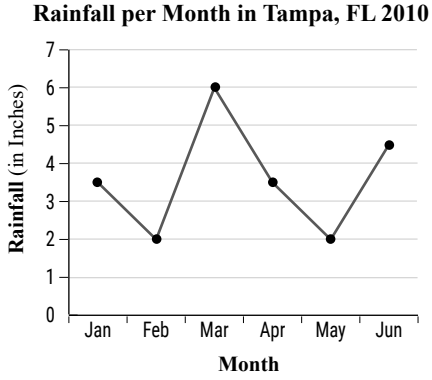
a. What amount will be allocated to each category?

b. What percent will be for expenditures other than salaries?

c. How much will be spent on maintenance and supplies

d. How much more will be spent on teachers' salaries than on ministration salaries?

7. The following line graph shows the total monthly rainfall in Tampa, Florida for the first 5 months of 2010.[1]



a. Which months had the least rainfall?

1 Source: weather.gov

b. What was the most rainfall in a month?

c. What month had the most rainfall?

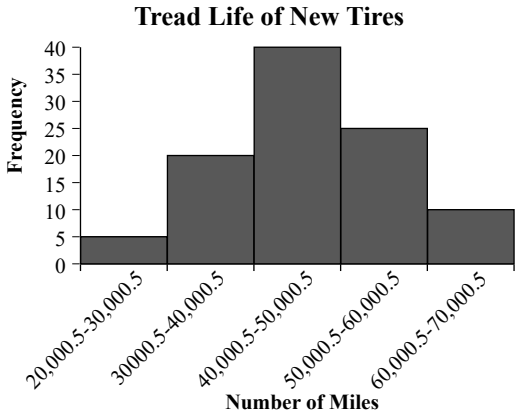
d. What was the mean rainfall over the six-month period (to the nearest hundredth)?

8. The following histogram summarizes the tread life for 100 types of new tires.

a. How many classes are represented?

b. What is the width of each class?

c. Which class has the highest frequency?



Writing & Thinking

9. State three properties or characteristics that should be true of all graphs so that they can communicate numerical data quickly and easily.

10. Compare and contrast a bar graph and a histogram.

Solution

$$101_2 = \left(1 \times 2^2\right) + \left(0 \times 2^1\right) + \left(1 \times 2^0\right)$$
$$101_2 = 4 + 0 + 1$$
$$101_2 = 5$$

7.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. In the number 21,057, the "1" represents 1000.

2. In the number 17,536, the number in the tens place is 1.

Practice

3. Given the number 284,065 which digit indicates the number of

a. tens?

b. ten thousands?

c. hundreds?

4. For the number 309,472 state the place value of

a. the digit 9.

b. the digit 4.

c. the digit 7.

5. Name the place value of each nonzero digit in the following number: 24,608.

6. Name the place value of each nonzero digit in the following number: 2,403,189,500.

Writing & Thinking

7. How are natural numbers and whole numbers different and how are they the same?

$$\begin{array}{r} {}^{3}\not{4}\,{}^{5+}\not{2}_{5} \\ -\ 3\ 3_{5} \\ \hline 4_{5} \end{array}$$

Borrow 5^{1} from the second column.

$(5 + 2) - 3 = 4$

No conversion necessary.

$$\begin{array}{r} {}^{3}\not{4}\,{}^{5+}\not{2}_{5} \\ -\ 3\ 3_{5} \\ \hline 0\ 4_{5} \end{array}$$

$3 - 3 = 0$

No conversion necessary.

$$\begin{array}{r} {}^{3}\not{4}\,{}^{5+}\not{2}_{5} \\ -\ 3\ 3_{5} \\ \hline 1\ 0\ 4_{5} \end{array}$$

$1 - 0 = 1$

No conversion necessary.

Thus, $142_5 - 33_5 = 104_5$.

7.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. A polygon is a geometric figure in a plane with two or more sides.

2. To find the perimeter of a rectangle, add the lengths of the four sides.

3. When subtracting, sometimes the digit being subtracted is larger than the digit it is being subtracted from and so "carrying" must occur.

4. If your bank account has a balance of \$743 and you want to withdraw \$115, you would use subtraction to find that the new balance would be \$628.

Practice

Simplify.

5. $\begin{array}{r} 15 \\ +43 \\ \hline \end{array}$

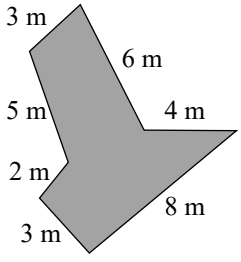
6. $\begin{array}{r} 981 \\ +46 \\ \hline \end{array}$

7. $\begin{array}{r} 275 \\ -131 \\ \hline \end{array}$

8. $\begin{array}{r} 543 \\ -167 \\ \hline \end{array}$

Calculate the perimeter of the geometric figure.

9.



Applications

Solve.

10. The Magley family has the following monthly budget: $815 mortgage; $69 electric; $47 water; and $122 phone bills (including cell phones). What is the family's budget for each month for these expenses?

11. A couple sold their house for $135,000. They paid the realtor $8100, and other expenses of the sale came to $800. If they owed the bank $87,000 for the mortgage, what were their net proceeds from the sale?

Writing & Thinking

12. List three properties of addition and give an example of each.

13. Explain when "carrying" should be used in addition with whole numbers and give an example.

$\overset{1}{1}02_3$ $2 \times 2 = 4_{10}$

$\times\ 2_3$ Convert 4_{10} to a base 3 number: $4_{10} = (1 \times 3^1) + (1 \times 3^0) = 11_3$

1_3 Write 5 and carry the 1.

$\overset{1}{1}02_3$ $(2 \times 0) + 1 = 1_3$

$\times\ 2_3$ No conversion necessary.

11_3

$\overset{1}{1}02_3$ $2 \times 1 = 2_3$

$\times\ 2_3$ No conversion necessary.

211_3

Thus, 102_3 multiplied by 2_3 is 211_3.

7.R.3 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The numbers being multiplied are called the divisors.

2. According to the multiplicative identity, $1 \cdot 25 = 52$.

3. According to the distributive property, $4 \cdot (7 + 2) = 4 \cdot 7 + 4 \cdot 2$.

4. The associative property of multiplication indicates that length can be multiplied by width or width can be multiplied by length to get the same answer.

Practice

Multiply.

5. 42×56

6. $20 \cdot 200$

Use your knowledge of the properties of multiplication to find the value of the variable that will make the statement true. State the property illustrated.

7. $(5 \cdot 10) \cdot y = 5 \cdot (10 \cdot 7)$

8. Rewrite $7(8 + 4)$ by using the distributive property then simplify.

Calculate the area of the given rectangle.

9.

Applications

Solve.

10. A group of 15 friends are gathering at a restaurant. The restaurant is having a special where each person can order a three-course meal for $35. If all 15 friends order this special, how much will the total bill going be?

11. A sandwich shop buys 372 loaves of bread for the week. If each loaf of bread has 24 slices, how many slices of bread were purchased?

Writing & Thinking

12. Explain, in your own words, what the zero-factor law indicates.

13. Name the property that uses both multiplication and addition and give an example of it.

7.R.4 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If a division problem has a nonzero remainder, then the divisor and quotient are factors of the dividend.

2. $13 \div 1 = 13$

3. $12 \div 0 = 12$

4. $\frac{0}{7}$ is undefined.

