

Section 5.R.5 Solving Linear Inequalities

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Intervals of Real Numbers

The set of all real numbers between a and b is called an _____.

Type of Interval	Algebraic Notation	Interval Notation	Graph
Open Interval	_____	(a, b)	
Closed Interval	$a \leq x \leq b$	_____	
_____	$\begin{cases} a \leq x < b \\ a < x \leq b \end{cases}$	$[a, b)$ $(a, b]$	 
Open Interval	$\begin{cases} x > a \\ x < b \end{cases}$	_____	 
Half-open Interval	_____	$[a, \infty)$ $(-\infty, b]$	 

Table 1

- In an **open interval**, _____.
- In a **closed interval**, _____.
- In a **half-open interval**, _____.

▮ Example 2 Graphing Intervals

Graph the half-open interval $0 < x \leq 4$.

Solution

Exercises

Graph each interval on a real number line.

1. $(-1, \infty)$

3. $(-\infty, 5]$

2. $[-2, 4)$

4. $[0, 3]$

Graph each interval on a real number line and tell what type of interval it is.

5. $x \leq -3$

7. $0 < x \leq 2.5$

6. $x > 4$

8. $-2 \leq x \leq 0$

Solving Linear Inequalities: $ax < c$ and $x + b < c$

Linear inequalities are inequalities that _____.

A **solution** to an inequality is any number that _____
_____.

Addition Principle for Solving Linear Inequalities

If A and B are algebraic expressions and C is a real number, then _____

_____ and _____

(If a real number is added to both sides of an inequality, the new inequality is _____
_____.)

Multiplication Principle for Solving Linear Inequalities

If A and B are algebraic expressions and C is a positive real number, then _____

_____ and _____

If A and B are algebraic expressions and C is a negative real number, then _____

_____ and _____

(In other words, if both sides are multiplied by a _____
_____.)

▮ Example 8 Solving an Inequality and Graphing the Solution Set

Solve the inequality $-3x \leq 12.3$ and graph the solution set. Write the solution set using interval notation.

Solution

Exercises

Solve each inequality and graph the solution set. Write each solution set using interval notation.

9. $x + 1 > 5$

14. $4x > 16$

10. $x - 3 < 2$

15. $3x < 27$

11. $3 + x \leq 7$

16. $5x \leq 15$

12. $5 + x \geq 11$

17. $-2x \geq 6$

13. $3 < 4 + x$

18. $10 > -5x$

Solving Linear Inequalities: $ax + b < c$ and $ax + b < cx + d$

Steps for Solving Linear Inequalities

1. Combine _____
2. Use the addition principle of inequality to _____

3. Use the multiplication (or division) principle of inequality to multiply (or divide) both sides by the coefficient of the variable so that _____
If this coefficient is negative, _____
4. A quick (and generally satisfactory) check is to _____

If the statement is false, _____

▣ Example 10 Solving Linear Inequalities

Solve the linear inequality $x - 3 > 3x + 4$ and graph the solution set. Write the solution set using interval notation.

Solution

Exercises

Solve each inequality and graph the solution set. Write each solution set using interval notation.

19. $2x + 3 < 5$

24. $3x - 1 \leq 11 - 3x$

20. $4x - 7 \geq 9$

25. $5x + 6 \geq 2x - 2$

21. $14 - 5x < 4$

26. $4 - 2x < 5 + x$

22. $1.5x + 9.6 < 12.6$

27. $\frac{5x}{3} + 2 > \frac{x}{3} - 1$

23. $x - 6 \leq 4 - x$

Applications of Linear Inequalities

▣ Example 13 Application: Using Inequalities

A math student has grades of 85, 98, 93, and 90 on four examinations. If he must average 90 or better to receive an A for the course, what scores can he receive on the final exam and earn an A? (Assume that the final exam counts the same as the other exams.)

Solution

Exercises

Solve.

- 28.** A statistics student has grades of 82, 95, 93, and 78 on four hour-long exams. He must average 90 or higher to receive an A for the course. What scores can he receive on the final exam and earn an A if:
- The final is equivalent to a single hour-long exam (100 points maximum)?
 - The final is equivalent to two hourly exams (200 points maximum)?
- 29.** A car salesman makes \$1000 each day that he works and makes approximately \$250 commission for each car he sells. If a car salesman wants to make at least \$3500 in one day, how many cars does he need to sell?