# Formulas and Tables

## Chapter 3

**Class Width** $= \dfrac{\text{maximum value} - \text{minimum value}}{\text{number of classes}}$

**Relative Frequency** $= \dfrac{\text{number in class}}{\text{total number of observations}}$

## Chapter 4

**Arithmetic Mean:** $\dfrac{1}{n}(x_1 + x_2 + \ldots + x_n)$

**Population Mean:** $\mu = \dfrac{1}{N}(x_1 + x_2 + \ldots + x_N) = \dfrac{\sum x_i}{N}$

**Sample Mean:** $\bar{x} = \dfrac{1}{n}(x_1 + x_2 + \ldots + x_n) = \dfrac{\sum x_i}{n}$

**Weighted Mean:**

$$\bar{x} = \frac{w_1x_1 + w_2x_2 + \cdots + w_nx_n}{w_1 + w_2 + \cdots + w_n} = \frac{\sum(w_i \cdot x_i)}{\sum w_i}$$

**Mean Absolute Deviation:** $\text{MAD} = \dfrac{\sum|x_i - \bar{x}|}{n}$

**Sample Variance:** $s^2 = \dfrac{\sum(x_i - \bar{x})^2}{n-1}$

**Population Variance:** $\sigma^2 = \dfrac{\sum(x_i - \mu)^2}{N}$

**Sample Standard Deviation:** $s = \sqrt{s^2} = \sqrt{\dfrac{\sum(x_i - \bar{x})^2}{n-1}}$

**Population Standard Deviation:**

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\sum(x_i - \mu)^2}{N}}$$

**Range =** maximum data value − minimum data value

**Location of the *P*th percentile:** $l = n\left(\dfrac{P}{100}\right)$

**Percentile of *x*** $= \dfrac{\text{number of data values} \le x}{\text{total number of data values}} \cdot 100$

**$Q_1$ (location of 25th percentile):** $l = n\left(\dfrac{25}{100}\right)$

**$Q_2$ (location of 50th percentile):** $l = n\left(\dfrac{50}{100}\right)$

**$Q_3$ (location of 75th percentile):** $l = n\left(\dfrac{75}{100}\right)$

**Interquartile Range** $= Q_3 - Q_1$

***z*-Score:** $z = \dfrac{x - \mu}{\sigma}$

**Outliers:** Data value greater than $Q_3 + 1.5 \cdot$ Interquartile Range or less than $Q_1 - 1.5 \cdot$ Interquartile Range.

**Empirical Rule** (Assumes the distribution of the data is bell-shaped):

**One sigma rule:** About 68% of the data should lie within one standard deviation of the mean.

**Two sigma rule:** About 95% of the data should lie within two standard deviations of the mean.

**Three sigma rule:** About 99.7% of the data should lie within three standard deviations of the mean.

**Chebyshev's Theorem:** The proportion of any data set lying within $k$ standard deviations of the mean is at least $1 - \dfrac{1}{k^2}$, for $k > 1$.

**Coefficient of Variation (for sample data):** $CV = \left(\dfrac{s}{\bar{x}} \cdot 100\right)\%$

**Coefficient of Variation (for a population):** $CV = \left(\dfrac{\sigma}{\mu} \cdot 100\right)\%$

**Mean of grouped data:** $\mu = \dfrac{\sum f_i M_i}{N}$

**Sample Mean of grouped data:** $\bar{x} = \dfrac{\sum f_i M_i}{n}$

**Variance of grouped data:** $\sigma^2 = \dfrac{\sum(M_i - \mu)^2 f_i}{N}$

**Sample Variance of grouped data:** $s^2 = \dfrac{\sum(M_i - \bar{x})^2 f_i}{n-1}$

**Midpoint** $= \dfrac{\text{lowerclass boundary} + \text{upperclass boundary}}{2}$

**Sample Proportion:** $\hat{p} = \dfrac{X}{n}$

**Population Proportion:** $p = \dfrac{X}{N}$

## Chapter 5

**Correlation Coefficient:**

$$r = \frac{1}{n-1}\left\{\sum_{i=1}^{n}\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)\right\}, \quad -1 \le r \le 1$$

**Simple Linear Regression Model:**

$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

**Residual (Error) =** observed y − predicted y $= y_i - \hat{y}_i$

**Estimated Simple Linear Regression Equation:** $\hat{y} = b_0 + b_1 x_i$

**Sum of Squared Errors (SSE):**

$\text{SSE} = \sum \text{error}_i^2 = \sum\left(y_i - \hat{y}_i\right)^2 = \sum\left(y_i - \left(b_0 + b_1 x_i\right)\right)^2$

