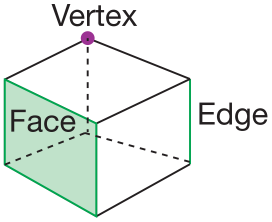
14.4 PROJECT

PLANAR POLYHEDRONS

In this section, you learned about planar graphs and how their numbers of v vertices, e edges, and f faces are related by Euler's formula: $v + f - e = 2$. You may have also learned about vertices, edges, and faces in geometry as part of the study of convex polyhedrons. A convex polyhedron is a solid made up of flat polygonal faces joined at their edges and vertices with the additional property that any line segment joining any two points on the surface of the polyhedron stays on or inside the polyhedron.

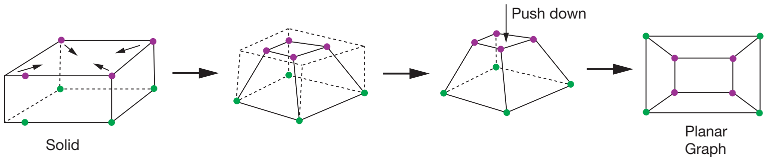
A cube is an example of a convex polyhedron.



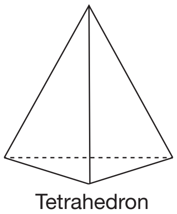
1. How many faces, edges, and vertices does a cube have?

2. Show that a cube satisfies Euler's formula.

The fact that a cube satisfies Euler's formula isn't a coincidence. We can turn a cube into a planar graph without changing the number of faces, edges, or vertices. The process is illustrated as follows.



3. Draw the planar graph that corresponds to the tetrahedron, illustrated as follows. (**Hint:** In this case, all you must do is push down on the top vertex.)



4. Show that a tetrahedron satisfies Euler's formula.

We can turn any convex polyhedron into a planar graph using a process like the one described for the cube. That is, Euler's formula $v+f-e=2$ is satisfied for all convex polyhedrons.

5. One of your classmates claims that they have constructed a convex polyhedron out of two triangles, two squares, six pentagons, and five octagons. Explain why such a convex polyhedron cannot exist. (**Hint:** Each vertex of a convex polyhedron must border at least three faces, and the sum of the degrees of all the vertices in a graph is equal to twice the number of edges.)