Practice

Divide.

5. $13\overline{)0}$

6. $0\overline{)51}$

7. $12\overline{)108}$

8. $11\overline{)4406}$

Applications

Solve.

9. One pint of Ben and Jerry's Crème Brûlée Ice Cream has 64 grams of fat. If there are 4 servings per pint, how many grams of fat are in each serving?

10. US Astronaut Peggy Whitson orbited the Earth 6032 times during her space flights on the International Space Station. If the International Space Station orbits the Earth 16 times per day, how many days was Petty Whitson in space?

Writing & Thinking

11. Explain how you would check a division problem that has a nonzero remainder.

12. Discuss how division is related to multiplication.

8.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. A number that is divisible by 10 is also divisible by 2 and 5.

2. 6801 is divisible by 9.

3. 7605 is divisible by 10.

4. 5,187,042 is divisible by 3.

Practice

Using the tests for divisibility, determine which of 2, 3, 4, 5, 6, 9, and 10 (if any) will divide exactly into each given number.

5. 105

6. 150

7. 331

8. 1234

Applications

Solve.

9. You are on a team that is participating in a charity walk with a goal to raise $12,400. Each team member agrees to raise the same amount of money. If the possible team sizes are 5, 6, 9, or 10 members, which team sizes allow the goal amount to be evenly split between the team members? How much money would each team member raise for each team size that can evenly split the goal amount?

10. A company is working on a project that will take 440 hours of work to complete. The manager in charge of the project has the option to have 4, 6, or 8 people work on the project. If the manager wants to evenly divide the work between the team members, which team size will evenly split the work hours? How many hours would each team member spend on the project for each team size that evenly splits the work hours?

Writing & Thinking

11. a. If a number is divisible by both 3 and 5, then it will be divisible by 15. Give two examples.

b. However, a number might be divisible by 3 and not by 5. Give two examples.

c. Also, a number might be divisible by 5 and not 3. Give two examples.

8.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. A prime number has exactly 1 factor.

2. A composite number has 2 or more factors.

3. 231 is a prime number.

4. All the factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30.

Practice

Determine whether each number is prime or composite. If the number is composite, find at least three factors of the number.

5. 47

6. 63

Find the prime factorization of each number. Use the tests for divisibility for 2, 3, 4, 5, 6, 9, and 10 whenever they help to find beginning factors.

7. 125

8. 150

Applications

Solve.

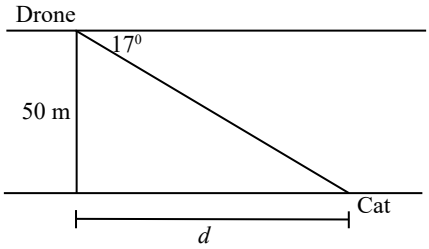
9. Twenty-four pencils are to be distributed evenly between the members of a group. What are the possible group sizes if each person in the group is to receive the same number of pencils?

10. A chocolatier makes 72 specialty truffles. She wants to sell packages that each have the same number of truffles. What are her options for the number of truffles that can be in a package?

Writing & Thinking

11. Are all odd numbers also prime numbers? Explain your answer.

12. Explain the difference between factors of a number and multiples of that number.



Notice that the angle of depression from the drone to the cat (15^o) is equal to the angle of elevation from the cat to the drone, θ, because they are alternate interior angles. From the sketch, we see that the length of the opposite side is 50 meters and the unknown length (d) is the adjacent side. Thus, we should use the tangent function, to $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$, find d.

$$\tan 15^\circ = \frac{50\,\text{m}}{d}$$
$$d = \frac{50\,\text{m}}{\tan 15^\circ}$$
$$d \approx 186.6\,\text{m}$$

Therefore, the horizontal distance between the cat and the drone is approximately 186 meters.

9.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The sum of the measures of two complementary angles is equal to the measure of one right angle.

2. The sum of the measures of complementary angles is greater than the sum of the measures of supplementary angles.

3. Adjacent angles are two angles that share a side.

4. If two lines in a plane are not parallel, then they are perpendicular.

5. A triangle with sides of 4 inches, 4 inches, and 3 inches is an isosceles triangle.

6. A triangle with three angles that each measure less than 90 degrees is an acute triangle.

Practice

7. Name the type of angle formed by the hands on a clock.

a.  **b.**  **c.**  **d.** 

a. at six o'clock

b. at three o'clock

c. at one o'clock

d. at five o'clock

8. Assume that $\angle 1$ and $\angle 2$ are complementary.

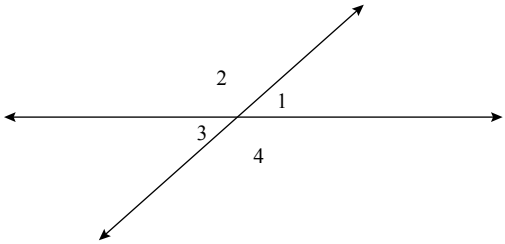
a. If $m\angle 1 = 15°$, what is $m\angle 2$?

b. If $m\angle 1 = 3°$, what is $m\angle 2$?

c. If $m\angle 1 = 45°$, what is $m\angle 2$?

d. If $m\angle 1 = 75°$, what is $m\angle 2$?

9. The figure shows two intersecting lines.



a. If $m\angle 1 = 30°$, what is $m\angle 2$?

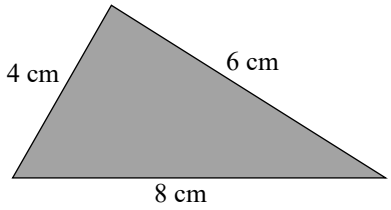
b. Is $m\angle 3 = 30°$? Give a reason for your answer other than the fact that $\angle 1$ and $\angle 3$ are vertical angles.

c. Name two pairs of congruent angles.

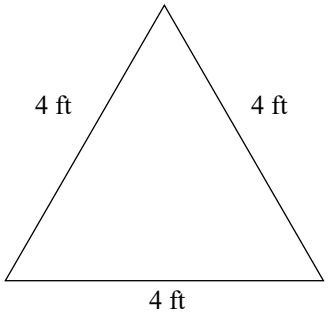
d. Name four pairs of adjacent angles.

Classify each triangle in the most precise way possible, given the indicated lengths of its sides and/or measures of its angles.

10.



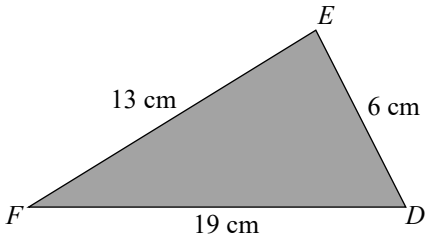
11.



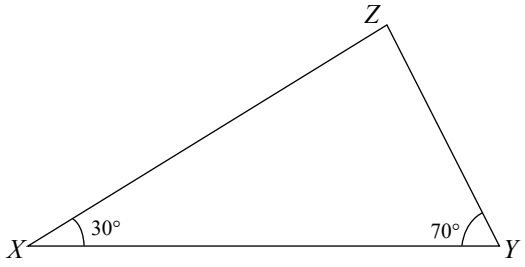
Applications

Solve.

12. Suppose the lengths of the sides of $\angle DEF$ are as shown in the figure. Is this possible? Explain your reasoning.



