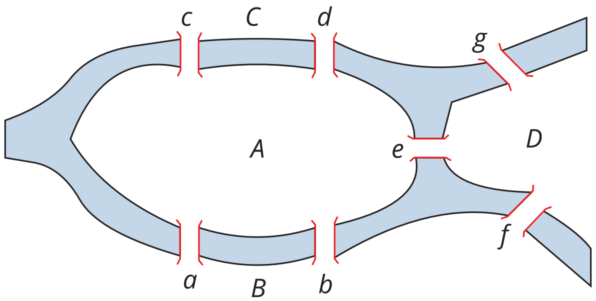
14.1 PROJECT

THE KÖNIGSBERG BRIDGES

In Section 14.1, you learned about the famous Königsberg bridge problem that asks whether it is possible to start at one of the land masses in the city and cross every bridge exactly once. Let's investigate a slight modification of the problem: is it possible to start your journey on one of the land masses, cross every bridge exactly once, and return to the *original* land mass?



A Simplified Diagram of Königsberg and Its Bridges

1. Draw a graph that models the situation. The land masses A, B, C, and D will be represented by vertices while the bridges a, b, c, d, e, f, and g will be represented by edges. Determine the degree of each vertex.

In graph theory, a **circuit** is a walk that starts and ends at the same vertex.

2. Find a circuit in the graph that was created in part 1.

We define an **Euler circuit** as a circuit that uses each edge exactly once. In other words, an Euler circuit starts at a vertex, uses every edge exactly once, and then returns to the same vertex.

3. Rephrase our version of the Königsberg bridge problem in terms of Euler circuits.

4. How many Euler circuits are in the graph?

Notice that in order to produce an Euler circuit, you must enter a vertex using one edge and leave that vertex using a different edge—that is, the edges that meet at a vertex must come in pairs. This indicates that a connected graph will have an Euler circuit when all of its vertices have even degrees.

5. Use this fact to justify why the modified Königsberg bridge problem does or does not have a solution.

6. Draw a graph that represents 4 land masses and 7 bridges that would have an Euler circuit and explain your answer.

7. Could you remove one of the Königsberg bridges to create an Euler circuit? If yes, which bridge should be removed? If not, explain why.