Answer Key

Chapter 1: Algebraic Expressions, Equations, and Inequalities

1.1 Exercises

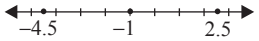
1. a. $19, 2^5$ **b.** $19, \frac{0}{15}, 2^5$

c. $19, \frac{0}{15}, 2^5, -33$

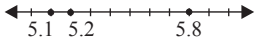
d. $19, -4.3, \frac{0}{15}, 2^5, -33$

e. $-\sqrt{3}$ **f.** All

3.



5.



7. $<, \leq$ **9.** $\leq, \geq$

11. $>, \geq$ **13.** $2a + b > c$

15. $2c \leq 3d$

17. $\{n \mid n \text{ is an integer and } 5 \leq n \leq 105\}$

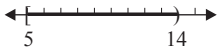
19. $\{2^n \mid n \text{ is a whole number}\}$

21. $\left\{\frac{1}{n} \middle| n \text{ is an odd integer}\right\}$

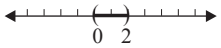
23. $[-3, 19)$ **25.** $(-\infty, 15)$

27. $(0, \infty)$

29.



31.



33.

35. $[-7, 7)$ **37.** $\varnothing$

39. $(-\infty, \infty)$ **41.** $\mathbb{Z}$

43. $\mathbb{Z}$ **45.** -11

47. $\sqrt{5} - \sqrt{3}$ **49.** 1

51. -15 **53.** 5

55. 6

57. $3x^2y^3, -2\sqrt{x+y}, 7z$

59. 1, 8.5, −14

61. $\frac{-5x}{2yz}, -8x^5y^3, 6.9z$

63. -98 **65.** $6\pi + 8$

67. -20 **69.** Commutative

71. Distributive **73.** Associative

75. Multiplicative cancellation; $\frac{1}{5}$

77. Additive cancellation; x

79. Multiplicative cancellation; 6

81. Additive cancellation; $-2x + y$

83.

85. 20 miles

87. If sugar = s,

$\{s \mid 3 \leq s \leq 4\} = [3, 4]$

If walnuts = w,

$\left\{w \middle| \frac{1}{2} \leq w \leq \frac{2}{3}\right\} = \left[\frac{1}{2}, \frac{2}{3}\right]$

89. 131.11 °C

91. Derek's BMI is 23.7 and his weight status is normal.

93. 187 ft

95. The union contains the citizens with brown hair or blue eyes and the intersection contains the citizens with brown hair and blue eyes.

97. Yes, all whole numbers are also integers, but the negative integers are not whole numbers, so not all integers are whole numbers.

1.2 Exercises

1. 16 **3.** 81

5. x^3 **7.** $\frac{3}{t^5}$

9. $2n^8$ **11.** $\frac{y^2}{s^3 z^{11}}$

13. $\frac{1}{9m^2 - 8n^6}$ **15.** $\frac{25m^8}{n^4}$

17. $27x^9$ **19.** $\frac{1}{5z^6 - 81x^{12}}$

21. -9.12×10^8 **23.** 32,000,000

25. 3.1536×10^7 **27.** 0.0003937

29. 2.8×10^{-3} **31.** -11

33. 1×10^{-8} **39.** -3

41. $-\frac{3}{5}$ **43.** 2

45. $\frac{2}{5}$ **47.** $|x^3|\sqrt{2y}$

49. $\frac{1}{2|xy^3|}$ **51.** $\frac{|x^3|y^2}{2}$

53. $2xy^2\sqrt[5]{x^2}$ **55.** $-\frac{|a|\sqrt{2}}{2}$

57. $\sqrt{6} + \sqrt{3}$ **59.** $\frac{x + 2\sqrt{xy} + y}{x - y}$

61. $\frac{1}{\sqrt{13} - \sqrt{t}}$ **63.** $3x\sqrt[3]{2x}$

65. Not possible **67.** $5x^2\sqrt[5]{x^3}$

69. $|x^3|$ **71.** $(3x^2 - 4)^2$

73. 4 **75.** $y\sqrt[3]{y^2}$

77. $\frac{1}{64}$ **79.** $6^{-\frac{1}{3}}$

81. $\sqrt[4]{125}$ **83.** y^4

89. $\pi r^2 h$ **91.** 585 m^3

93. $14(N + M)$ ft^2

95. $bh + bl + hl + l\sqrt{b^2 + h^2}$

97. 1651 cm^2; no

103. Because a root is the same as a fractional exponent.

1.3 Exercises

1. Not a polynomial

3. Degree 11; polynomial of four terms

5. Degree 0 monomial

7. Degree 4 binomial

9. Degree 2 trinomial

11. Degree 5 binomial

13. $-x^{13} + 7x^{11} - 4x^{10} + 9$

a. 13 **b.** -1

15. $2s^6 - 10s^5 + 4s^3$ **a.** 6 **b.** 2

17. $9y^6 - 3y^5 + y - 2$ **a.** 6 **b.** 9

19. πz^5+8z^2-2z+1 **a.** 5 **b.** π

21. $-4x^3y-6y-x^2z$

23. $x^2y+xy^2+6x-6y$

25. $-3ab$

27. xy^2-x^2y-y

29. $3a^3b^3+21a^3b^2+2a^2b^2+14a^2b$
$-3ab^3-21ab^2$

31. $3a^2-2ab-8b^2$

33. $6x^2+33xy-18y^2$

35. $7y^4-34xy^2-5x^2$

37. $6x^3y^3-3x^3y+36x^2y^3+4x^2y^2$
$-18x^2y+24xy^2$

39. $9a^2+6ab+b^2$

41. $4x^2-9y^2$

43. $x^2+4xy+4y^2$

45. $\frac{1}{x^2}-y^2$

47. $m\left(4mn+16m^2+7\right)$

49. $6\left(a-b^2\right)$

51. $2x\left(x^5-7x^2+4\right)$

53. $\left(x^3-y\right)\left(x^3-y-1\right)$

55. $4y^2\left(3y^4-2-4y^3\right)$

57. $\left(a^2+b\right)(a-b)$

59. $z(1+z)\left(1+z^2\right)$

61. $(n-2)\left(x^2+y\right)$

63. $(a-5b)(x+5y)$

65. $(2x-11)(2x+11)$

67. $(7a-12b)(7a+12b)$

69. $\left(5x^2y-3\right)\left(5x^2y+3\right)$

71. $(x-10y)\left(x^2+10xy+100y^2\right)$

73. $\left(m^2+5n^3\right)\left(m^4-5m^2n^3+25n^6\right)$

75. $\left(3x^2-2y^4z\right)$
$\times\left(9x^4+6x^2y^4z+4y^8z^2\right)$

77. $\left(4y^2z-3x^4\right)\left(4y^2z+3x^4\right)$

79. $\left(7y^3+3xz^2\right)$
$\times\left(49y^6+21xy^3z^2+9x^2z^4\right)$

81. $(x+5)(x-3)$

83. $(x-1)^2$

85. $(x-2)^2$

87. $(y+7)^2$

89. $(x+11)(x+2)$

91. $(y-8)(y-1)$

93. $(5a+3)(a-8)$

95. $(x+6)(5x-3)$

97. $(16y-9)(y-1)$

99. $(4a-3)(2a+1)$

101. $(4y-5)(3y-1)$

103. $2x(2x-1)^{\frac{-3}{2}}$

105. $a^{-3}\left(7a^2-2b\right)$

107. $2y^{-5}\left(5y^3-x\right)$

109. $(5x+7)^{\frac{4}{3}}(5x+6)$

111. $y^{-4}\left(7y^3+5\right)$

113. No; a variable in the denominator is equivalent to a variable with a negative exponent.

115. **a.** Yes; degree = 4; leading coefficient = 2; terms = 4

b. Yes; degree = 3; leading coefficient = 2; terms = 3

1.4 Exercises

1. $\frac{2x+1}{x-5}$; $x\neq -3, 5$

3. $x(x-1)$; $x\neq -3$

5. $\frac{x+6}{x+5}$; $x\neq -5, 1$

7. $\frac{1}{x^2-x+1}$; $x\neq -1$

9. $2x+1$; $x\neq -5$

11. $2x-3$; $x\neq -7$

13. $\frac{x^3+9x^2+11x+19}{(x-3)(x+5)}$

15. $\frac{13x}{(x-3)(x+5)}$

17. $\frac{x^3+4x^2-7x+18}{(x+3)(x-3)}$

19. $\frac{x^2+11x+17}{x+3}$

21. $\frac{x+2}{x-6}$

23. $y-1$

25. $(x+2)(2x+3)$

27. $\frac{y-8}{y+8}$ **29.** $5y^2-2y-3$

31. -6 **33.** $\frac{x^2+9}{6x-3}$

35. $\frac{2x^2}{x+1}$ **37.** $\frac{s-r}{r^2s+s}$

39. $\frac{m+n}{mn}$ **41.** $\frac{x}{y}$

43. x^2y^2 **45.** $\frac{11x}{7y}$

47. $\frac{5z-3x}{z^2}$ **49.** $\frac{x-2}{x+2}$

51. $\frac{\left(z^2-11z+54\right)(z-9)}{(z-2)}$

53. $\frac{2y^2+5y-4}{y+1}$

1.5 Exercises

1. $5i$ **3.** $-3i\sqrt{3}$

5. $4i\sqrt{2x}$ **7.** $i\sqrt{29}$

9. $1-3i$ **11.** $8-6i$

13. $-5+6i$ **15.** $16-30i$

17. i **19.** -11

21. $40-42i$ **23.** -9

25. $1+5i$ **27.** $-1-4i$

29. $7i$ **31.** $3+i$

33. $-i$ **35.** $-i$

37. $10-2i$ **39.** $\frac{14}{37}+\frac{10}{37}i$

41. $\frac{21}{17}-\frac{1}{17}i$ **43.** $-5+2i\sqrt{6}$

45. 8 **47.** $-\frac{7}{3}i$

49. $22+10i\sqrt{3}$ **51.** $6+3j$ ohms

53. $11-2j$ ohms

1.6 Exercises

1. $t=-5$ **3.** $y=-1$

5. $w=-3$ **7.** $\mathbb{R}$ (Identity)

9. $\varnothing$ (Contradiction)