13. In the triangle shown, $m\angle X = 30°$ and $m\angle Y = 70°$.



a. What is $m\angle Z$?

b. What kind of triangle is $\angle XYZ$?

c. Which side is opposite $\angle X$?

d. Which sides include $\angle X$?

e. Is $\angle XYZ$ a right triangle?

Writing & Thinking

14. Explain, in your own words, the relationships between vertex, ray, angle, and line.

We will label the distance of the ladder from the tree as one leg, $a = 12$, and the length of the ladder as the hypotenuse, $c = 20$. Then the height of the branch can be found by solving the following equation for b.

$$\begin{aligned} c^2 &= a^2 + b^2 && \text{Pythagorean Theorem} \\ 400 &= 144 + b^2 \\ 400 - 114 &= b^2 \\ b &= \sqrt{256} \\ b &= 16 \end{aligned}$$

9.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. 49 is a perfect square.

2. In the expression $\sqrt{81}$, the number 9 is the radicand.

3. The Pythagorean Theorem can be used to find the length of the longest side of a right triangle if the lengths of the two legs are known.

4. The Pythagorean Theorem works for any type of triangle.

Practice

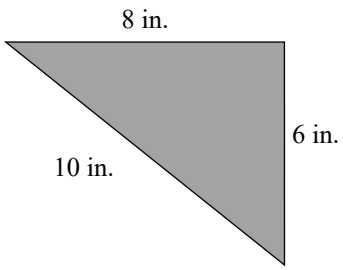
Evaluate each expression.

5. $\sqrt{36}$

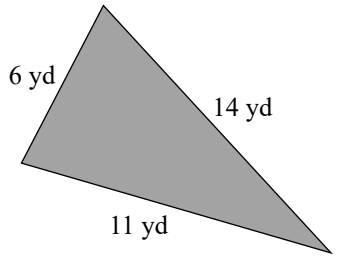
6. $\sqrt{225}$

Use the Pythagorean Theorem to determine whether or not each triangle is a right triangle.

7.

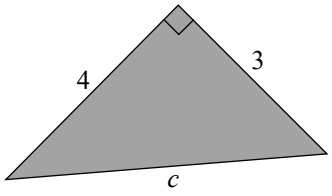


8.



Find the hypotenuse for the given right triangle accurate to the nearest hundredth.

9.

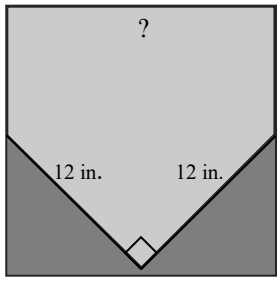


Applications

Solve.

10. The base of a fire engine ladder is 30 feet from a building and reaches to a third floor window 50 feet above ground level. Find the length of the ladder to the nearest hundredth of a foot.

11. The shape of home plate in the game of baseball can be created by cutting off two triangular pieces at the corners of a square, as shown in the figure. If each of the triangular pieces has a hypotenuse of 12 inches and legs of equal length, what is the length of one side of the original square, to the nearest tenth of an inch?



Writing & Thinking

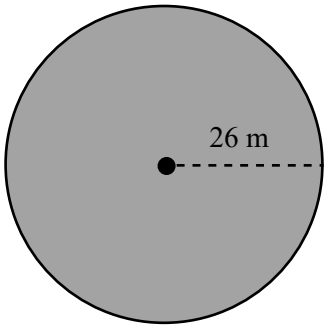
12. Explain the connection between a perfect square and its square root. Give an example.

Looking Ahead

The evaluation skills you just reviewed will be put to good use when solving problems involving geometric formulas like the one below.

Example Preview

Given the following figure of a table top, how many square centimeters of paint would be required to cover the top's surface? Use $\pi = 3.14$ and round your answer to the nearest hundredth, if necessary.



Solution

To calculate how many square centimeters of paint would be required to cover the top's surface, we use the formula for the area of a circle, $A = \pi r^2$, with $r = 26$ cm.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi(26 \text{ cm})^2 \\ &= \pi(676 \text{ cm}^2) \\ &\approx (3.14)(676 \text{ cm}^2) \\ &\approx 2122.64 \text{ cm}^2 \end{aligned}$$

9.R.3 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. A variable that does not appear to have an exponent has an exponent of 1.

2. Like terms have the same coefficients.

Practice

Simplify each expression and then evaluate the expression for $y = 3$ and $a = -2$.

3. $5y + 4 - 2y$

4. $\frac{3a + 5a}{-2} + 12a$

Applications

Solve.

5. An apartment management company owns a property with 100 units. The company has determined that the profit made per month from the property can be calculated using the equation $P = -10x^2 + 1500x - 6000$, where x is the number of units rented per month. How much profit does the company make when 80 units are rented?

6. A ball is thrown upward from an initial height of 96 feet with an initial velocity of 16 feet per second. After t seconds, the height of the ball can be described by the expression $-16t^2 + 16t + 96$. What is the height of the ball after 3 seconds?

Writing & Thinking

7. Discuss like and unlike terms and give an example of each.

8. Explain the difference between -13^2 and $(-13)^2$.

9.R.4 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. When using formulas, typically it does not matter if capital or lower case letters are used: $A = a$, $C = c$, etc.

2. If the perimeter and length are known, $P = 2l + 2w$ can be used to find the width of a rectangle.

3. Rate of interest is stated as an annual rate in percent form.

Practice

Solve each formula for the indicated variable.

4. $P = a + b + c$; solve for b.

5. $P = 3s$; solve for s.

6. $I = Prt$; solve for t.

7. $A = P(1 + rt)$; solve for r.

Applications

In the following application problems, read the descriptions carefully and then substitute the values given in the problem for the corresponding variables in the formulas. Evaluate the resulting expression for the unknown variable.

Velocity

If an object is shot upward with an initial velocity v_0 in feet per second, the velocity v in feet per second is given by the formula $v = v_0 - 32t$, where t is time in seconds. (v_0 is read "v sub zero." The $_0$ is called a subscript.)

8. An object projected upward with an initial velocity of 106 feet per second has a velocity of 42 feet per second. How many seconds have passed?

Investments

The total amount of money in an account with P dollars invested in it is given by the formula $A = P + Prt$, where r is the rate expressed as a decimal and t is time (one year or part of a year).

9. If \$1000 is invested at 6% interest, find the total amount in the account after 6 months.

Cost

The total cost C of producing x items can be found by the formula $C = ax + k$, where a is the cost per item and k is the fixed costs (rent, utilities, and so on).

10. Find the total cost of producing 30 items if each costs \$15 and the fixed costs are \$580.

Solution

Because the cards are being drawn with replacement, the first card drawn has no effect on the next card drawn, and so on. So the events are independent, and we can use the Multiplication Rule for Independent Events.

Let A be the event "draw a diamond," B the event "draw a black card," and C the event "draw a ten." Recall that a standard deck of 52 cards contains 13 diamonds, 26 black cards, and 4 tens. So, $P(A)=\frac{13}{52}, P(B)=\frac{26}{52},$ and $P(C)=\frac{4}{52}.$

Then, applying the Multiplication Rule for independent events, the probability that the first card drawn will be a diamond, the second card a black card, and the third card a ten is

$$P(A)\cdot P(B)\cdot P(C)=\frac{13}{52}\cdot\frac{26}{52}\cdot\frac{4}{52}=\frac{1}{104}\approx 0.009615$$

10.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. In $\frac{11}{13}$, the denominator is 11.