**Slope (predicted):** $b_1 = \dfrac{n\left(\sum x_i y_i\right) - \sum x_i \sum y_i}{n\left(\sum x_i^2\right) - \left(\sum x_i\right)^2}$

***y*-Intercept (predicted):**

$b_0 = \dfrac{1}{n}\left(\sum y_i - b_1 \sum x_i\right) = \bar{y} - b_1 \bar{x}$

**Mean Square Error:** $s_e^2 = \dfrac{\sum\left(y_i - \hat{y}_i\right)^2}{n-2} = \dfrac{\text{SSE}}{n-2}$

**Standard Error:** $s_e = \sqrt{s_e^2} = \sqrt{\dfrac{\sum\left(y_i - \hat{y}_i\right)^2}{n-2}} = \sqrt{\dfrac{\text{SSE}}{n-2}}$

**Total Sum of Squares (Total SS):** $\sum\left(y_i - \bar{y}\right)^2$

**Sum of Squares of Regression (SSR):** $\text{SSR} = \text{Total SS} - \text{SSE}$

**Coefficient of Determination:**

$$R^2 = \left(\frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{\left[n\sum x_i^2 - \left(\sum x_i\right)^2\right]\left[n\sum y_i^2 - \left(\sum y_i\right)^2\right]}}\right)^2$$

$$= \frac{\text{SSR}}{\text{Total SS}} = 1 - \frac{\text{SSE}}{\text{Total SS}},\quad 0 \le R^2 \le 1$$

# Chapter 6

**Relative Frequency of *A*:** $A = \dfrac{k}{n}$

**Probability of Event A:**

$P(A) = \dfrac{\text{number of outcomes in } A}{\text{total number of outcomes in the sample space}}$

**Probability of a Complement:** $P\left(A^c\right) = 1 - P(A)$

**Odds:** The odds in favor of an event $A$ occurring is given by $\dfrac{P(A)}{P(\text{not } A)} = \dfrac{P(A)}{P\left(A^c\right)}$. The odds against an event $A$ occurring is given by $\dfrac{P(\text{not } A)}{P(A)} = \dfrac{P\left(A^c\right)}{P(A)}$.

**Union of Mutually Exclusive Events:** $P(A \cup B) = P(A) + P(B)$

**Addition Rule:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

**Conditional Probability:** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

**Independent Events Rule:**

$P(A \mid B) = P(A)$ and $P(B \mid A) = P(B)$

**Multiplication Rule for Independent Events:**

$P\left(A_1 \cap A_2 \cap \ldots \cap A_n\right) = P\left(A_1\right) \cdot P\left(A_2\right) \cdot \ldots \cdot P\left(A_n\right)$

**Mulitplication Rule for Dependent Events:**

$P(A \cap B) = P(A) \cdot P(B \mid A) = P(B) \cdot P(A \mid B)$

***n* Factorial:** $n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$

**Permutation:** ${}_nP_k = \dfrac{n!}{(n-k)!}$

**Distinguishable Permutation:** given $n_1$ alike, $n_2$ alike,..., $n_k$ alike, $\dfrac{n!}{\left(n_1! n_2! n_3! \ldots n_k!\right)}$

**Combination:** ${}_nC_k = \dfrac{n!}{(n-k)!k!}$

**Bayes' Theorem:**

$$P\left(B_i \middle| A\right) = \frac{P\left(A \middle| B_i\right) P\left(B_i\right)}{P\left(A \middle| B_1\right) P\left(B_1\right) + P\left(A \middle| B_2\right) P\left(B_2\right) + \cdots + P\left(A \middle| B_k\right) P\left(B_k\right)}$$

**The Fundamental Counting Principle:** If $E_1$ is an event with $n_1$ possible outcomes and $E_2$ is an event with $n_2$ possible outcomes, the number of ways the events can occur in sequence is $n_1 \cdot n_2$.

# Chapter 7

**Expected Value of a Discrete Random Variable:** $\mu = E(X) = \sum\left[x \cdot p(x)\right]$

**Variance of a Discrete Random Variable:**

$\sigma^2 = V(X) = \sum (x - \mu)^2 \cdot p(x)$

**Standard Deviation of a Discrete Random Variable:** $\sigma = \sqrt{V(X)} = \sqrt{\sum (x-\mu)^2 \cdot p(x)}$

**Binomial Probability Distribution:**

$P(X = x) = {}_nC_x p^x (1-p)^{n-x}$

**Expected Value of a Binomial Random Variable:** $\mu = E(X) = np$

**Variance of a Binomial Random Variable:** $\sigma^2 = V(X) = np(1-p)$

**Standard Deviation of a Binomial Random Variable:** $\sigma = \sqrt{V(X)} = \sqrt{np(1-p)}$

**Poisson Probability Distribution:**

$$P(X=x)=\left(\frac{e^{-\lambda}\lambda^{x}}{x!}\right), \text{ for } x=0,1,2,\ldots$$