11. $m=7$ **13.** $x=3.7$

15. $x=1.05$ **17.** $y=-5$

19. $\mathbb{R}$ (Identity) **21.** $\mathbb{R}$ (Identity)

23. $x = 3$

25. $\varnothing$ (Contradiction)

27. $y = -\frac{1}{3}, -3$ **29.** $x = \frac{1}{3}$

31. $x = -311, 420$

33. $x = -\frac{4}{5}, 2$

35. $\varnothing$ (Contradiction)

37. $x = -2, 2$ **39.** $x = 5$

41. $x = -\frac{1}{2}$ **43.** $x = \frac{1}{4}$

45. $x = \frac{1}{7}$ **47.** $r = \frac{C}{2\pi}$

49. $a = \frac{v^2 - v_0^{\,2}}{2x}$

51. $F = \frac{9}{5}C + 32$

53. $h = \frac{A - 2lw}{2w + 2l}$

55. $m = \frac{2K}{v^2}$

57. $\frac{19}{3}$ hours, or 6 hours and 20 minutes

59. 13.5 miles **61.** \$390

63. 2 gallons 44%, 1 gallon 50%

65. 24 child tickets, 15 adult tickets

67. 7.5%

69. 26 feet by 26 feet

71. 53, 55, and 57

73. 36.4% **75.** $x \approx 0.72$

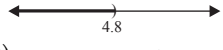
77. $x \approx 13.11$

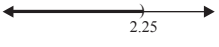
1.7 Exercises

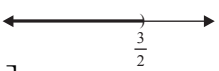
1. $\{-9, 3.14, -2.83, 1, -3, 4\}$

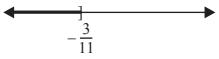
3. $\{-2.83, 1, -3\}$

5. $(-\infty, -3]$ 

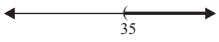
7. $(-\infty, 4.8)$ 

9. $(-\infty, 2.25)$ 

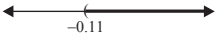
11. $\left(-\infty, \frac{3}{2}\right)$ 

13. $\left(-\infty, -\frac{3}{11}\right]$ 

15. $(7, \infty)$ 

17. $(35, \infty)$ 

19. $(-3, \infty)$ 

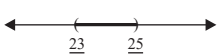
21. $(-0.11, \infty)$ 

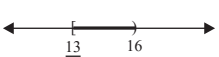
23. $(1, 5]$ 

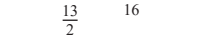
25. $(-10, 6]$ 

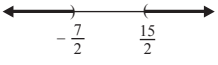
27. $[-8, -2)$ 

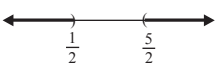
29. $(21, 69]$ 

31. $\left(\frac{23}{7}, \frac{25}{7}\right)$ 

33. $\left[\frac{13}{2}, 16\right)$ 

35. $\left(-\frac{5}{3}, 1\right]$ 

37. $\left(-\infty, -\frac{7}{2}\right) \cup \left(\frac{15}{2}, \infty\right)$ 

39. $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$ 

41. $\varnothing$

43. $(-\infty, 2) \cup (6, \infty)$ 

45. $\varnothing$

47. $\varnothing$

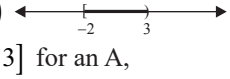
49. $[-4, 0]$ 

51. $(-\infty, \infty)$

53. $(3, 15)$ 

55. $(-1, 3]$ 

57. $(-\infty, \infty)$ 

59. $[-2, 3)$ 

61. $[73, 113]$ for an A, $(113, 115)$ for an A+.

63. $(1140, 1600]$

1.8 Exercises

1. $\left\{\frac{3}{2}, -1\right\}$ **3.** $\left\{\frac{-3}{2}, -3\right\}$

5. $\{2\}$ **7.** $\left\{\pm\frac{3}{2}\right\}$

9. $\left\{2 \pm i\sqrt{5}\right\}$ **11.** $\left\{-\frac{5}{3}, \frac{1}{3}\right\}$

13. $\{-5, 9\}$ **15.** $\{-5, 2\}$

17. $\left\{-\frac{7}{2}, \frac{9}{2}\right\}$ **19.** $\{-16, -6\}$

21. $\{0.17 \pm 0.86i\}$

23. $\left\{0, \frac{2}{3}\right\}$ **25.** $\left\{-\frac{3}{2}, 5\right\}$

27. Two real solutions

29. One real solution

31. $\{8, 14\}$ **33.** $\left\{-3, \frac{3}{10}\right\}$

35. $\left\{\frac{-1 \pm i\sqrt{2}}{2}\right\}$ **37.** $\{1 \pm 17i\}$

39. $\{0, 6\}$ **41.** $\{3, 5\}$

43. $\left\{-1, 2, \frac{1 \pm i\sqrt{7}}{2}\right\}$

45. $\{-3, 4\}$ **47.** $\left\{\pm\sqrt{2}, \pm i\sqrt{5}\right\}$

49. $\left\{1 \pm 2i, 1 \pm \sqrt{3}\right\}$

51. $\left\{\frac{1}{8}, 27\right\}$ **53.** $\{\pm 2i, \pm 3\}$

55. $\{-1, \pm 2, 3\}$ **57.** $\left\{1, -\frac{8}{27}\right\}$

59. $\{\pm 1, 3\}$ **61.** $\{\pm 2, \pm 3i\}$

63. $\left\{\pm 2, -\frac{6}{5}\right\}$ **65.** $\left\{\pm\frac{3}{2}, \pm\frac{3i}{2}\right\}$

67. $\left\{-\frac{5}{2}, 0, \frac{4}{7}\right\}$

69. $\left\{-\frac{4}{3}, \frac{2 \pm 2i\sqrt{3}}{3}\right\}$

71. $\left\{-\frac{5}{3}, 0, 1\right\}$ **73.** $\{1\}$

75. $\{4\}$ **77.** $\{0, 2, 3\}$

79. $\left\{-\frac{1}{5}, \frac{1}{7}\right\}$ **81.** $\left\{-1, 0, \frac{2}{5}\right\}$

83. $\left\{-\frac{1}{3}, -\frac{1}{5}\right\}$ **85.** 3 seconds

87. 3 seconds **89.** 2.6 seconds

91. $(x - 3 - 2i)(x - 3 + 2i)$

93. $\left(2x + 3 - 2\sqrt{2}\right)\left(2x + 3 + 2\sqrt{2}\right)$

95. $b = -5$ and $c = -24$

97. $b = -4$, $c = -12$, and $d = 0$

99. $a = 1$, $c = -36$, and $d = -144$

101. $a = 15$, $b = -16$, and $c = -5$

1.9 Exercises

1. $\left\{-\frac{3}{2}\right\}$ **3.** $\left\{3 \pm \sqrt{10}\right\}$

5. $\left\{-3 \pm \sqrt{6}\right\}$ **7.** $\{-2\}$

9. $\varnothing$

11. $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

13. $\left\{\frac{5}{2}, \frac{7}{2}\right\}$

15. $\left\{1, -\sqrt{5}, 2 + \sqrt{5}\right\}$

17. $\{0\}$ **19.** $\varnothing$

21. $\{1\}$ **23.** $\left\{\frac{2}{3}\right\}$

25. $\varnothing$ **27.** $\left\{\frac{29}{8}\right\}$

29. $\{6\}$ **31.** $\varnothing$

33. $\{-2, 1\}$ **35.** $\{1\}$

37. $\{10\}$ **39.** $\{4\}$

41. $\{2\}$ **43.** $\{-32\}$

45. $\left\{\pm\frac{125}{343}\right\}$ **47.** $\{-2, 5\}$

49. $\{7, 10\}$

51. $a = \pm\sqrt{c^2 - b^2}$

53. $m = \frac{k}{\omega^2}$ **55.** $v = \pm\sqrt{\frac{Fr}{m}}$

57. $h = \pm\sqrt{\frac{m}{23}}$ **59.** $c = \pm\sqrt{\frac{2gm}{r}}$

61. $b = \pm\sqrt{c^2 - a^2}$

63. $a = \sqrt[3]{\frac{uP^2}{4\pi^2}}$

65. $\frac{4}{3}$ minutes, or 1 minute and 20 seconds

67. 4 hours and 12 hours

69. 90 minutes **71.** 9.1 hours

Chapter 1 Project

1. Alex: 81.7; Ashley: 90.3; Barron: 81.0; Elizabeth: 83.1; Gabe: 90.5; Lynn: 82.4

3. The final exam contributes a higher percentage of points to the final grade than does the semester project.

Chapter 1 Review Exercises

1. a. 2^3 **b.** $2^3, 0$

c. $-\sqrt{4}$, 2^3, 0

d. All except $\sqrt{17}$

e. $\sqrt{17}$ **f.** All

3. $[4, 17)$ **5.** -7

7. 4 **9.** -1

11. $\frac{x^2}{2y}$, $12.1x$, $-\sqrt{y+5}$

13. $\frac{4\pi}{3} - 36$ **15.** 51

17. Commutative Property

19. Zero-Factor Property

21. $[5, 8)$ **23.** $-\frac{t^9}{2s^7}$

25. $\frac{18y^2}{x^4z^5}$ **27.** 6.952×10^7

29. 2.0×10^{-8} **31.** 5

33. $5x^{10}$ **35.** $-\frac{4y}{x^3}$

37. $\frac{2y\sqrt[3]{9x^2y}}{3}$ **39.** $-\sqrt{2} - \sqrt{6}$

41. $3|x|\sqrt{2xy} - 2x\sqrt[3]{2xy}$

43. $\frac{1}{x^{\frac{7}{4}}}$

45. $m^4 - 5m^3 + 3m^2 + 2$

47. $3x^3 - 4x^2y^3 + 3xy - 4y^4$

49. $(x+3)(x-4)$

51. $(2a+1)(3a-5)$

53. $(6x^3 + y)(6x^3 - y)$

55. $(2x - 5y)(x + 3)$

57. $(3x-2y)^{\frac{2}{3}}\left[(3x-2y)^{\frac{2}{3}} - 1\right]$

59. $\frac{x+3}{x-3}$, $x \neq 0, \pm 3$

61. $\frac{-2}{x}$ **63.** $\frac{b-a}{4a+4b}$

65. $-x - y$ **67.** 3

69. $5 + 9i$ **71.** $4 + i$

73. $-\frac{7}{25} + \frac{24}{25}i$ **75.** $62 - 16i\sqrt{2}$

77. $\varnothing$ (Contradiction)