2. $\frac{17}{0}$ is undefined.

3. To find $\frac{1}{2}$ of $\frac{2}{9}$ requires multiplication.

4. The statement $\frac{1}{3}\cdot\frac{2}{5}=\frac{2}{5}\cdot\frac{1}{3}$ is an example of the associative property of multiplication.

Practice

Raise each fraction to higher terms as indicated.

5. $\frac{1}{7} = \frac{?}{35}$

6. $\frac{3}{5} = \frac{?}{60}$

Reduce each fraction to lowest terms.

7. $\frac{5}{23}$

8. $\frac{29}{39}$

Simplify and reduce to lowest terms.

9. $\frac{7}{3} \cdot \frac{1}{8}$

10. $\frac{7}{15} \cdot \frac{3}{8}$

11. $\frac{48}{7} \div \frac{48}{27}$

12. $\frac{11}{10} \div \frac{27}{26}$

Applications

Solve.

13. A bus is carrying 90 passengers, which is $\frac{9}{10}$ of the capacity of the bus. What is the capacity of the bus?

14. There are 3000 students at Canyon High School and $\frac{1}{4}$ of these students are seniors. If $\frac{3}{5}$ of the seniors are opposed to the school forming a rock climbing team and $\frac{9}{10}$ of the remaining students (not seniors) are also opposed to forming a rock climbing team, how many students are in favor of this idea?

Writing & Thinking

15. If two fractions are between 0 and 1, can their product be more than 1? Explain.

10.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The LCM of 15 and 25 is 50.

2. The first five multiples of 9 are 9, 18, 27, 36, and 45.

3. The first five multiples of 4 are 4, 8, 12, 20, and 24.

4. When given larger numbers, the most efficient way to find the LCM is to use the prime factorization method.

Practice

Find the LCM of each set of numbers.

5. 6, 10

6. 3, 4, 8

7. For 14, 35, and 49, **a.** find the LCM and **b.** state how many times each number divides into the LCM.

For each equation, find the missing numerator that will make the fractions equivalent.

8. $\frac{5}{8} = \frac{?}{24}$

9. $\frac{5}{12} = \frac{?}{108}$

Applications

Solve.

10. Three security guards meet at the front gate for coffee before they walk around inspecting buildings at a manufacturing plant. The guards take 15, 20, and 30 minutes, respectively, for the inspection trip.

a. If they start at the same time, in how many minutes will they meet again at the front gate for coffee?

b. How many trips will each guard have made?

11. A fruit production company has three packaging facilities, each of which uses different-sized boxes as follows: 24 pieces/box, 36 pieces/box, and 45 pieces/box.

a. Assuming that the truck provides the same quantity of uniformly-sized pieces of fruit to all three packaging facilities, what is the minimum number of pieces of fruit that will be delivered so that no fruit will be left over?

b. How many boxes will each facility package?

Writing & Thinking

12. Explain, in your own words, why each number in a set divides evenly into the LCM of that set of numbers.

13. Explain why simply multiplying two numbers together will not necessarily find the LCM of those numbers. Give an example of when it would find the LCM and an example when it would not.

There are 10 students who live on-campus, so the probability of choosing a student who lives on-campus is

$$P(\text{on-campus}) = \frac{10}{17}.$$

We also need to know the probability that a randomly selected student is both a freshman and lives on-campus. There are 10 students who are both a freshman and live on-campus. So, the probability of choosing a student who is in both groups is

$$P(\text{freshman and on-campus}) = \frac{3}{17}.$$

Now, using the Addition Rule for Probability, we can calculate the probability that a randomly chosen member of the Student Government Board is a freshman or lives in on-campus housing.

$$\begin{aligned} P(\text{freshman or on-campus}) &= \frac{5}{17} + \frac{10}{17} - \frac{3}{17} \\ &= \frac{5+10-3}{17} \\ &= \frac{12}{17} \\ &\approx 0.705882 \end{aligned}$$

Therefore, the probability that a randomly chosen member of the Student Government Board is a freshman or lives in on-campus housing is 0.705882.

10.R.3 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The final step in adding fractions is to reduce, if possible.

2. The process for finding the LCD is the same as the process for finding the LCM.

3. When subtracting fractions, simply subtract the numerators and the denominators.

4. Subtraction of fractions requires that the fractions have the same denominators.

Practice

Perform the indicated operation and reduce your answer.

5. $\frac{5}{32}+\frac{15}{32}$

6. $\frac{11}{21}+\frac{5}{35}$

7. $\frac{6}{18}+\frac{2}{6}$

8. $\frac{8}{11}-\frac{4}{11}$

9. $\frac{11}{18}-\frac{2}{9}$

10. $\frac{17}{20}-\frac{9}{15}$

Applications

Solve.

11. If your income is \$4820 a month and you plan to budget $\frac{3}{4}$ of your income for rent and $\frac{1}{20}$ of your income for food, how much do you plan to spend each month on these two items? Simplify your answer.

12. Three letters weigh $\frac{1}{2}$ ounces, $\frac{1}{3}$ ounces, and $\frac{5}{6}$ ounces. What is the total weight of the letters? Simplify your answer.

Writing & Thinking

13. Explain the steps to follow when adding or subtracting fractions with unlike denominators

10.R.4 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. When a decimal number is changed to a fraction, the denominator will be the power of 10 that names the rightmost digit of the decimal number.

2. When a decimal number is changed to a fraction, the numerator can be determined by using the whole number that is formed by all the digits of the decimal number.

3. Fractions can always be converted to decimal form without losing accuracy.

4. In decimal form, $\frac{1}{3}$ is repeating and nonterminating.

Practice

Change each decimal number to a fraction or mixed number in lowest terms.

5. 0.18

6. 2.75

Change each fraction to a decimal number rounded to the nearest hundredth.

7. $\frac{20}{3}$

8. $\frac{40}{9}$

Simplify the expression by first writing all of the numbers in decimal form. Round to the nearest hundredth, if necessary.

9. $\frac{1}{4}+0.25+\frac{1}{5}$

10. Arrange $0.76, \frac{3}{4}, \frac{7}{10}$ in order from smallest to largest.

Applications

Solve.

11. A rectangle measures 6.4 inches in length, and has a width that measures $\frac{2}{5}$ of the length. Find the perimeter of the rectangle.

12. A loaf of bread weighs 21.6 ounces. Mauricio cut off a third of the loaf to save for later and then cut the remaining portion into 16 equal slices. What was the weight of each slice of the 16 slices he cut?

Writing & Thinking

13. Describe the process used to change a terminating decimal number to a fraction.

14. List 2 different ways to solve this problem: $\frac{1}{2} + 3.67 - \frac{1}{8}$. State which method you prefer and why.

This gives 8 possible outcomes for the first three points: {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}. Notice that only 1 of the outcomes consists of the coin landing on tails three times (TTT). Thus, the probability of the older player winning all three of the points is calculated as follows.

$$P(\text{Tails Three Times}) = \frac{n(E)}{n(S)} = \frac{1}{8} = 0.125$$

Notice that the probability of the coin landing on heads all three times is also $\frac{1}{8}$.

10.R.5 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The individual result of an experiment is a probability.