**Poisson Expected Value:** $\mu=\lambda$

**Poisson Variance:** $\sigma^2=\lambda$

**Poisson Standard Deviation:** $\sigma=\sqrt{\sigma^2}=\sqrt{\lambda}$

**Hypergeometric Probability Distribution:**

$$P(X=x)=\frac{{}_{k}C_{x}\,{}_{N-k}C_{n-x}}{{}_{N}C_{n}}, \text{ where maximum of}$$
$(0, n+k-N) \le x \le$ minimum of $(k,n)$

**Expected Value of a Hypergeometric Random Variable:** $\mu=E(X)=n\cdot\frac{k}{N}$

**Variance of a Hypergeometric Random Variable:** $\sigma^2=V(X)=\left[n\cdot\frac{k}{N}\cdot\left(1-\frac{k}{N}\right)\cdot\frac{(N-n)}{(N-1)}\right]$

# Chapter 8

**Uniform Probability Density Function:**

$$f(x)=\begin{cases}\frac{1}{b-a}, & \text{for } a\le x\le b\\ 0 & \text{otherwise}\end{cases}$$

**Expected Value for a Continuous Uniform Random Variable:** $\mu=E(x)=\frac{a+b}{2}$

**Standard Deviation for a Continuous Uniform Random Variable:** $\sigma=\frac{b-a}{\sqrt{12}}$

**Standardizing a Normal Random Variable:** $z=\frac{x-\mu}{\sigma}$

# Chapter 9

**Estimator of $\mu$:** An unbiased estimator of $\mu$ is $\bar{x}$.

**Estimator of $p$:** An unbiased estimator of $p$ is $\hat{p}$.

**Estimator of $\sigma^2$:** An unbiased estimator of $\sigma^2$ is $s^2$.

**Variance of $\bar{x}$ (population of infinite size):** $\sigma_{\bar{x}}^2=\frac{\sigma^2}{n}$

**Variance of $\bar{x}$ (population of finite size):** $\sigma_{\bar{x}}^2=\frac{(N-n)}{(N-1)}\cdot\frac{\sigma^2}{n}$

**Central Limit Theorem – Mean of the Sample Means:** $\mu_{\bar{x}}=E(\bar{x})=\mu$

**Central Limit Theorem – Variance of the Sample Means:** $\sigma_{\bar{x}}^2=\frac{\sigma^2}{n}$

**Sample Proportion:** $\hat{p}=\frac{x}{n}$

**Mean of the Sample Proportion (population of infinite size):** $\mu_{\hat{p}}=E(\hat{p})=p$

**Variance of the Sample Proportion (population of infinite size):** $\sigma_{\hat{p}}^2=\frac{p(1-p)}{n}\approx\frac{\hat{p}(1-\hat{p})}{n}$

**Mean of the Sample Proportion (population of finite size):** $\mu_{\hat{p}}=E(\hat{p})=p$

**Variance of the Sample Proportion (population of finite size):**

$$\sigma_{\hat{p}}^2=\frac{N-n}{N-1}\cdot\frac{p(1-p)}{n}\approx\frac{N-n}{N-1}\cdot\frac{\hat{p}(1-\hat{p})}{n}$$

# Chapter 10

***z*-Transformation:** $z=\frac{\bar{x}-\mu}{\sigma_{\bar{x}}}$

**Mean Squared Error for the Sample Mean** $\text{MSE}(\bar{x})=E(\bar{x}-\mu)^2$

**Confidence Interval for the Population Mean ($\sigma$ is known):** $\bar{x}\pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

***t*-test statistic:** $t=\frac{(\bar{x}-\mu)}{\frac{s}{\sqrt{n}}}$

**Degrees of Freedom (*df*) for *t*-distribution:**
$df=$ number of sample observations $-\ 1=n-1$

**Confidence Interval for the Population Mean ($\sigma$ is unknown):** $\bar{x}\pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$

**Critical Value for a *t*-distribution:** $t_{\alpha/2,n-1}$

**Minimum Sample Size to Select $\mu$:**

$$n=\left(\frac{z_{\alpha/2}\sigma}{error}\right)^2 \text{ (use } s \text{ if } \sigma \text{ is not known)}$$

**Confidence Interval for the Population Proportion:** $(np \geq 10,\ n(1-p) \geq 10)$:

$$\hat{p} \pm z_{\alpha/2}\sigma_{\hat{p}}, \text{ where } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

**Minimum Sample Size to Estimate the Population Proportion:** $n \approx \dfrac{z_{\alpha/2}^2 p(1-p)}{error^2}$ (use $p = 0.5$ if there is no estimate of $p$ available)