79. $x = 6.25$ **81.** $x = 3, 4$

83. $z = -\frac{10}{7}, 0$ **85.** $x = -3, 4$

87. $c = \frac{2A}{h} - b$ **89.** $C = \frac{5}{9}(F - 32)$

91. \$85

93. $(4, \infty)$ 

95. $(1, \infty)$ 

97. $[-7, 4)$ 

99. $(-5, -1)$ 

101. $(-\infty, -6] \cup [8, \infty)$

103. $(-\infty, 4] \cup [10, \infty)$

105. $\left\{-\frac{2}{5}, 3\right\}$ **107.** $\{2 \pm 3i\}$

109. $\left\{4 \pm \sqrt{2}\right\}$ **111.** $\left\{3 \pm \sqrt{7}i\right\}$

113. $\left\{-4, \frac{5}{2}\right\}$ **115.** $\left\{\frac{19 \pm \sqrt{701}}{17}\right\}$

117. $\left\{\pm 1, \pm\sqrt{2}\right\}$ **119.** $\{-6, 4\}$

121. $\left\{\pm\sqrt{2}, 4\right\}$ **123.** $\{1, \pm 2i\}$

125. $\{-1, 0, 4\}$ **127.** $\left\{\frac{3}{2}, 2\right\}$

129. $b = -2$ and $c = -8$

131. -5 **133.** $\{0, 3\}$

135. $\{-5\}$ **137.** $\{-4\}$

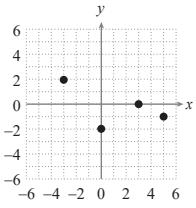
139. $\{2\}$ **141.** $\left\{-\frac{3}{2}, 5\right\}$

143. $\{3\}$ **145.** $r = \sqrt{\frac{3V}{\pi h}}$

Chapter 2: Equations and Inequalities in Two Variables

2.1 Exercises

1.



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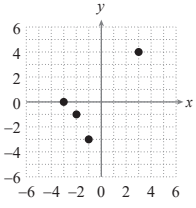

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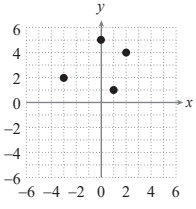

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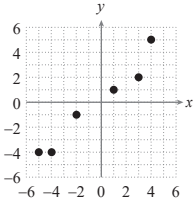


9. III **11.** IV

13. Positive x-axis

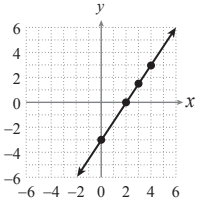
15. III **17.** IV

19. II **21.** IV

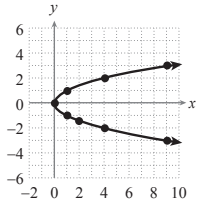
23. I

25. Negative y-axis

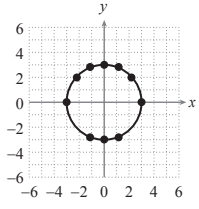
27. $\left\{(0,-3),(2,0),\left(3,\frac{3}{2}\right),(4,3)\right\}$



29. $\left\{(0,0),(1,\pm1),(4,\pm2),(9,\pm3),\left(2,-\sqrt{2}\right)\right\}$



31. $\left\{(0,\pm3),(\pm3,0),\left(-1,\pm2\sqrt{2}\right),\left(1,\pm2\sqrt{2}\right),\left(\pm\sqrt{5},2\right)\right\}$



33. $\sqrt{34},\ \left(\frac{-7}{2},\frac{1}{2}\right)$

35. $\sqrt{58},\ \left(\frac{3}{2},\frac{7}{2}\right)$

37. $2\sqrt{2},(-1,-1)$

39. $4\sqrt{34},(3,-8)$

41. $10,(1,-6)$

43. $3\sqrt{13},\ \left(2,\frac{1}{2}\right)$

45. $10\sqrt{2},\ (3,3)$

47. $x=2$ or 18

49. $x=10,\ y=1$

51. 12

53. $2\sqrt{29}+\sqrt{26}+5\sqrt{2}$

55. 54

57. 1.25 kilometers

59. **a.** 249.19 meters

b. $\left(\frac{133}{2},\frac{709}{2}\right)$

61. area $=\frac{15}{2}$ **63.** area $=25$

65. area $=17$ **67.** area $=48$

69. $x=[-5,6];\ y=[-8,9]$

71. $x=[-3,6];\ y=[-4,5]$

73. $x=[-6,8];\ y=[-9,7]$

2.2 Exercises

1. $(x+4)^2+(y+3)^2=25$

3. $(x-7)^2+(y+9)^2=9$

5. $x^2+y^2=6$

7. $\left(x-\sqrt{5}\right)^2+\left(y-\sqrt{3}\right)^2=16$

9. $(x-7)^2+(y-2)^2=4$

11. $(x+3)^2+(y-8)^2=2$

13. $(x-4)^2+(y-8)^2=10$

15. $x^2+y^2=85$

17. $\left(x+\frac{7}{2}\right)^2+\left(y-\frac{17}{2}\right)^2=\frac{53}{2}$

19. $(x+6)^2+\left(y-\frac{3}{2}\right)^2=\frac{125}{4}$

21. $\left(x+\frac{13}{2}\right)^2+(y+7)^2=\frac{365}{4}$

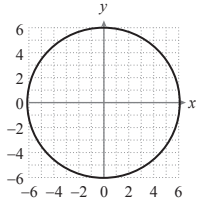
23. $(x-4)^2+(y-3)^2=25$

25. $(x-2)^2+y^2=4$

27. $(x-2)^2+(y-4)^2=49$

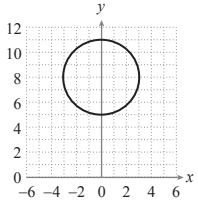
29. $(x+3)^2+(y+2)^2=64$

31.



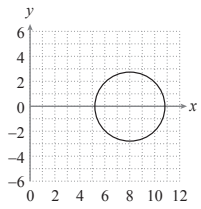
$(0,0),\ r=6$

33.



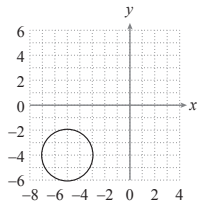
$(0,8),\ r=3$

35.



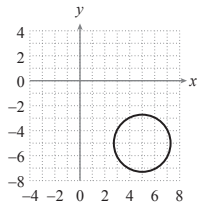
$(8,0),\ r = 2\sqrt{2}$

37.



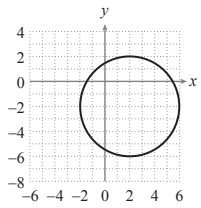
$(-5,-4),\ r = 2$

39.



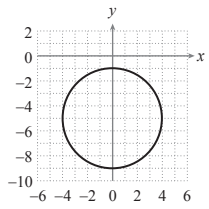
$(5,-5),\ r = \sqrt{5}$

41.



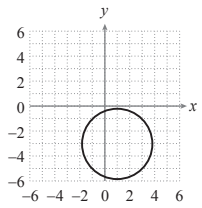
$(2,-2),\ r = 4$

43.



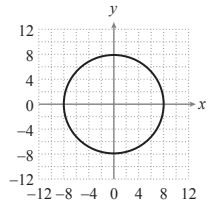
$(0,-5),\ r = 4$

45.



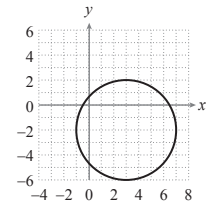
$(1,-3),\ r = 2\sqrt{2}$

47.



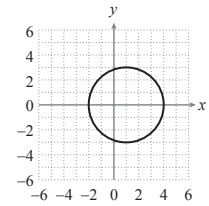
$(0,0),\ r = 8$

49.



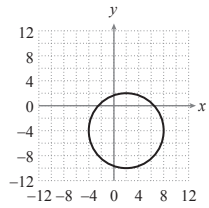
$(3,-2),\ r = 4$

51.



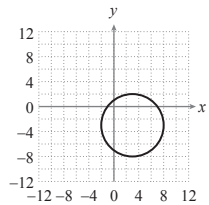
$(1,0),\ r = 3$

53.



$(2,-4),\ r = 6$

55.

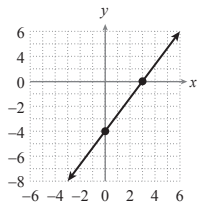


$(3,-3),\ r = 5$

2.3 Exercises

1. Yes	**3.** No	**5.** No
7. No	**9.** Yes	**11.** Yes
13. Yes	**15.** No	**17.** No
19. No	**21.** No	**23.** Yes

25.



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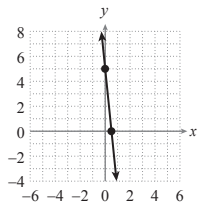

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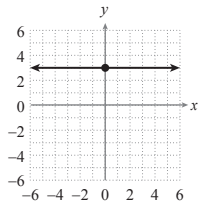

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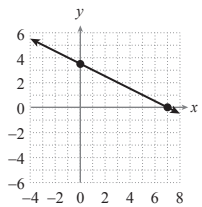

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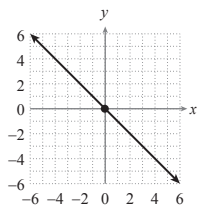

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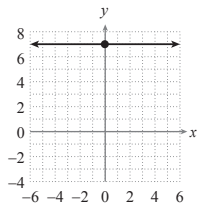

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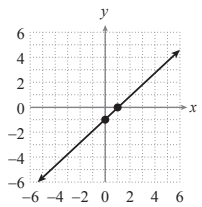

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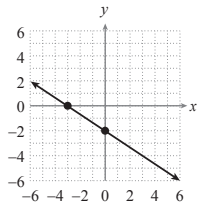


41. e **43.** c **45.** f

47. $a = P - b - c$

49. $j = 24{,}000 + 9b$;

$b = \dfrac{j - 24{,}000}{9}$; Yes

2.4 Exercises

1. -4 **3.** 0

5. Undefined **7.** $\dfrac{2}{3}$

9. $\dfrac{1}{6}$ **11.** -7

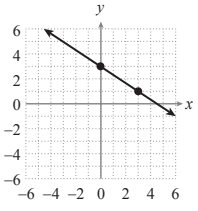
13. -3 **15.** $-\dfrac{9}{13}$

17. $-\dfrac{1}{4}$ **19.** 0

21. Undefined **23.** 2

25. $\dfrac{7}{6}$ **27.** $-\dfrac{5}{2}$

29.