2. An event is some or all of the outcomes from the sample space.

3. A single result of an experiment is an outcome.

4. Each branch of a tree diagram represents a separate possible outcome.

Applications

For each experiment, draw a tree diagram illustrating the possible outcomes and list the outcomes in the sample space.

5. Four marbles are in a box: one red, one white, one blue, and one purple. One ball is chosen.

6. There are three flavors of potato chips to choose from: original, BBQ, and cheddar. One flavor is chosen.

Writing & Thinking

7. List at least three activities that are experiments of chance.

8. Explain the benefits of using a tree diagram to determine probability.

a. Which degree type occurred the most?

b. How many total students were enrolled during the fall 2019 semester?

Solution

a. The degree type that occurred the most was baccalaureate with 13,294 students.

b. To find the total students enrolled, we need to find the sum of students enrolled in each degree type.

$$33 + 933 + 13{,}294 + 1600 + 675 + 429 + 2254 = 19{,}218$$

Therefore, there were 19,218 students enrolled during the fall 2019 semester.

11.R.1 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. Graphs should always be clearly labeled, easy to read, and have appropriate titles.

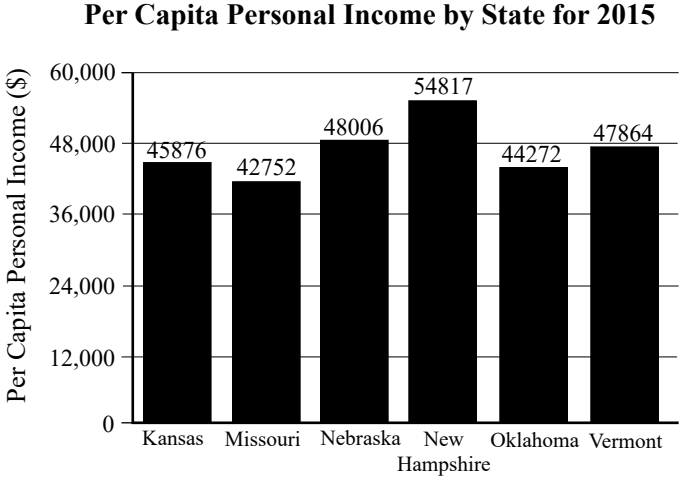
2. Circle graphs show trends over a period of time.

3. Line graphs are used to indicate a trend over a period of time.

4. Pictographs are used to illustrate percentages as part of a whole.

Practice

5. The following bar graph shows the per capita personal incomes for six states in 2015. Use this bar graph to answer the questions.



a. Find the lowest per capita personal income for the six states shown. Round your answer to the nearest hundredth, if necessary.

b. Find the highest per capita personal income for the six states shown. Round your answer to the nearest hundredth, if necessary.

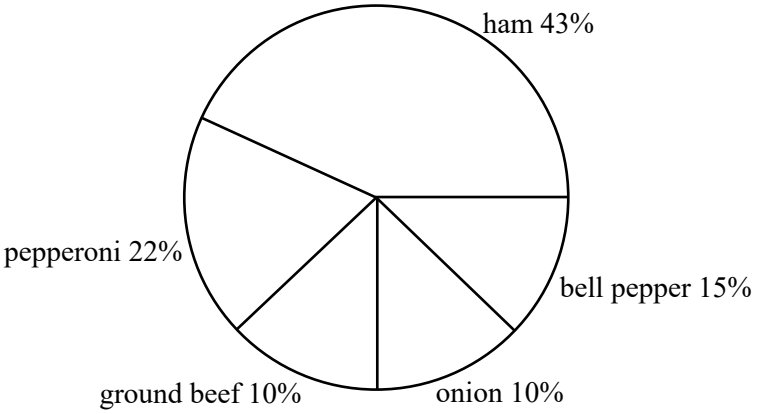
c. Which state of the six shown has the highest per capita personal income?

d. What is the difference in per capita personal income between New Hampshire and Missouri? Round your answer to the nearest hundredth, if necessary.

e. If a single resident of Missouri makes $40,000 in 2015, what percent of the per capita personal income for Missouri was his/her salary? Round your answer to the nearest hundredth of a percent, if necessary.

6. The Pizza Pie 'N Go sells about 1600 one-topping pizzas each month. The circle graph displays the most requested one-topping pizzas, by percentage, for one month.

Most Popular One-Topping Pizzas



a. Find the number of ham pizzas sold each month. Round your answer to the nearest integer.

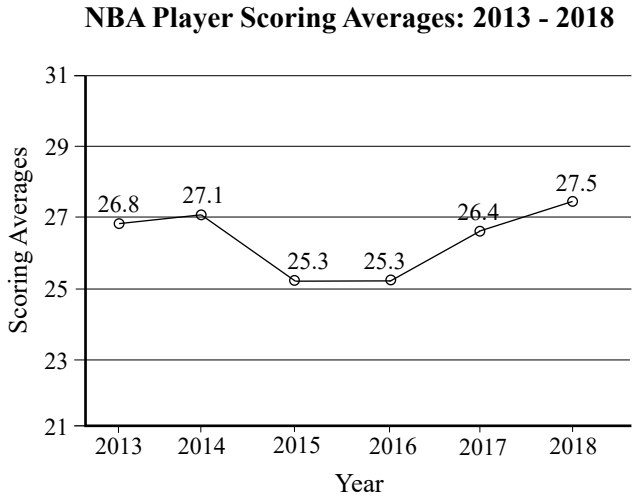
b. Find the number of pepperoni pizzas sold each month. Round your answer to the nearest integer.

c. Find the number of ground beef pizzas sold each month. Round your answer to the nearest integer.

d. Find the number of onion pizzas sold each month. Round your answer to the nearest integer.

e. Find the number of bell pepper pizzas sold each month. Round your answer to the nearest integer.

7. The following line graph shows the per game scoring averages for Lebron James from the 2013 NBA season to the 2018 NBA season. Use the line graph to answer the questions.



a. Find the lowest per game scoring average for the six seasons shown.

b. Find the highest per game scoring average for the six season shown.

Writing & Thinking

8. State three properties or characteristics that should be true of all graphs so that they can communicate numerical data quickly and easily.

11.R.2 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. In creating a vertical bar graph, a bar's width should vary based on the number it represents.

2. The first step in constructing a vertical bar graph is to draw a vertical and a horizontal axis.

3. Bar graphs can have either vertical or horizontal bars.

4. Titles are unnecessary for circle graphs.

Applications

For each set of data, construct the specified graph.

5. Construct a bar graph that represents the following data.

Largest Islands of the World

Island	Area in Square Miles (nearest ten thousand)
Greenland	840,000
New Guinea	310,000
Borneo	290,000
Madagascar	230,000
Baffin	200,000
Sumatra	180,000
Honshu	90,000
Great Britain	90,000

6. Construct a bar graph that represents the following data.

10 Top Films by Domestic Box Office Earnings, 2016

Motion Picture	Box Office(in millions of dollars)
Rogue One: A Star Wars Story	$532.2
Finding Dory	$486.3
Captain America: Civil War	$408.1
The Secret Life of Pets	$368.4
The Jungle Book (2016)	$364.1
Deadpool	$363.1
Zootopia	$341.3
Batman v Superman: Dawn of Justice	$330.4
Suicide Squad	$325.1
Sing	$270.3

Source: Information courtesy of Box Office Mojo. Used with permission. www.boxofficemojo.com

7. Construct a circle graph that represents the following data.

Percent of Population with Particular Blood Types

Type of Blood	Percent of Population
O positive (O^+)	38%
O negative (O^-)	7%
A positive (A^+)	34%
A negative (A^-)	6%
B positive (B^+)	9%
B negative (B^-)	2%
AB positive (AB^+)	3%
AB negative (AB^-)	1%

Source: AABB.org

8. Construct a circle graph that represents the following data.

Sources for World Electricity Generation in 2014

Source of Energy	Percent
Coal/Peat	40.8%
Natural Gas	21.6%
Hydro	16.4%
Nuclear	10.6%
Oil	4.3%
Other	6.3%

Source: Key World Energy Statistics, 2016. International Energy Agency.