**Confidence Interval for the Population Variance:**

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

# Chapter 11

**Hypothesis Test Statistic for population mean ($\sigma$ is known):** $z = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}}$, where $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

**Hypothesis Test Statistic for population mean ($\sigma$ is unknown):** $t = \dfrac{\bar{x} - \mu_0}{s_{\bar{x}}}$, where $s_{\bar{x}} = \dfrac{s}{\sqrt{n}}$

**Hypothesis Test Statistic for population proportion:** $z = \dfrac{\hat{p} - p}{\sigma_{\hat{p}}}$, where $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}}$

**Hypothesis Test Statistic for a population variance:**

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} \quad \text{with degrees of freedom} = n-1$$

# Chapter 12

**Hypothesis Test Statistic and Confidence Interval (comparing two population means with $\sigma_1$ and $\sigma_2$ known):**

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - \mu_{\bar{x}_1 - \bar{x}_2}}{\sigma_{\bar{x}_1 - \bar{x}_2}} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

**Hypothesis Test Statistic and Confidence Interval (comparing two population means with $\sigma_1$ and $\sigma_2$ unknown and unequal):**

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}},$$

**Hypothesis Test Statistic and Confidence Interval (comparing two population means with $\sigma_1$ and $\sigma_2$ unknown and assumed equal)**

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

**Pooled Sample Variance:** $s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

**Hypothesis Test Statistic and Confidence Interval (paired difference):** $t = \dfrac{\bar{x}_d - \mu_d}{\frac{s_d}{\sqrt{n}}}$

$$\bar{x}_d \pm t_{\alpha/2}\frac{s_d}{\sqrt{n}}$$

**Hypothesis Test Statistic and Confidence Interval (comparing two population proportions):**

$$z = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1} + \frac{\bar{p}(1-\bar{p})}{n_2}}}$$

$$(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

**Pooled Sample Proportion:** $\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

**Hypothesis Test Statistic and Confidence Interval (comparing two population variances):** $F = \dfrac{s_1^2}{s_2^2}$.

$$\left(\frac{s_1^2}{s_2^2} \cdot \frac{1}{F_{\alpha/2}}\right) < \frac{\sigma_1^2}{\sigma_2^2} < \left(\frac{s_1^2}{s_2^2} \cdot \frac{1}{F_{(1-\alpha/2)}}\right)$$

# Chapter 13

**Simple Linear Regression Model:** $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

**Estimated Simple Linear Regression Equation:** $y = b_0 + b_1 x$

**Standard Deviation of the Sample Estimate $b_1$:** $s_{b_1} = \sqrt{\dfrac{s_e^2}{\Sigma(x_i - \bar{x})^2}}$ (see Chapter 5 for $s_e^2$)

**Confidence Interval for $\beta_1$:** $b_1 \pm t_{\alpha/2, n-2} \cdot s_{b_1}$

**Confidence Interval for the Average Value of $y$ Given $x$:** $\hat{y}_p \pm t_{\alpha/2, n-2} \cdot s_e \sqrt{\frac{1}{n} + \frac{(x_p - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$

**Confidence Interval for the Predicted Value of $y$ Given $x$:** $\hat{y}_p \pm t_{\alpha/2, n-2} \cdot s_e \sqrt{1 + \frac{1}{n} + \frac{(x_p - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$

**$t$-Statistic:** $t = \frac{b_1 - 0}{s_{b_1}} = \frac{b_1}{s_{b_1}}$

# Chapter 14

**Multiple Regression Model:**

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + \beta_k x_{ki} + \varepsilon_i$$

**Estimated Multiple Regression Equation:**

$$\hat{y}_i = b_0 + b_1 x_{1i} + b_2 x_{2i} + \cdots + b_k x_{ki}$$

**F-Statistic:** $F = \frac{\frac{SSR}{k}}{\frac{SSE}{n-(k+1)}} = \frac{\text{Mean Squared Regression}}{\text{Mean Squared Error}}$

**Confidence Interval in a Multiple Regression Model:** $b_i \pm t_{\alpha/2, n-(k+1)} s_{b_i}$, $i = 1, \ldots, k$

**Adjusted $R^2$:** $R_a^2 = 1 - \left(\frac{n-1}{n-k-1}\right) \frac{\text{SSE}}{\text{Total SS}}$

**Test Statistic for Testing the Hypothesis $\beta_i \neq 0$:** $t = \frac{b_i - 0}{s_{b_i}} = \frac{b_i}{s_{b_i}} \quad i = 1, \ \ldots, \ k$