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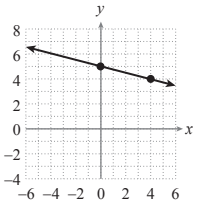

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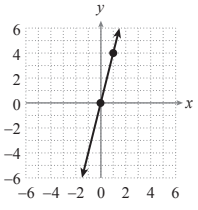

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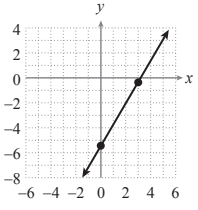


37. $y = \dfrac{3}{4}x - 3$ **39.** $y = -\dfrac{5}{2}x - 7$

41. $y = -5x - 9$ **43.** $3x - 2y = 3$

45. $y = 5$ **47.** $10x - y = 31$

49. $3x + y = 26$ **51.** $4x + 3y = 5$

53. $x = 2$ **55.** $y = -1$

57. $2x + 7y = 52$

59. $y = 5$ **61.** $15x - 8y = 0$

63. c **65.** e **67.** d

69. a. \$2225 **b.** \$2100 **c.** \$0.25

71. \$325

2.5 Exercises

1. $y = 4x + 9$ **3.** $y = 3x - 11$

5. $y = -9$ **7.** $y = x$

9. $y = \dfrac{7}{6}x + \dfrac{53}{6}$

11. Yes **13.** Yes **15.** Yes

17. No **19.** No **21.** No

23. No **25.** No **27.** Yes

29. No

31. $y = -\dfrac{1}{3}x - 1$ **33.** $y = 7$

35. $y = -\dfrac{1}{4}x - \dfrac{3}{4}$

37. $y = x + 3$ **39.** $y = -3x + 28$

41. No **43.** No **45.** No

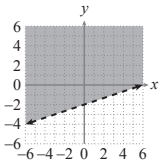
47. No **49.** Yes **51.** No

53. No **55.** No **57.** Yes

59. $41\dfrac{2}{3}$ ft

2.6 Exercises

1.



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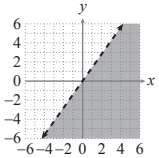

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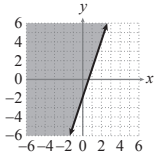
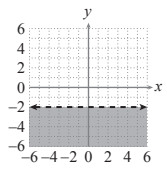


7.



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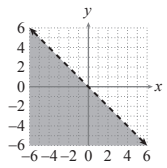

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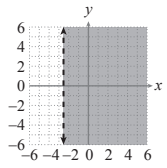
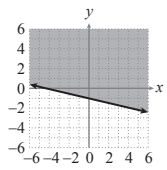


13.



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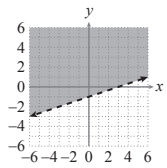

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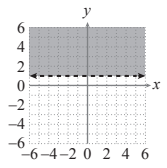

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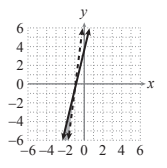

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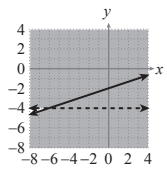

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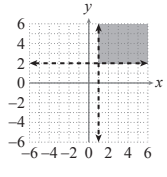

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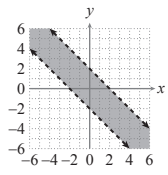

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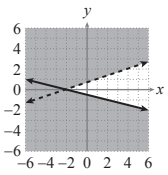

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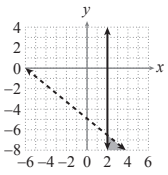

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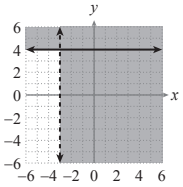

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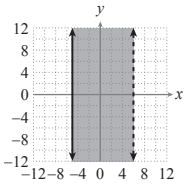

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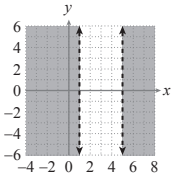

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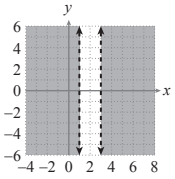

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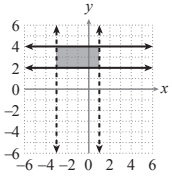

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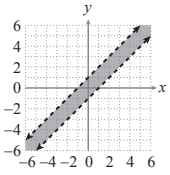

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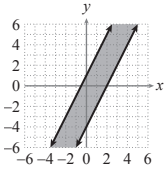

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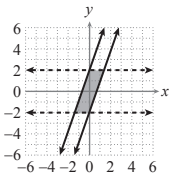
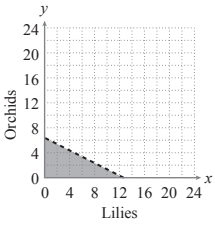


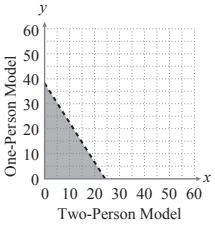
47. h **49.** b

51. g **53.** c

55. $12x + 22y < 150$



57. $73x + 46y < 1750$



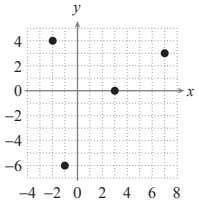
Chapter 2 Project

1. Shelbyville Tower: approx. 47.5 ft; Brockton Tower: 75 ft; Springfield Tower: approx. 60.6 ft; Ogdenville Tower: approx. 109.5 ft

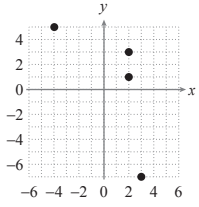
3. No; attachment point: 29.4 ft from the top

Chapter 2 Review Exercises

1.

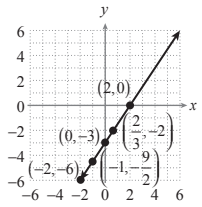


3.



5. Positive x-axis

7.



$(2,0), (0,-3), \left(-1,-\frac{9}{2}\right), \left(\frac{2}{3},-2\right), (-2,-6)$

9. a. $\sqrt{2}$ **b.** $\left(\frac{5}{2},-\frac{13}{2}\right)$

11. a. $2\sqrt{13}$ **b.** $(-5,3)$

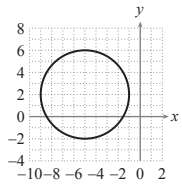
13. 2 **15.** 24

17. $\left(x-\sqrt{5}\right)^2+\left(y+\sqrt{2}\right)^2=16$

19. $(x-2)^2+(y+1)^2=20$

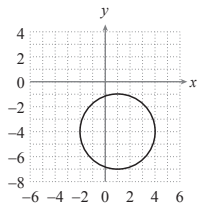
21. Center: $(-3,1)$; Radius: $2\sqrt{2}$

23.



$r = 4$; $(h,k) = (-5,2)$

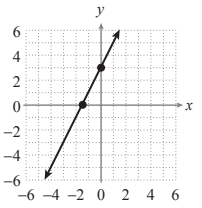
25.



$r = 3$; $(h,k) = (1,-4)$

27. No **29.** Yes **31.** No

33.



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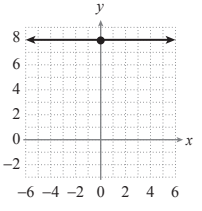

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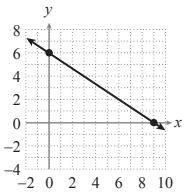
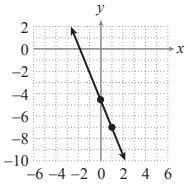


39. 12

41. Undefined

43.



45. $x - y = 5$

47. $y = \frac{5}{9}x - 2$

49. $9x - 2y = 31$

51. $W = 0.08s + 2800$

53. Perpendicular

55. $y = 3x + 10$

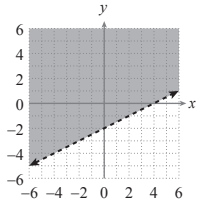
57. $y = 2x - 3$

59. $y = -\frac{4}{3}x + 6$

61. $x = 7$

63. Yes

65.



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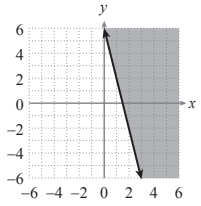

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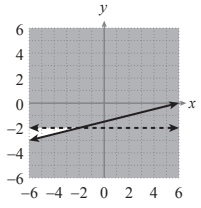
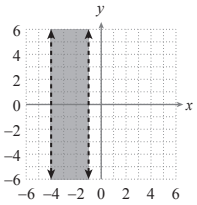


71.



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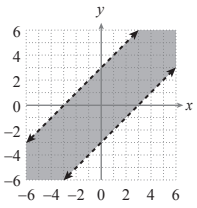
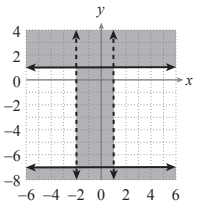


75.