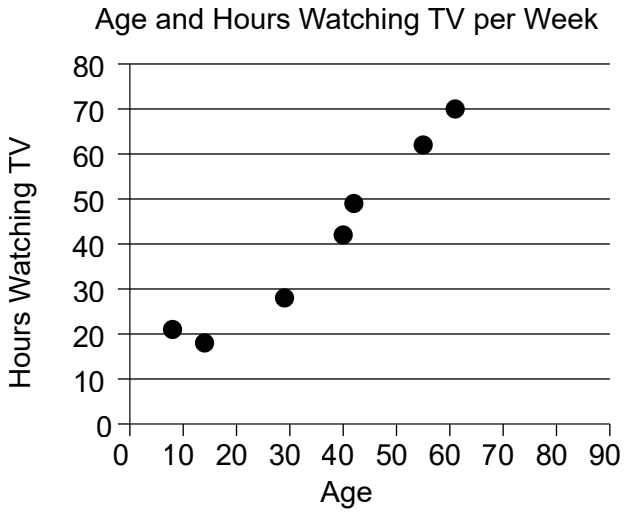
Writing & Thinking

9. While most graphs can be created through the use of a computer, give at least one benefit from constructing a graph yourself.

10. List three mistakes a person might make when constructing a graph.

Solution

Begin by creating a scatter plot of the data to determine if there is a correlation between age and the number of hours a person watches TV per week.



Notice that there is an upward trend in the data. Therefore, the correlation in words is positive.

11.R.3 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. The y-intercept is the point where a line crosses the y-axis.

2. The terms ordered pair and point are used interchangeably.

3. A horizontal line does not have a y-intercept.

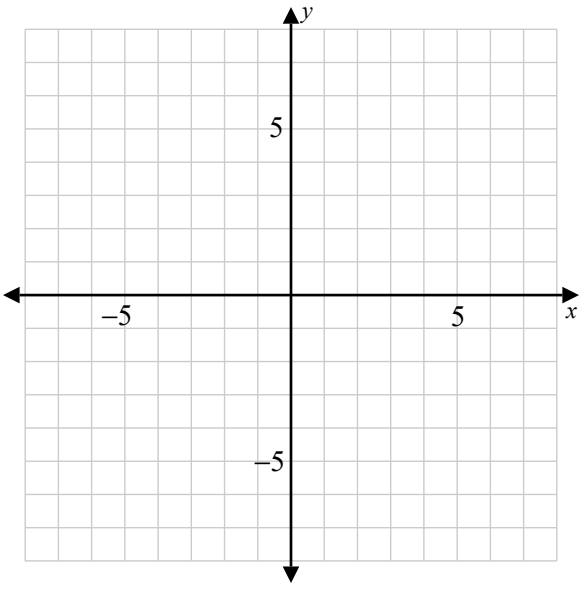
4. All x-intercepts correspond to an ordered pair of the form .

Practice

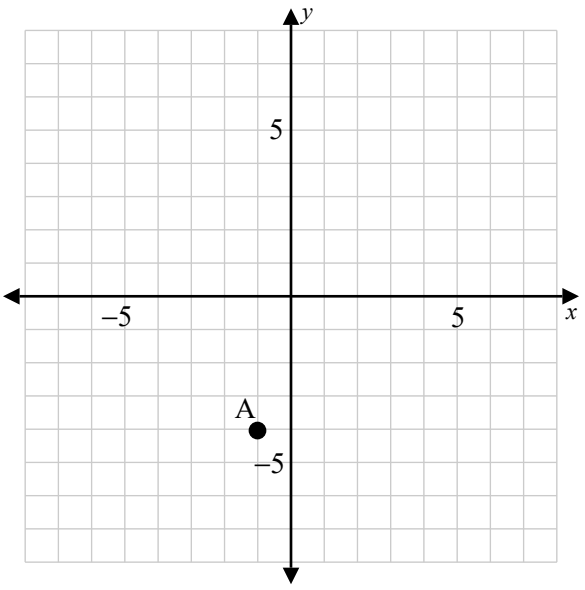
5. Consider the following.

A (2, −1)

a. Plot the given point on the graph.



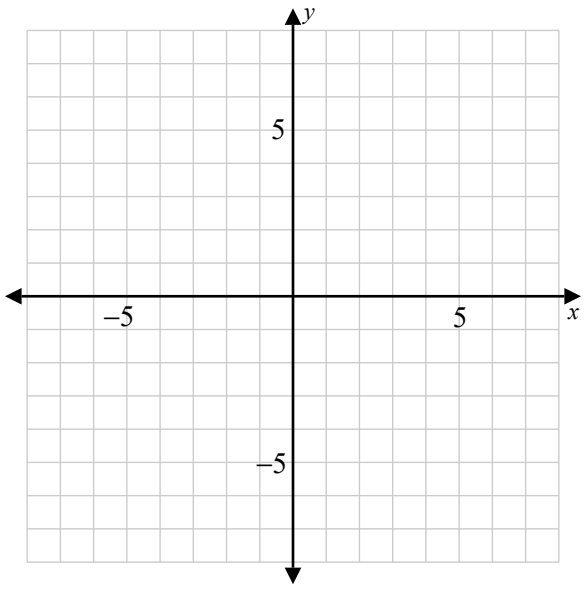
b. Identify the coordinates of the point A on the graph.



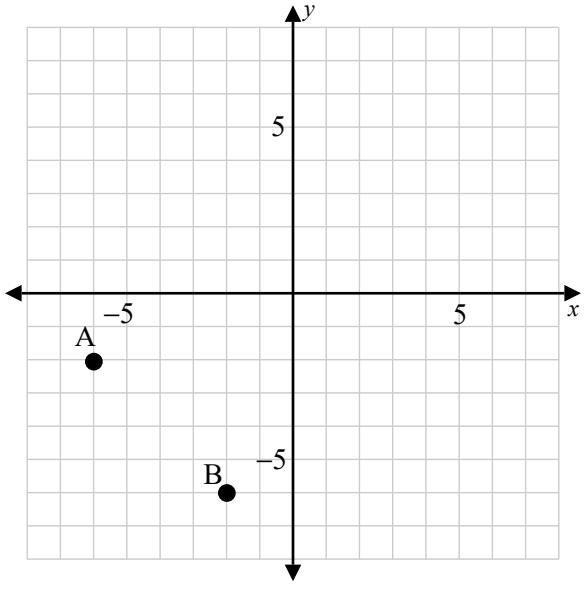
6. Consider the following.

A (−5, −4) B (−1, 1)

a. Plot the given points on the graph.



b. Identify the coordinates of the points A and B on the graph.