# Chapter 15

**Total Sum of Squares One-Way ANOVA:**

Total SS = SSTreatment + SSError

$$\sum_{j=1}^{k} \sum_{i=1}^{n_j} \left(x_{ij} - \bar{\bar{x}}\right)^2 = \sum_{j=1}^{k} n_j \left(\bar{x}_j - \bar{\bar{x}}\right)^2 + \left\{ \sum_{i=1}^{n_1} (x_{i1} - \bar{x}_1)^2 + \sum_{i=1}^{n_2} (x_{i2} - \bar{x}_2)^2 + \cdots + \sum_{i=1}^{n_k} (x_{ik} - \bar{x}_k)^2 \right\}$$

***F*-Statistic One-Way ANOVA:**

$$F = \frac{\text{MST}}{\text{MSE}} = \frac{\dfrac{\sum_{j=1}^{k} n_j \left(\bar{x}_j - \bar{\bar{x}}\right)^2}{k-1}}{\dfrac{\sum_{i=1}^{n_1} (x_{i1} - \bar{x}_1)^2 + \sum_{i=1}^{n_2} (x_{i2} - \bar{x}_2)^2 + \cdots + \sum_{i=1}^{n_k} (x_{ik} - \bar{x}_k)^2}{N-k}}$$

**Mean Square for Treatment (MST):** MST = SST/$(k-1)$

$$\text{MST} = \frac{\dfrac{\left(\sum_{i=1}^{n_1} x_{i1}\right)^2}{n_1} + \dfrac{\left(\sum_{i=1}^{n_2} x_{i2}\right)^2}{n_2} + \cdots + \dfrac{\left(\sum_{i=1}^{n_k} x_{ik}\right)^2}{n_k} - \dfrac{\left(\sum_{j=1}^{k} \sum_{i=1}^{n_j} x_{ij}\right)^2}{N}}{k-1}$$

**Mean Square for Error (MSE):** MSE = SSE/$(N-k)$

$$\text{MSE} = \frac{\sum_{j=1}^{k} \sum_{i=1}^{n_j} x_{ij}^2 - \left\{ \dfrac{\left(\sum_{i=1}^{n_1} x_{i1}\right)^2}{n_1} + \dfrac{\left(\sum_{i=1}^{n_2} x_{i2}\right)^2}{n_2} + \cdots + \dfrac{\left(\sum_{i=1}^{n_k} x_{ik}\right)^2}{n_k} \right\}}{N-k}$$

**Sum of Squares Two-Way ANOVA (Randomized Block):** Total SS = SSTreatment + SSBlock + SSError

***F*-Statistic for Two-Way ANOVA (Randomized Block):** $F = \frac{\text{MST}}{\text{MSE}} = \frac{\frac{\text{SST}}{k-1}}{\frac{\text{SSE}}{(k-1)(b-1)}}$

**Sum of Squares Two-way ANOVA (with Interaction):** Total SS = SSFactorA +SSFactorB + SSInteraction +SSError

***F*-Statistic for Interaction between Factors:**

$$F = \frac{\frac{\text{SSAB}}{(a-1)(b-1)}}{\frac{\text{SSE}}{ab(r-1)}} = \frac{\text{MSAB}}{\text{MSE}}$$

***F*-Statistic for Main Effect for Factor A:**

$$F = \frac{\frac{\text{SSA}}{(a-1)}}{\frac{\text{SSE}}{ab(r-1)}} = \frac{\text{MSA}}{\text{MSE}}$$

***F*-Statistic for Main Effect for Factor B:**

$$F = \frac{\frac{\text{SSB}}{(b-1)}}{\frac{\text{SSE}}{ab(r-1)}} = \frac{\text{MSB}}{\text{MSE}}$$

# Chapter 16

**Chi-Square Test Statistic for Goodness of Fit:**

$$\chi^2 = \sum_{i=1}^{k} \frac{\left[n_i - E(n_i)\right]^2}{E(n_i)}$$

**Chi-Square Test Statistic for Association:**

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left[n_{ij} - E\left(n_{ij}\right)\right]^2}{E\left(n_{ij}\right)}$$

# Chapter 17

**Sign Test ($n \le 25$):** $X$ = the number of times the less frequent sign occurs

**Sign Test ($n > 25$):** $z = \dfrac{X + 0.5 - \left(\frac{n}{2}\right)}{\frac{\sqrt{n}}{2}}$

**Wilcoxon Signed-Rank Test ($n \le 25$):** $H_a$: >, then $T = T_+$ = the sum of the ranks of the positive differences; $H_a$: <, then $T = T_-$ = the sum of the ranks of the negative differences; $H_a$: ≠, then $T = \text{Min}(T_+, T_-)$.

