

10.3 PROJECT

THE BIRTHDAY PARADOX

It is often the case that we can gauge how likely an event is by simply thinking of our past experiences and comparing it to other events in our daily lives. For instance, most people would agree that the probability of buying one lottery ticket and winning the jackpot is much lower than the probability of rolling one die and getting a six, which is 1 in 6. We could use data or computations to confirm our intuition about these situations. In fact, if a lottery was set up where you choose 6 numbers (in any order) from a possible pool of 49 numbers, your chances of winning the jackpot are 1 in 13,983,816.

Sometimes, however, our intuition betrays us. In this activity, we will investigate a classic probability problem called the birthday paradox.

Consider a room that has 25 people in it.

1. Do you think that the probability of at least two people sharing a birthday (same month and same day) is above or below 50%? Would you say the probability is below 10% or above 90%? Explain your reasoning.
2. If we assume that there are 365 days in a year, what is the smallest number of people in a room that would guarantee that at least two people in the room share a birthday?

If we have two people in a room, they either share a birthday or they don't. These are complementary events, and we can write the following equation.

$$P(\text{Same Birthday}) = 1 - P(\text{Different Birthday})$$

Since there are 365 days in a year, there are $365(364) = 132,860$ different ways that the two people can have different birthdays. There are also $365(365) = 133,225$ possible pairs of birthdays. From this information, we get the following probability.

$$P(\text{Same Birthday}) = 1 - \frac{\text{Different Birthdays}}{\text{All Possible Birthdays}} = 1 - \frac{365(364)}{365(365)} \approx 0.0027$$

Hence, with two people in a room, the probability that they share a birthday is 0.27%.

In a room with three people, we can use the same argument. Either no birthday is shared or at least two people share a birthday. We would have the following probability.

$$P(\text{At Least Two People with the Same Birthday}) = 1 - \frac{365(364)(363)}{365(365)(365)}$$

3. Complete the computation above. Did the probability increase or decrease by adding just one extra person? By how much?
4. Now, follow the same reasoning and write an expression for the probability of at least two people sharing a birthday in a room with 25 people.
5. Use a computer to determine the value of the expression you found in part 4. (**Hint:** The website wolframalpha.com has a good computation engine.)
6. How does the answer found in part 5 compare to your answer from part 1?