77. $3x + 4y \geq 1500$

Chapter 3: Relations, Functions, and Their Graphs

3.1 Exercises

1. Dom $= \{-2\}$, Ran $= \{5, 3, 0, -9\}$

3. Dom $= \{\pi, -2\pi, 3, 1\}$, Ran $= \{2, 4, 0, 7\}$

5. Dom $= \mathbb{Z}$, Ran = even integers

7. Dom $= \mathbb{Z}$, Ran $= \{\ldots, -2, 1, 4, \ldots\}$

9. Dom = Ran $= \mathbb{R}$

11. Dom $= [0, \infty)$, Ran $= \mathbb{R}$

13. Dom $= \mathbb{R}$, Ran $= \{-1\}$

15. Dom $= \{0\}$, Ran $= \mathbb{R}$

17. Dom $= [-3, 1]$, Ran $= [0, 4]$

19. Dom $= [0, 3]$, Ran $= [1, 5]$

21. Dom $= [-1, 3]$, Ran $= [-4, 3]$

23. Dom = All males with siblings, Ran = All people who have brothers

25. Not a function; $(-2, 5)$ and $(-2, 3)$

27. Function

29. Not a function; $(6, -1)$ and $(6, 4)$

31. Not a function; $(-1, 0)$ and $(-1, 4)$

33. Function **35.** Function

37. Function

39. Not a function; $(-1, -1)$ and $(-1, 1)$

41. Function **43.** Function

45. Not a function; $(1, -2)$ and $(1, 2)$

47. $f(x) = -6x^2 + 2x, f(-1) = -8$

49. $f(x) = \frac{-x + 10}{3}, f(-1) = \frac{11}{3}$

51. $f(x) = -2x - 10, f(-1) = -8$

53. 1

55. 3

57. $x = -3, 2, 3$

59. **a.** 10 **b.** $x^2 + x - 2$
c. $2ax + 3a + a^2$ **d.** $x^4 + 3x^2$

61. **a.** 8 **b.** $3x - 1$
c. $3a$ **d.** $3x^2 + 2$

63. **a.** -2 **b.** $-6x + 16$
c. $-6a$ **d.** $-6x^2 + 10$

65. **a.** $i - 3$ **b.** $\sqrt{2 - x} - 3$
c. $\sqrt{1 - x - a} - \sqrt{1 - x}$
d. $\sqrt{1 - x^2} - 3$

67. $2x + h - 5$

69. $\frac{-1}{(x + h + 2)(x + 2)}$

71. $5(2x + h)$

73. 2

75. $\frac{\sqrt{x + h} - \sqrt{x}}{h}$

77. Dom = Cod = Ran $= \mathbb{R}$

79. Dom = Cod = Ran $= \mathbb{Z}$

81. Dom = Cod $= \mathbb{N}$, Ran $= \{6, 7, 8, \ldots\}$

83. $[1, \infty)$

85. $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$

87. $\mathbb{R}$

89. $\left(-\infty,\frac{1}{3}\right)\cup\left(\frac{1}{3},\infty\right)$

91. $(-\infty,2)\cup(2,\infty)$

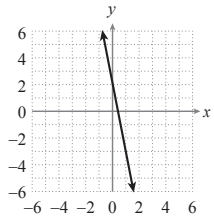
93. $[-6,\infty)$

95. $(-\infty,0)\cup(0,\infty)$

97. A function is a special relation in which every element of the domain is paired with exactly one element of the range.

3.2 Exercises

1.



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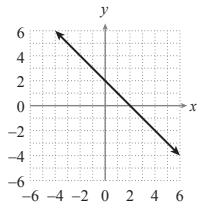
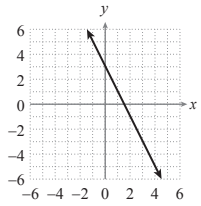
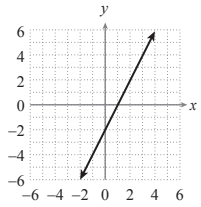


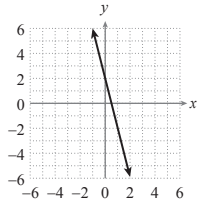
5.



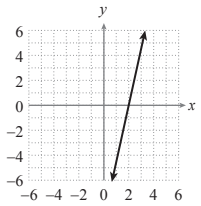
7.



9.



11.



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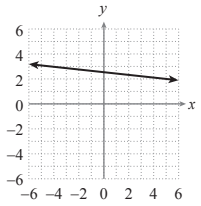
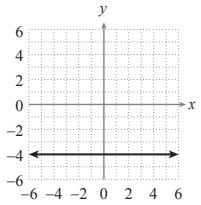


15.

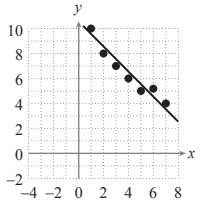


17. b **19.** a

21. $f(x)=-x+3$

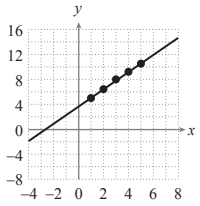
23. $f(x)=2x-3$

25. a. $y=-0.93x+10.14$



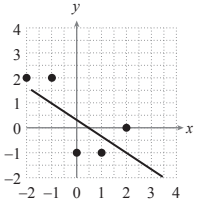
b. $r\approx-0.969$

27. a. $y=1.36x+3.82$



b. $r\approx0.998$

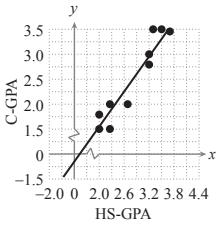
29. a. $y=-0.7x+0.4$



b. $r\approx-0.730$

31. a. $y=1.20x-0.85$

b.

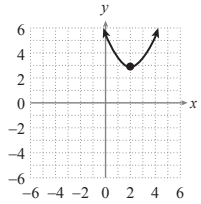


c. $r\approx0.951$

33. Neither. They are equal because the correlation is the absolute value.

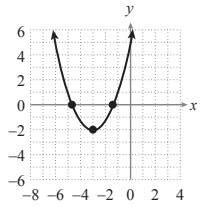
3.3 Exercises

1. Vertex: $(2,3)$; no x-int.



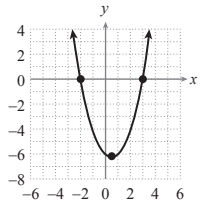
3. Vertex: $(-3,-2)$;

x-int.: $x=-3\pm\sqrt{2}$

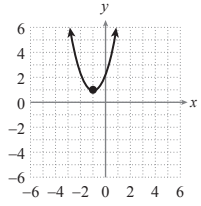


5. Vertex: $\left(\frac{1}{2},-\frac{25}{4}\right)$;

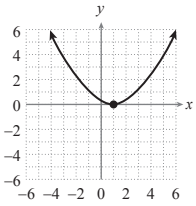
x-int.: $x=-2,3$



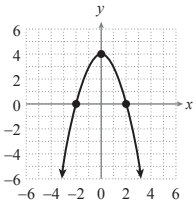
7. Vertex: $(-1,1)$; no x-int.



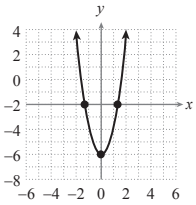
9. Vertex: $(1,0)$; x-int.: $x=1$



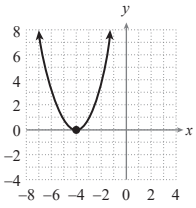
11. Vertex: $(0,4)$; x-int.: $x=-2,2$



13. Vertex: $(0,-6)$; x-int.: $x=\pm\dfrac{\sqrt{6}}{2}$



15. Vertex: $(-4,0)$; x-int.: $x=-4$



17. b **19.** d

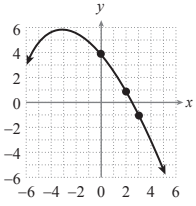
21. a. $f(x)=-\dfrac{1}{2}(x+2)(x-1)$

b. $\left(-\dfrac{1}{2},\dfrac{9}{8}\right)$

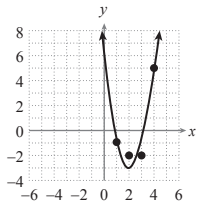
23. a. $f(x)=3(x+1)(x-3)$

b. $(1,-12)$

25. $f(x)=-0.167x^2-1.17x+4$



27. $f(x)=2x^2-8.2x+5.5$



29. a. $f(x)=x^2-2x+2$

b. $(1,1)$

31. a. $f(x)=-x^2+2x-3$

b. $(1,-2)$

33. 5 and 5 **35.** $(2,1)$

37. The dimensions should be 5 inches by 10 inches by 10 feet

39. 49 people; \$2401

41. 12 and 24 **43.** 375 units

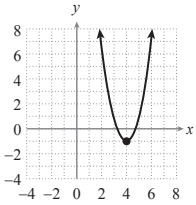
45. 25 sets of golf clubs

47. 112 feet **49.** 164 feet

51. $h(t)=-15.88t^2+60.17t+7$; the maximum height of the baseball was approximately 64 feet.

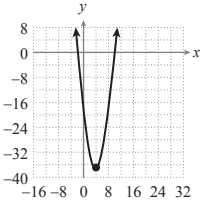
53. Vertex: $(4,-1)$;

x-int.: $x=\dfrac{8\pm\sqrt{2}}{2}$

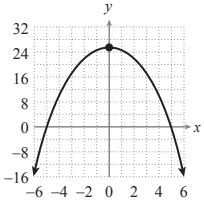


55. Vertex: $(4,-36)$;

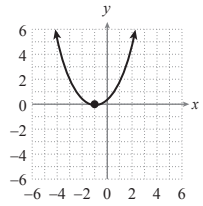
x-int.: $x=-2,10$



57. Vertex: $(0,25)$; x-int.: $x=-5,5$

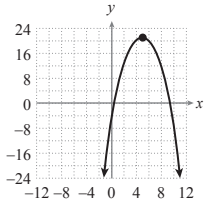


59. Vertex: $(-1,0)$; x-int.: $x=-1$



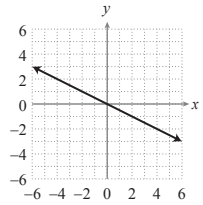
61. Vertex: $(5,21)$;

x-int.: $x=5\pm\sqrt{21}$



3.4 Exercises

1.