**Wilcoxon Signed-Rank Test ($n > 25$):**

$$z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$$, where $T = \text{Min}(T_+, T_-)$

**Wilcoxon Rank-Sum Test**
**($n_1 \le 10$):** $T = T_X$ = the rank sum of the smaller sample

**Wilcoxon Rank-Sum Test ($n_1$ >10):**

$$z = \frac{T_X - \frac{n_1(n_1 + n_2 + 1)}{2}}{\sqrt{\frac{n_1 n_2(n_1 + n_2 + 1)}{12}}}$$, where $T_X$ = the rank sum of the smaller sample

**Spearman Rank Correlation Coefficient:**

$$r_s = 1 - \frac{6T}{n \cdot (n^2 - 1)}, \text{where } T = \sum_{i=1}^{n}[R(X_i) - R(Y_i)]^2$$

**Rank Correlation Test ($n \le 30$):** Reject null hypothesis if $|r_s| >$ critical value in Appendix A Table L.

**Rank Correlation Test ($n > 30$):** Reject null hypothesis if $|r_s| > \dfrac{z_{\alpha/2}}{\sqrt{n-1}}$

**Runs Test for Randomness ($\alpha$ = 0.05, $m \le 20$, and $n \le 20$):** $R$ (number of runs)

**Runs Test for Randomness ($\alpha$ = 0.05, $m > 20$, and $n > 20$):** $z = \dfrac{(R - \mu_R)}{\sigma_R}$, where $\mu_R = 1 + \dfrac{2mn}{N}$,

$$\sigma_R = \sqrt{\frac{2mn(2mn - N)}{\left[N^2(N-1)\right]}}$$

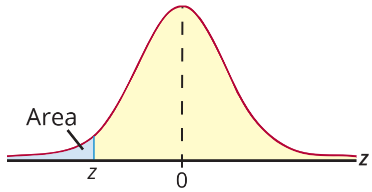
**Kruskal-Wallis Test:** $H = \dfrac{12}{N(N+1)}\sum_{i=1}^{k}\dfrac{R_i^2}{n_i} - 3(N+1)$,

where $N = \sum_{i=1}^{k} n_i$, $R_i = \sum_{j=1}^{n_i} r_{ij}$

Reject null hypothesis if $H \ge k_\alpha$, where value of $k_\alpha$ is found using the chi-square table in Appendix A Table G with $(k - 1)$ degrees of freedom and a predetermined value of $\alpha$.

# A Standard Normal Distribution

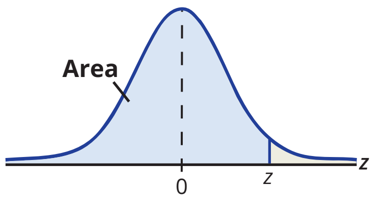
Numerical entries represent the probability that a standard normal random variable is between $-\infty$ and $z$ where $z = \dfrac{x - \mu}{\sigma}$.