7. Consider the following equation.

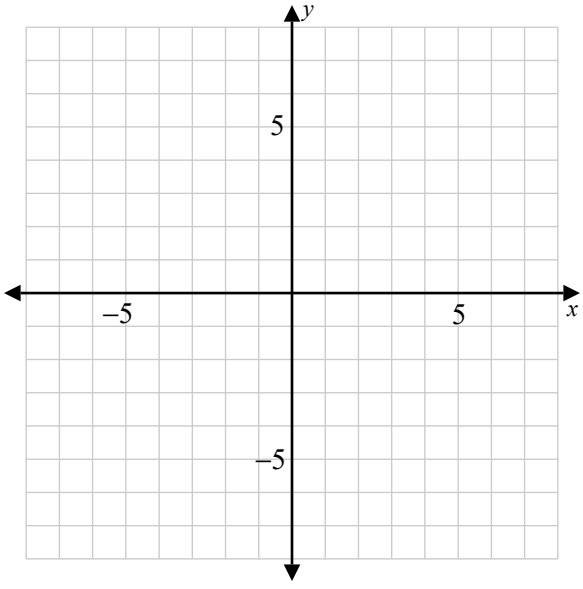
$5x + 5y = 18$

a. Determine the missing coordinate in the ordered pair (2,?) so that it will satisfy the given equation.

b. Determine the missing coordinate in the ordered pair so that it will satisfy the given equation.

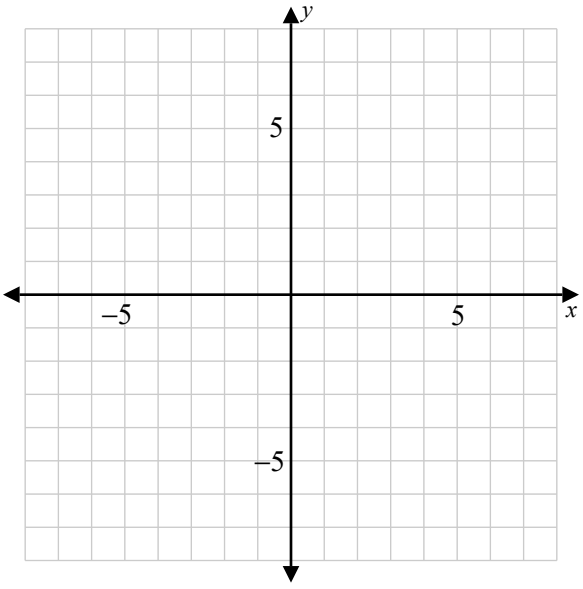
8. Graph the line by plotting any two ordered pairs with integer value coordinates that satisfy the equation.

$-9x + 9y = 0$



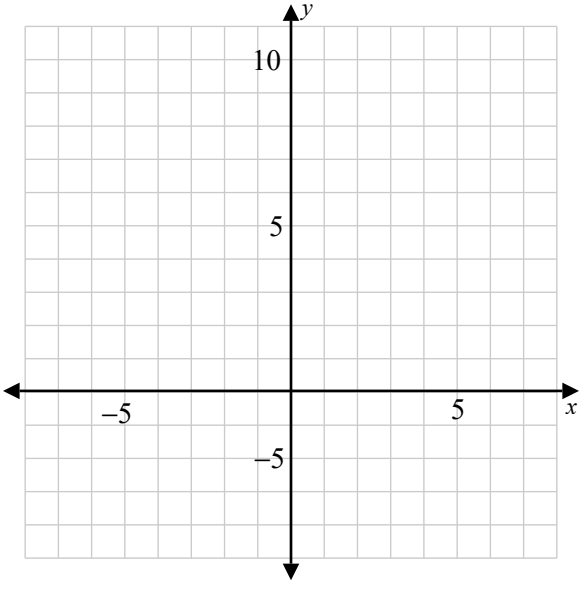
9. Graph the line by plotting any two ordered pairs that satisfy the equation.

$y = -\frac{1}{5}x - 1$



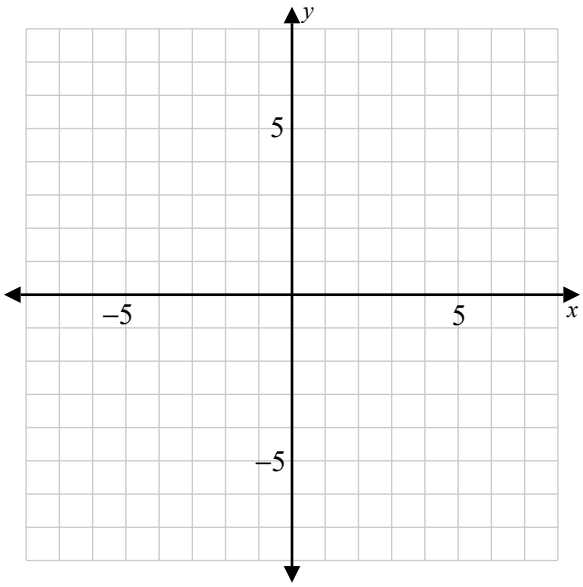
10. Graph the line by plotting the x-intercept and y-intercept.

$-10x + 5y = 50$



11. Graph the line by plotting the x-intercept and y-intercept.

$-7x + 6y = 16$



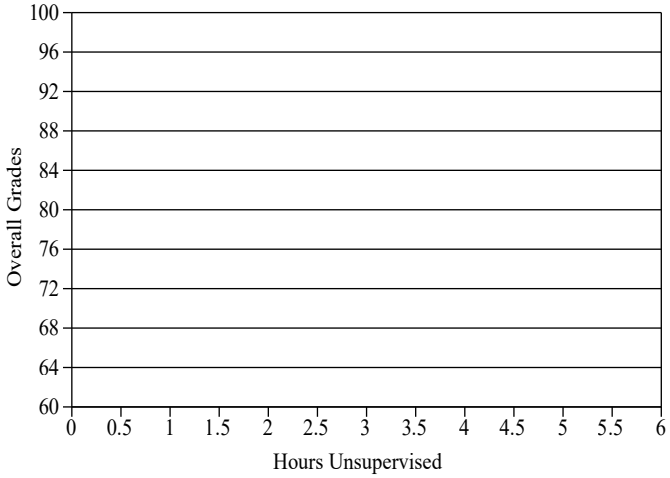
Applications

Solve.

12. The following table gives the average number of hours 7 junior high students were left unsupervised each day and their corresponding overall grade averages.

Hours Unsupervised	0	0.5	1	2.5	3.5	4.5	5
Overall Grades	96	92	88	80	76	72	68

Draw a scatter plot of the given data.