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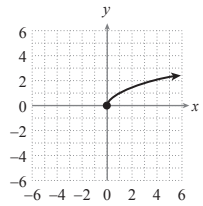

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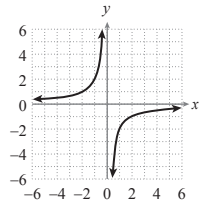

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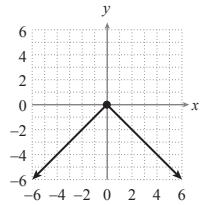

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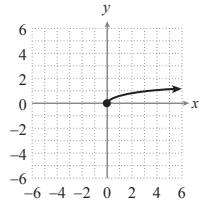

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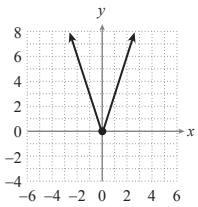

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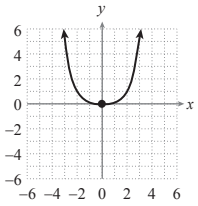

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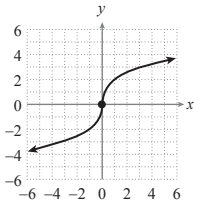

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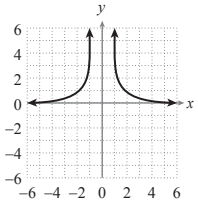

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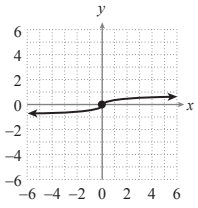

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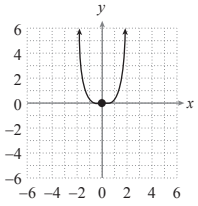

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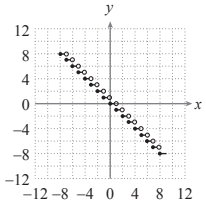

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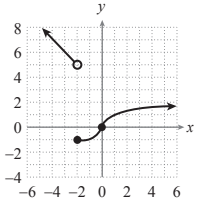

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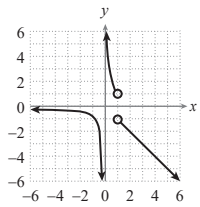

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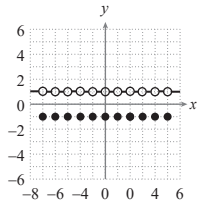

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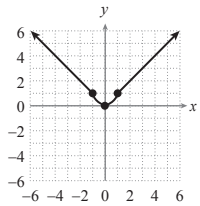

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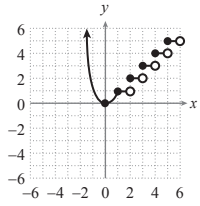

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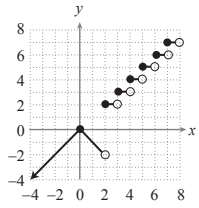


37. j **39.** a **41.** i

43. e **45.** f

3.5 Exercises

1. $A = kbh$ **3.** $W = \dfrac{k}{d^2}$

5. $r = \dfrac{k}{t}$ **7.** $x = ky^3z^2$

9. $y = 18\sqrt{5}$ **11.** $y = 60\sqrt[3]{2}$

13. $y = 0.75$ **15.** $y = 0.0024$

17. $z = 112$ **19.** $z = 48$

21. $a = 10\sqrt{3}$ **23.** $a = 36$

25. $a = 108$ **27.** 256 feet

29. 20.60 **31.** 6.7 meters

33. 1.25 centimeters

35. 34.54 inches **37.** 164.7872 in.^{2}

39. 9 watts **41.** \$43

43. 17.28 ohms **45.** 19.66 inches

47. 210 cubic inches

49. $P(\sigma, \varepsilon) = \dfrac{\sigma^2}{2\varepsilon}$

3.6 Exercises

1. a. If x represents the side length of each square cut from a corner,

$$V(x) = (60 - 2x)(20 - 2x)x$$
$$= 4x^3 - 160x^2 + 1200x$$

for $0 \le x \le 10$.

b. Yes; for example, $V(4) = 2496 \text{ cm}^3$.

c. Approximately 2525 cm^3, obtained by letting $x = 4.5$ cm.

d. A height of 5 cm will lead to a width of 10 cm and consequently a length of 50 cm, so the ratio of length to width will be 5:1.

3. a. $V(t) = P\left(1 - \dfrac{2t}{9}\right)$ for $0 \le t \le 3$

b. $\dfrac{9}{4}$ years, or 2 years and 3 months

c. $V(1) = \dfrac{7}{9}P$

5. a. $d = \sqrt{(1+m^2)x^2 + 2bmx + b^2}$

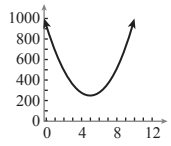
b. $d = \sqrt{x^2 + b^2}$

c. $d = \sqrt{(1+m^2)}\ |x|$

d. $d = 2|x|$

7. a. $s(x) = 30x^2 - 300x + 1000$ for $0 \le x \le 10$

b.



c. 1000, 250

9. a. 9.25×10^6

b. 1.45 times, or 145%

c. They aren't weightless, but instead in free fall. Astronauts fall toward Earth at the same rate as the craft they're in, making it appear as though they're floating.

11. a. $A = 6V^{\frac{2}{3}}$

b. $A = 600 \text{ mm}^2$

13. a. $C = 8x + 4y = 8x + \dfrac{7200}{x}$, where x is the length of fence along the boundary

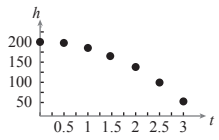
b. 30 ft

15. a. $R(x) = x(30 - 3x)$,
$C(x) = 36 + 6x$,
$P(x) = -3x^2 + 24x - 36$

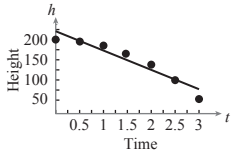
b. 2 **c.** 4

d. \$18 for \$12 maximum profit

17. a.

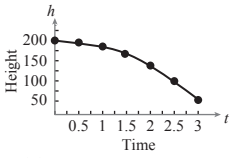


b.



$h(t) = -48t + 220$;
approximately 4.58 s; 220 ft

c.

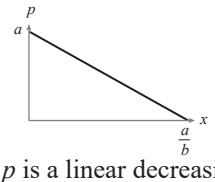


$h(t) = -16t^2 + 200$;
approximately 3.54 s; 200 ft

d. The quadratic model is more accurate.

Chapter 3 Project

1. a.



b. p is a linear decreasing function of x.

c. $x = 0$ corresponds to $p = a$, so a price of a corresponds to no sales.

d. $p = 0$ corresponds to $x = \dfrac{a}{b}$, which is the maximum feasible sales figure.

e. $R(x) = xp = x(a - bx)$
$= -bx^2 + ax$

f. R is a quadratic function.

g. The graph of R is a downward-opening parabola.

Chapter 3 Review Exercises

1. Dom $= \{-2, -3\}$,
Ran $= \{-9, -3, 2, 9\}$; No

3. Dom $= \mathbb{R}$, Ran $= \{2\}$; Yes

5. Dom $= \mathbb{R}$, Ran $= \mathbb{R}$; Yes

7. Dom $= [0, \infty)$, Ran $= [4, \infty)$; Yes

9. Dom $= \{-2, 4\}$,
Ran $= \{-1, 5\}$; Yes

11. $f(x) = 3\sqrt{x + 11} - 4$;
$f(-2) = 5$

13. 4

15. $x = -1, \quad x = 3$

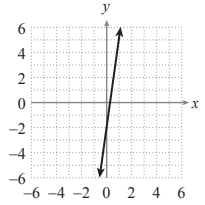
17. $\sqrt{x + h}$

19. $\sqrt[3]{(x + h)^2}$

21. Dom $= \mathbb{N}$, Cod $= \mathbb{R}$,
Ran $= \left\{\dfrac{3}{4}, \dfrac{3}{2}, \dfrac{9}{4}, \ldots\right\}$

23. $\mathbb{R}$

25.



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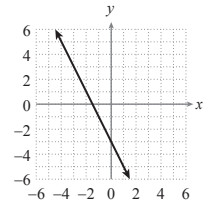

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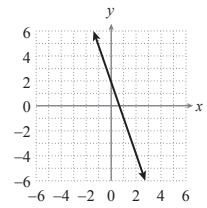
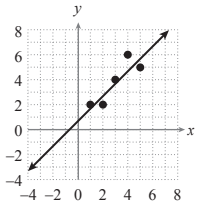


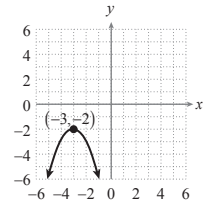
31. $f(x) = 2x - 1$

33. a. $y = x + 0.8$



b. $r \approx 0.884$

35.



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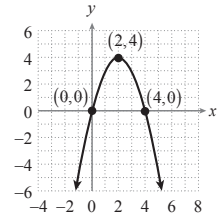

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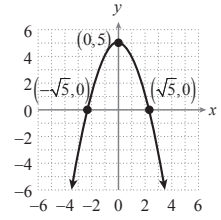
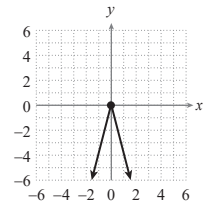


41. a. $f(x) = x^2 - 3x + 1$

b. $(1.5, -1.25)$

43.



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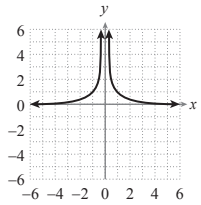

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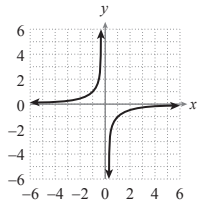

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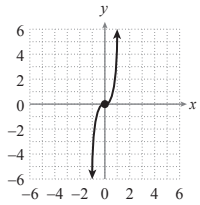

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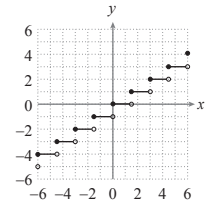

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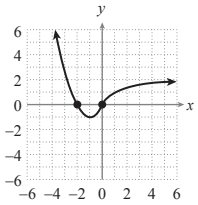
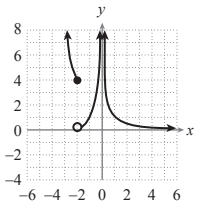
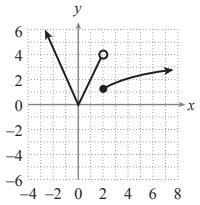


55.



57.



59. $y = \dfrac{ka^3}{\sqrt{b}}$

61. $y = 72$

63. 7.44×10^7 m

65. a. $C(x) = \dfrac{12{,}000}{x} + 13x$, where x is the length of fence along the road.

b. Approximately 30.38 ft (along the road) by 39.50 ft (not along the road); approximately \$789.94

Chapter 4: Working with Functions

4.1 Exercises

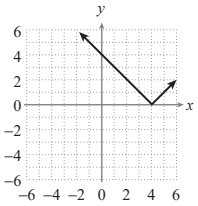
1. $f(x) = x^2$ **3.** $f(x) = \sqrt[3]{x}$

5. $f(x) = \sqrt{x}$ **7.** $f(x) = \dfrac{1}{x^2}$

9. $f(x) = x^3$ **11.** $f(x) = |x|$

13. $f(x) = \sqrt{x}$ **15.** $f(x) = x^3$

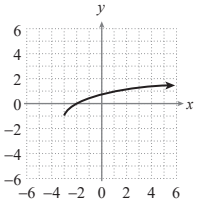
17.