| $z$ | 0.09 | 0.08 | 0.07 | 0.06 | 0.05 | 0.04 | 0.03 | 0.02 | 0.01 | 0.00 |
|---|---|---|---|---|---|---|---|---|---|---|
| –3.4 | 0.0002 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 |
| –3.3 | 0.0003 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0005 | 0.0005 | 0.0005 |
| –3.2 | 0.0005 | 0.0005 | 0.0005 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0007 | 0.0007 |
| –3.1 | 0.0007 | 0.0007 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0009 | 0.0009 | 0.0009 | 0.0010 |
| –3.0 | 0.0010 | 0.0010 | 0.0011 | 0.0011 | 0.0011 | 0.0012 | 0.0012 | 0.0013 | 0.0013 | 0.0013 |
| –2.9 | 0.0014 | 0.0014 | 0.0015 | 0.0015 | 0.0016 | 0.0016 | 0.0017 | 0.0018 | 0.0018 | 0.0019 |
| –2.8 | 0.0019 | 0.0020 | 0.0021 | 0.0021 | 0.0022 | 0.0023 | 0.0023 | 0.0024 | 0.0025 | 0.0026 |
| –2.7 | 0.0026 | 0.0027 | 0.0028 | 0.0029 | 0.0030 | 0.0031 | 0.0032 | 0.0033 | 0.0034 | 0.0035 |
| –2.6 | 0.0036 | 0.0037 | 0.0038 | 0.0039 | 0.0040 | 0.0041 | 0.0043 | 0.0044 | 0.0045 | 0.0047 |
| –2.5 | 0.0048 | 0.0049 | 0.0051 | 0.0052 | 0.0054 | 0.0055 | 0.0057 | 0.0059 | 0.0060 | 0.0062 |
| –2.4 | 0.0064 | 0.0066 | 0.0068 | 0.0069 | 0.0071 | 0.0073 | 0.0075 | 0.0078 | 0.0080 | 0.0082 |
| –2.3 | 0.0084 | 0.0087 | 0.0089 | 0.0091 | 0.0094 | 0.0096 | 0.0099 | 0.0102 | 0.0104 | 0.0107 |
| –2.2 | 0.0110 | 0.0113 | 0.0116 | 0.0119 | 0.0122 | 0.0125 | 0.0129 | 0.0132 | 0.0136 | 0.0139 |
| –2.1 | 0.0143 | 0.0146 | 0.0150 | 0.0154 | 0.0158 | 0.0162 | 0.0166 | 0.0170 | 0.0174 | 0.0179 |
| –2.0 | 0.0183 | 0.0188 | 0.0192 | 0.0197 | 0.0202 | 0.0207 | 0.0212 | 0.0217 | 0.0222 | 0.0228 |
| –1.9 | 0.0233 | 0.0239 | 0.0244 | 0.0250 | 0.0256 | 0.0262 | 0.0268 | 0.0274 | 0.0281 | 0.0287 |
| –1.8 | 0.0294 | 0.0301 | 0.0307 | 0.0314 | 0.0322 | 0.0329 | 0.0336 | 0.0344 | 0.0351 | 0.0359 |
| –1.7 | 0.0367 | 0.0375 | 0.0384 | 0.0392 | 0.0401 | 0.0409 | 0.0418 | 0.0427 | 0.0436 | 0.0446 |
| –1.6 | 0.0455 | 0.0465 | 0.0475 | 0.0485 | 0.0495 | 0.0505 | 0.0516 | 0.0526 | 0.0537 | 0.0548 |
| –1.5 | 0.0559 | 0.0571 | 0.0582 | 0.0594 | 0.0606 | 0.0618 | 0.0630 | 0.0643 | 0.0655 | 0.0668 |
| –1.4 | 0.0681 | 0.0694 | 0.0708 | 0.0721 | 0.0735 | 0.0749 | 0.0764 | 0.0778 | 0.0793 | 0.0808 |
| –1.3 | 0.0823 | 0.0838 | 0.0853 | 0.0869 | 0.0885 | 0.0901 | 0.0918 | 0.0934 | 0.0951 | 0.0968 |
| –1.2 | 0.0985 | 0.1003 | 0.1020 | 0.1038 | 0.1056 | 0.1075 | 0.1093 | 0.1112 | 0.1131 | 0.1151 |
| –1.1 | 0.1170 | 0.1190 | 0.1210 | 0.1230 | 0.1251 | 0.1271 | 0.1292 | 0.1314 | 0.1335 | 0.1357 |
| –1.0 | 0.1379 | 0.1401 | 0.1423 | 0.1446 | 0.1469 | 0.1492 | 0.1515 | 0.1539 | 0.1562 | 0.1587 |
| –0.9 | 0.1611 | 0.1635 | 0.1660 | 0.1685 | 0.1711 | 0.1736 | 0.1762 | 0.1788 | 0.1814 | 0.1841 |
| –0.8 | 0.1867 | 0.1894 | 0.1922 | 0.1949 | 0.1977 | 0.2005 | 0.2033 | 0.2061 | 0.2090 | 0.2119 |
| –0.7 | 0.2148 | 0.2177 | 0.2206 | 0.2236 | 0.2266 | 0.2296 | 0.2327 | 0.2358 | 0.2389 | 0.2420 |
| –0.6 | 0.2451 | 0.2483 | 0.2514 | 0.2546 | 0.2578 | 0.2611 | 0.2643 | 0.2676 | 0.2709 | 0.2743 |
| –0.5 | 0.2776 | 0.2810 | 0.2843 | 0.2877 | 0.2912 | 0.2946 | 0.2981 | 0.3015 | 0.3050 | 0.3085 |
| –0.4 | 0.3121 | 0.3156 | 0.3192 | 0.3228 | 0.3264 | 0.3300 | 0.3336 | 0.3372 | 0.3409 | 0.3446 |
| –0.3 | 0.3483 | 0.3520 | 0.3557 | 0.3594 | 0.3632 | 0.3669 | 0.3707 | 0.3745 | 0.3783 | 0.3821 |
| –0.2 | 0.3859 | 0.3897 | 0.3936 | 0.3974 | 0.4013 | 0.4052 | 0.4090 | 0.4129 | 0.4168 | 0.4207 |
| –0.1 | 0.4247 | 0.4286 | 0.4325 | 0.4364 | 0.4404 | 0.4443 | 0.4483 | 0.4522 | 0.4562 | 0.4602 |
| 0.0 | 0.4641 | 0.4681 | 0.4721 | 0.4761 | 0.4801 | 0.4840 | 0.4880 | 0.4920 | 0.4960 | 0.5000 |