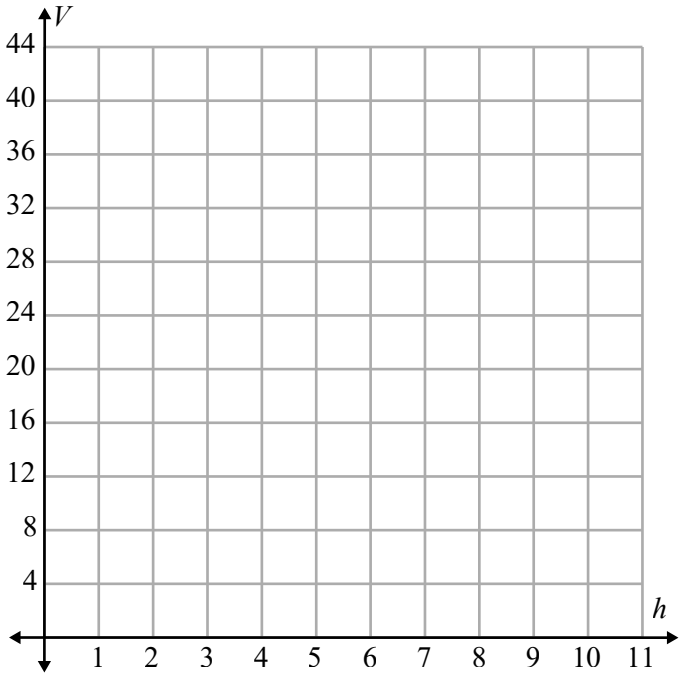


13. Consider the equation $V = 4h$ where V is the volume (in cubic centimeters) of a box with a variable height h in centimeters and a fixed base of area 4 cm^2.

a. Complete the table below so that each ordered pair will satisfy the equation.

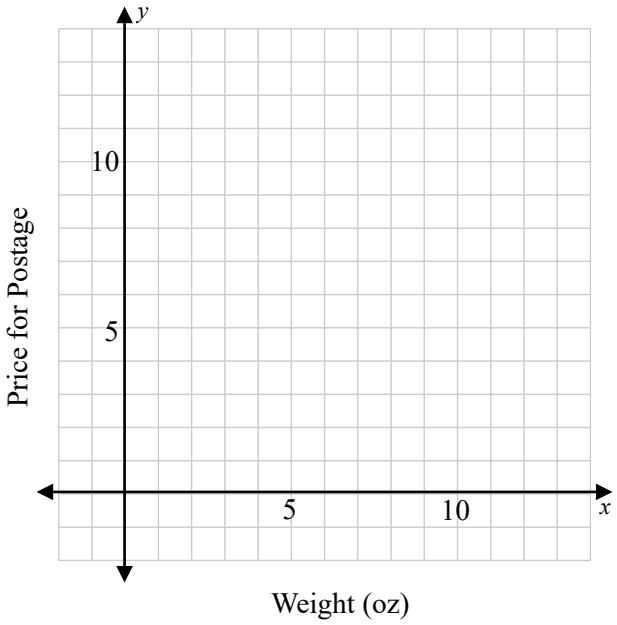
h	V
2	
6	
7	
9	

b. Plot the points corresponding to the ordered pairs from the previous part.



14. To mail a letter to Vienna, Austria the post office charges a flat rate of \$6.50 and an additional \$0.25 for every ounce the letter weighs. The cost of mailing a letter is determined by the equation $y = 0.25x + 6.50$, where y is the cost of the postage and x is the weight of the letter in ounces.

a. Graph the equation by finding two points that satisfy the equation and plotting them on the graph.



b. Use the graph to estimate to the nearest quarter of a dollar (\$0.25) the cost to send a letter that weighs 4 ounces.

Solution

The slope is the coefficient of the variable x in the regression equation. Therefore, the slope is 0.697. Notice that the slope is positive, which indicates a positive relationship between the hours spent on homework and the grades on the first test. For every 1 hour increase in time spent on homework, the grade on the first test increases by 0.697 points.

11.R.4 Exercises

Practice

Find the slope of the line determined by each pair of points.

1. $(1, -2); (1, 4)$

2. $(-3, 7); (4, -1)$

Determine whether the equation $x = -3$ represents a horizontal line or a vertical line and give its slope.

3. $x = -3$

Write each equation in slope-intercept form. Find the slope and y-intercept, and then use them to draw the graph.

4. $y = 2x - 1$

5. $3y - 9 = 0$

Find an equation in slope-intercept form for the line passing through (0,3) with the slope $m = -\frac{1}{2}$.

6. $(0, 3)$; $m = -\frac{1}{2}$

Applications

Solve.

7. John bought his new car for $35,000 in the year 2014. He knows that the value of his car has depreciated linearly. If the value of the car in 2017 was $23,000, what was the annual rate of depreciation of his car? Show this information on a graph. (When graphing, use years as the *x*-coordinates and the corresponding values of the car as the *y*-coordinates.)

8. The number of people in the United States with mobile cellular phones was about 198 million in 2011 and about 232 million in 2016. If the growth in the usage of mobile cellular phones was linear, what was the approximate rate of growth per year from 2011 to 2016. Show this information on a graph. (When graphing, use years as the *x*-coordinates and the corresponding numbers of users as the *y*-coordinates.)[1]

1 Source: https://www.statista.com/statistics/231612/number-of-cell-phone-users-usa/

Writing & Thinking

9. a. Explain in your own words why the slope of a horizontal line must be 0.

b. Explain in your own words why the slope of a vertical line must be undefined.

x_i	$x_i - \bar{x}$	$\left(x_i - \bar{x}\right)^2$
7	–9	81
11	–5	25
13	–3	9
14	–2	4
14	–2	4
17	1	1
19	3	9
19	3	9
20	4	16
20	4	16
22	6	36

If we take the sum of the third column, we find $\sum\left(x_i - \bar{x}\right)^2 = 210.$ We can now complete the calculation for the sample standard deviation.

$$s^2 = \frac{\sum\left(x_i - \bar{x}\right)^2}{n-1}$$

$$s = \sqrt{\frac{\sum\left(x_i - \bar{x}\right)^2}{n-1}}$$

$$s = \sqrt{\frac{210}{11-1}}$$

$$\approx 4.58$$

11.R.5 Exercises

Concept Check

True/False. Determine whether each statement is true or false. If a statement is false, explain how it can be changed so the statement will be true. (**Note:** There may be more than one acceptable change.)

1. If a number is squared and the principal square root of the result is found, that square root is always equal to the original number.

2. The simplest form of a radical expression can be found by using prime factorization.

3. The fraction $\frac{\sqrt{2}}{3}$ is in simplest form.

Practice

Evaluate each radical expression.

4. $\sqrt{-25}$

5. $-\sqrt{4}$

6. $\sqrt{\frac{36}{49}}$

7. Estimate the radical by identifying which two consecutive integers it falls between.

$\sqrt{53}$

Simplify each expression.

8. $\sqrt{20}$

9. $-\sqrt{\frac{25}{81}}$

Applications

Solve.

10. The following two formulas are used in electricity.

$$\begin{aligned} P &= I^2R \\ E^2 &= PR \end{aligned} \quad \text{where} \begin{cases} P = \text{power (in watts)} \\ I = \text{current (in amperes)} \\ E = \text{voltage (in volts)} \\ R = \text{resistance (in ohms, } \Omega) \end{cases}$$

What is the current in amperes of a light bulb that produces 200 watts of power and has a 10 Ω resistance? Round your answer to two decimal places, if necessary.

11. A nut company is determining how to package their new type of party mix. The marketing department is experimenting with different-sized cans for the party mix packaging. The designers use the equation $r = \sqrt{\frac{V}{h\pi}}$ to determine the radius of the can for a certain height h and volume V. The company decides they want the can to have a volume of 972π cm^3. Find the radius of the can if the height is 4 cm. Keep your answers in simplified radical form.

Writing & Thinking

12. Under what conditions is the expression $\sqrt{a}$ not a real number?