Dom $= \mathbb{R}$,

Ran $= [0, \infty)$

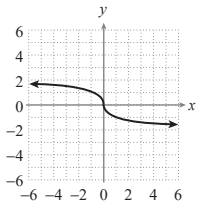
19.



Dom $= [-3, \infty)$,

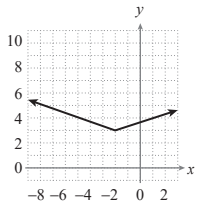
Ran $= [-1, \infty)$

21.



Dom = Ran = $\mathbb{R}$

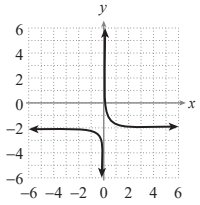
23.



Dom $= \mathbb{R}$,

Ran $= [3, \infty)$

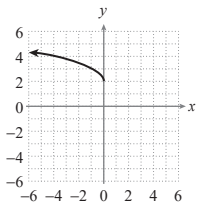
25.



Dom $= (-\infty, 0) \cup (0, \infty)$,

Ran $= (-\infty, -2) \cup (-2, \infty)$

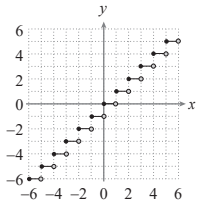
27.



Dom $= (-\infty, 0]$,

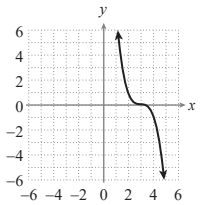
Ran $= [2, \infty)$

29.



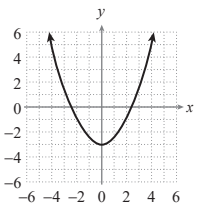
Dom $= \mathbb{R}$, Ran $= \mathbb{Z}$

31.



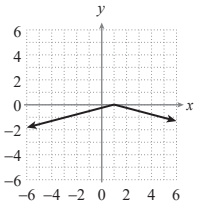
Dom = Ran = $\mathbb{R}$

33.



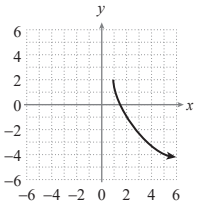
Dom $= \mathbb{R}$, Ran $= [-3,\infty)$

35.



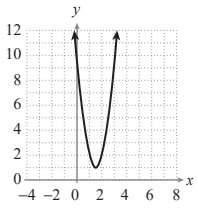
Dom $= \mathbb{R}$, Ran $= (-\infty,0]$

37.



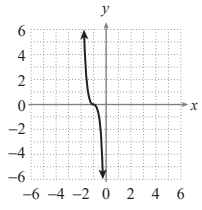
Dom $= [1,\infty)$, Ran $= (-\infty,2]$

39.



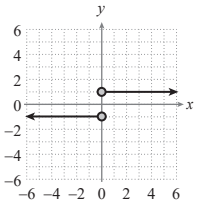
Dom $= \mathbb{R}$, Ran $= [1,\infty)$

41.



Dom $= \mathbb{R}$, Ran $= \mathbb{R}$

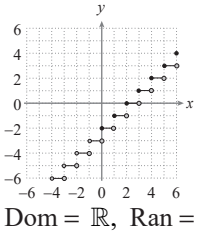
43.



Dom $= (-\infty,0)\cup(0,\infty)$,

Ran $= \{-1,1\}$

45.



Dom $= \mathbb{R}$, Ran $= \mathbb{Z}$

47. $f(x)=(x-4)^2+2$

49. $f(x)=(-x-2)^2=(x+2)^2$

51. $f(x)=(-x+1)^3$

53. $f(x)=-\sqrt{x+5}$

55. $f(x)=\sqrt{-\dfrac{x}{2}}+3$

57. $f(x)=-|x-8|-2$

59. $f(x)=-\sqrt{-(x+1)}$

61. $f(x)=\sqrt{2x}$

63. $f(x)=-\sqrt{x+4}$

65. $f(x)=1-(x-3)^3$

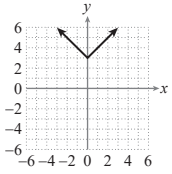
73. $f(x)=|x+4|-1$

75. $f(x)=-\sqrt{6-x}+2$

77. $f(x)=6-(x-3)^2$

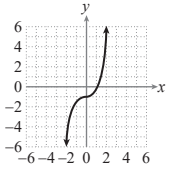
4.2 Exercises

1.



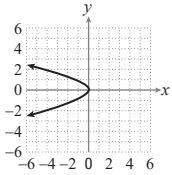
Even function; y-axis symmetry

3.



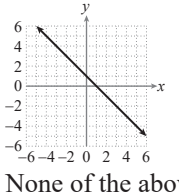
Neither; no symmetry

5.



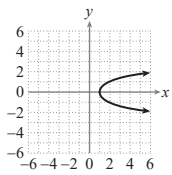
Not a function; x-axis symmetry

7.



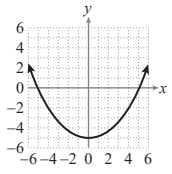
None of the above

9.



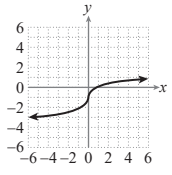
Not a function; x-axis symmetry

11.



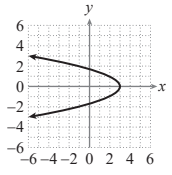
Even function; y-axis symmetry

13.



None of the above

15.



Not a function; x-axis symmetry

17. Inc. on $(-\infty,2)$,
Dec. on $(2,\infty)$

19. Inc. on $(-\infty,-3)$,
Dec. on $(-3,\infty)$

21. Constant on $\mathbb{R}$

23. Dec. on $(-\infty,4)$,
Inc. on $(4,\infty)$

25. Dec. on $(-\infty,-3)$,
Inc. on $(-3,-1)$,
Constant on $(-1,\infty)$

27. **a.** local min at -1, local max at 2

b. value at -1 is 0, value at 2 is 3

29. **a.** local min at -1, local max at 2

b. value at -1 is -12,
value at 2 is 15

31. **a.** local min at -2,
local max at 0, local min at 3

b. value at -2 is $\dfrac{5}{3}$, value at 0 is 7, value at 3 is $-\dfrac{35}{4}$

33. **a.** local min at 5

b. value at 5 is 2

35. a. local max at –2

b. value at –2 is 1

37. a. local min at 3

b. value at 3 is –2

39. 11 **41.** 2

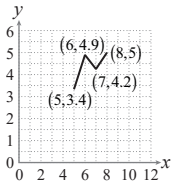
43. $\dfrac{2-\sqrt{2}}{2}$

45. $2c+h$ **47.** $-\dfrac{1}{12}$

49. a. $[a,b]$ **b.** $[b,c],[b,d]$

c. $[a,c],[a,d],[c,d]$

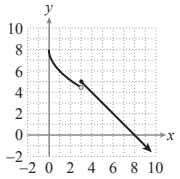
51.



Inc. on $(5,6)$ and $(7,8)$

Dec. on $(6,7)$

53.



Dec. on $(0,3)\cup(3,\infty)$

55. The average rate of change is m for any interval.

57. $2cp+hp+q$; The difference quotient approaches $2cp+q$ as h gets very small.

59. Nothing; Exercise 49a illustrates this conclusion.

4.3 Exercises

1. a. 2 **b.** –8 **c.** –15 **d.** $-\dfrac{3}{5}$

3. a. –3 **b.** –1 **c.** 2 **d.** 2

5. a. 12 **b.** 18 **c.** –45 **d.** –5

7. a. 3 **b.** 1 **c.** 2 **d.** 2

9. a. 6 **b.** 0 **c.** 9 **d.** 1

11. a. 5 **b.** –1 **c.** 6 **d.** $\dfrac{2}{3}$

13. a. 3 **b.** 5 **c.** –4 **d.** –4

15. a. $|x|+\sqrt{x}$, Dom $=[0,\infty)$

b. $\dfrac{|x|}{\sqrt{x}}$, Dom $=(0,\infty)$

17. a. x^2+x-2, Dom $=\mathbb{R}$

b. $\dfrac{1}{x+1}$,

Dom $:(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

19. a. x^3+3x-8, Dom $=\mathbb{R}$

b. $\dfrac{3x}{x^3-8}$,

Dom $=(-\infty,2)\cup(2,\infty)$

21. a. $-2x^2+|x+4|$, Dom $=\mathbb{R}$

b. $\dfrac{-2x^2}{|x+4|}$,

Dom $=(-\infty,-4)\cup(-4,\infty)$

23. 2 **25.** 0 **27.** 8

29. 3 **31.** 1 **33.** $\dfrac{1}{3}$

35. a. $\dfrac{4-2x}{3x}$,

Dom $=(-\infty,0)\cup(0,\infty)$

b. $\dfrac{3}{4x-2}$,

Dom $=\left(-\infty,\dfrac{1}{2}\right)\cup\left(\dfrac{1}{2},\infty\right)$

37. a. $|x^3-2|$, Dom $=\mathbb{R}$

b. $|x-3|^3+1$, Dom $=\mathbb{R}$

39. a. $\sqrt{\dfrac{x-1}{2}}$, Dom $=[1,\infty)$

b. $\dfrac{\sqrt{x-1}+1}{2}$, Dom $=[1,\infty)$

41. a. $-3x^2-4$, Dom $=\mathbb{R}$

b. $9x^2-12x+6$, Dom $=\mathbb{R}$

43. a. $\sqrt{x^2-1}$,

Dom $=(-\infty,-1]\cup[1,\infty)$

b. $x-1$, Dom $=\mathbb{R}$

45. $g(x)=\dfrac{2}{x}$, $h(x)=5x-1$,

$f(x)=g(h(x))$

47. $g(x)=x+\sqrt{x}-5$,

$h(x)=x+2$, $f(x)=g(h(x))$

49. $g(x)=\dfrac{\sqrt{x}}{x^2}$, $h(x)=x-3$,

$f(x)=g(h(x))$

51. $g(x)=x-3$, $h(x)=|x^2+3x|$,

$f(x)=g(h(x))$

53. $g(x)=\sqrt{x+5}$

55. $g(x)=-x^3-7$

57. $V=3\pi r^3$

59. $V=\dfrac{1}{12}\pi r^2t^2$

61. $(f\circ g)(x)=\sqrt[3]{\dfrac{-x^3}{3x^2-9}}$,

$(f\circ g)(-x)=\sqrt[3]{\dfrac{x^3}{3x^2-9}}$

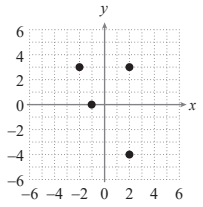
$=-(f\circ g)(x)$

63. Yes **65.** Yes **67.** No

69. No **71.** No

4.4 Exercises

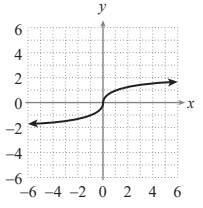
1.