# B Standard Normal Distribution

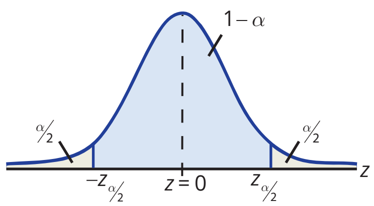
Numerical entries represent the probability that a standard normal random variable is between $-\infty$ and $z$ where $z = \dfrac{x - \mu}{\sigma}$.




| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **0.0** | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| **0.1** | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| **0.2** | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| **0.3** | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| **0.4** | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| **0.5** | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| **0.6** | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| **0.7** | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| **0.8** | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| **0.9** | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| **1.0** | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| **1.1** | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| **1.2** | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| **1.3** | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| **1.4** | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| **1.5** | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| **1.6** | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| **1.7** | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| **1.8** | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| **1.9** | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| **2.0** | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| **2.1** | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| **2.2** | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| **2.3** | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| **2.4** | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| **2.5** | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| **2.6** | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| **2.7** | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| **2.8** | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| **2.9** | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| **3.0** | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| **3.1** | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| **3.2** | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| **3.3** | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| **3.4** | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |

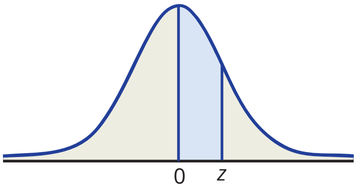
## Critical Values

| Level of Confidence | $z_{\alpha/2}$ |
|---|---|
| **0.80** | 1.28 |
| **0.90** | 1.645 |
| **0.95** | 1.96 |
| **0.98** | 2.33 |
| **0.99** | 2.575 |



# C Standard Normal Distribution

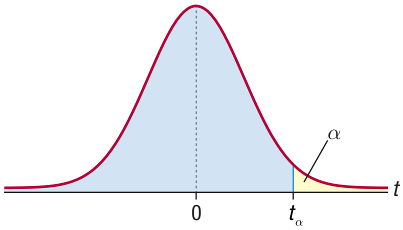
Numerical entries represent the probability that a standard normal random variable is between 0 and $z$ where $z = \frac{x - \mu}{\sigma}$.




| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **0.0** | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| **0.1** | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| **0.2** | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| **0.3** | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| **0.4** | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| **0.5** | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| **0.6** | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| **0.7** | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| **0.8** | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| **0.9** | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| **1.0** | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| **1.1** | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| **1.2** | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| **1.3** | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| **1.4** | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| **1.5** | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| **1.6** | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| **1.7** | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| **1.8** | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| **1.9** | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| **2.0** | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| **2.1** | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| **2.2** | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| **2.3** | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| **2.4** | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| **2.5** | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| **2.6** | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| **2.7** | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| **2.8** | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| **2.9** | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| **3.0** | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |
| **3.1** | 0.4990 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |
| **3.2** | 0.4993 | 0.4993 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4995 | 0.4995 | 0.4995 |
| **3.3** | 0.4995 | 0.4995 | 0.4995 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4997 |
| **3.4** | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4998 |

# D Critical Values of *t*

Numerical entries represent the value of $t$ such that the area to the right of $t$ is equal to $\alpha$.




| Degrees of Freedom | Area to the Right of the Critical Value $t_{0.200}$ | $t_{0.100}$ | $t_{0.050}$ | $t_{0.025}$ | $t_{0.010}$ | $t_{0.005}$ |
|---|---|---|---|---|---|---|
| 1 | 1.376 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 1.061 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 0.978 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 0.941 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 0.920 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | 0.906 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 0.896 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 0.889 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 0.883 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 0.879 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | 0.876 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | 0.873 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | 0.870 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | 0.868 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | 0.866 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | 0.865 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | 0.863 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | 0.862 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | 0.861 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | 0.860 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | 0.859 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | 0.858 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | 0.858 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | 0.857 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | 0.856 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | 0.856 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | 0.855 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | 0.855 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | 0.854 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |
| 30 | 0.854 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 |
| 40 | 0.851 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 |
| 50 | 0.849 | 1.299 | 1.676 | 2.009 | 2.403 | 2.678 |
| 60 | 0.848 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 |
| 70 | 0.847 | 1.294 | 1.667 | 1.994 | 2.381 | 2.648 |
| 80 | 0.846 | 1.292 | 1.664 | 1.990 | 2.374 | 2.639 |
| 90 | 0.846 | 1.291 | 1.662 | 1.987 | 2.368 | 2.632 |
| 100 | 0.845 | 1.290 | 1.660 | 1.984 | 2.364 | 2.626 |
| 120 | 0.845 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 |
| ∞ | 0.842 | 1.282 | 1.645 | 1.96 | 2.326 | 2.576 |