Dom $=\{2,-1,-2\}$,

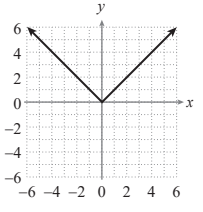
Ran $=\{-4,3,0\}$

3.



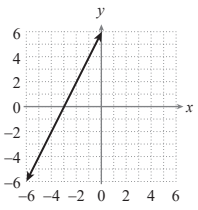
Dom = Ran = $\mathbb{R}$

5.



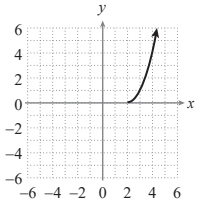
Dom = $\mathbb{R}$, Ran = $[0,\infty)$

7.



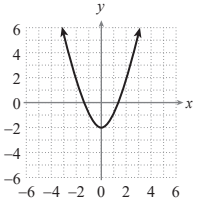
Dom = $\mathbb{R}$, Ran = $\mathbb{R}$

9.



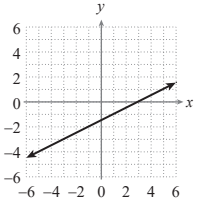
Dom = $[2,\infty)$, Ran = $[0,\infty)$

11.



Dom = $\mathbb{R}$, Ran = $[-2,\infty)$

13.



Dom = $\mathbb{R}$, Ran = $\mathbb{R}$

15. Not a one-to-one function
$f(-1)=f(1)=1$

17. Restrict to $[0,\infty)$

19. Inverse exists

21. Inverse exists

23. Inverse exists

25. Restrict to $[2,\infty)$

27. Restrict to $[12,\infty)$

29. $f^{-1}(x)=(x+2)^3$

31. $r^{-1}(x)=\dfrac{-2x-1}{3x-1}$

33. $F^{-1}(x)=(x-2)^{\frac{1}{3}}+5$

35. $V^{-1}(x)=2x-5$

37. $h^{-1}(x)=(x+2)^{\frac{5}{3}}$

39. $J^{-1}(x)=\dfrac{x-2}{3x}$

41. $h^{-1}(x)=(x-6)^{\frac{1}{7}}$

43. $r^{-1}(x)=\dfrac{x^5}{2}$ **45.** $f^{-1}(x)=\dfrac{x^3}{54}$

47. $f^{-1}(x)=\sqrt{x-2}+3$
or $f^{-1}(x)=-\sqrt{x-2}+3$

49. $f^{-1}(x)=\sqrt[4]{x+2}-1$
or $f^{-1}(x)=-\sqrt[4]{x+2}-1$

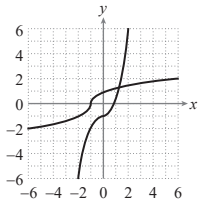
61. f **63.** c **65.** d

67. 184 72 96 96 72 160 144 8
72 112 160 120 32 8 200

69. REMEMBER
YOUR
SUNBLOCK

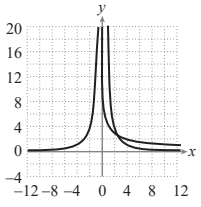
71. BEACH FUN IN THE SUN

73.



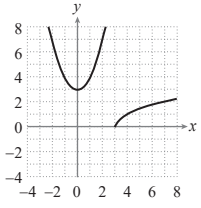
Dom = $(-\infty,\infty)$, Ran = $(-\infty,\infty)$

75.



Dom = $(0,\infty)$, Ran = $(0,\infty)$

77.



Dom = $[3,\infty)$, Ran = $[0,\infty)$

Chapter 4 Project

1. $A(r)=\pi r^2$

3. $A(t)=6.76\pi t^2$

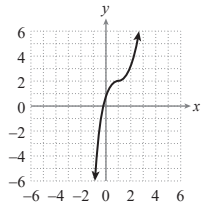
5. $r(5.5)=14.3$ km

7. $A(5.5)\approx 642.4$ km^2

9. Approximately 286.7 km^2/hr

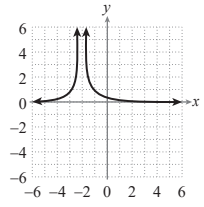
Chapter 4 Review Exercises

1.



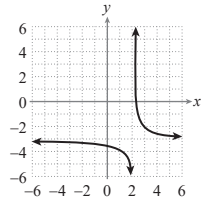
Dom = Ran = $\mathbb{R}$

3.



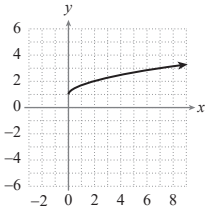
Dom = $(-\infty,-2)\cup(-2,\infty)$,
Ran = $(0,\infty)$

5.



Dom = $(-\infty,2)\cup(2,\infty)$,
Ran = $(-\infty,-3)\cup(-3,\infty)$

7.



Dom = $[0,\infty)$, Ran = $[1,\infty)$

9. $f(x)=(x-1)^2-2$

11. $f(x)=-\sqrt{x}+4$

13. Even function;
y-axis symmetry

15. Dec. on $(-\infty,2)$,
Inc. on $(2,\infty)$

17. **a.** Local min at 1,
local max at 2, local min at 3

b. Value at 1 is 1,
value at 2 is 4, value at 3 is 1

19. $-\dfrac{1}{3}$ **21.** -2

23. **a.** 3 **b.** 5 **c.** -4 **d.** -4

25. **a.** -2 **b.** 18 **c.** -80 **d.** $-\dfrac{4}{5}$

27. a. $\dfrac{1}{x-2}+\sqrt[3]{x}$,
Dom $=(-\infty,2)\cup(2,\infty)$

b. $\dfrac{1}{\sqrt[3]{x}(x-2)}$,
Dom $=$
$(-\infty,0)\cup(0,2)\cup(2,\infty)$

29. a. $x^2+\sqrt[3]{x}-5$, Dom $=\mathbb{R}$

b. $\dfrac{x^2-4}{\sqrt[3]{x}-1}$,
Dom $=(-\infty,1)\cup(1,\infty)$

31. $-\dfrac{9}{2}$ **33.** $-\dfrac{2}{3}$

35. a. $\dfrac{1}{\sqrt{x-2}}$, Dom $=(2,\infty)$

b. $\dfrac{1}{\sqrt{x-4}}+2$, Dom $=(4,\infty)$

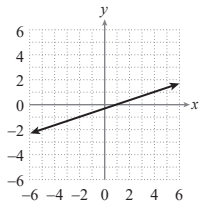
37. a. $3\sqrt{x-3}$, Dom $=[3,\infty)$

b. $\sqrt{3x-3}$, Dom $=[1,\infty)$

39. $g(x)=\dfrac{\sqrt{x}}{x^2}$, $h(x)=x+2$,
$f(x)=g(h(x))$

41. $g(x)=\dfrac{2}{x}+1$

43.



Dom $=$ Ran $=\mathbb{R}$

45. $r^{-1}(x)=\dfrac{x+2}{7x}$

47. $f^{-1}(x)=(x+6)^5$

49. $f^{-1}(x)=\dfrac{-x-3}{x-2}$

51. $f^{-1}(x)=\dfrac{x-3}{8}$

Chapter 5: Polynomial and Rational Functions

5.1 Exercises

19. Yes **21.** Yes **23.** Yes

25. $1\pm 2i$ **27.** $-3, \dfrac{1}{2}$

29. $\pm\sqrt{3}, \pm\sqrt{5}$ **31.** $-\dfrac{5}{2}$

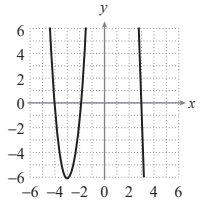
33. $0, 4\pm 3i$ **35.** $\pm 1, \pm 2i\sqrt{2}$

37. 7th-degree; lead coef. $=4$;
$j(x)\to-\infty$ as $x\to-\infty$
$j(x)\to\infty$ as $x\to\infty$

39. 5th-degree; lead coef. $=-6$;
$h(x)\to\infty$ as $x\to-\infty$
$h(x)\to-\infty$ as $x\to\infty$

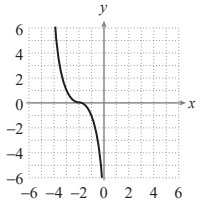
41. 4th-degree; lead coef. $=-2$;
$f(x)\to-\infty$ as
$x\to-\infty$ and $x\to\infty$

43.



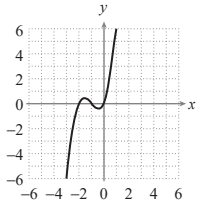
$g(x)\to\infty$ as $x\to-\infty$;
$g(x)\to-\infty$ as $x\to\infty$
x-int: $(-4,0),(-2,0),(3,0)$;
y-int: $(0,24)$

45.



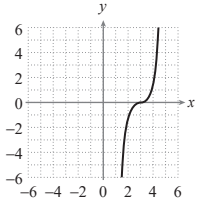
$h(x)\to\infty$ as $x\to-\infty$;
$h(x)\to-\infty$ as $x\to\infty$
x-int: $(-2,0)$; y-int: $(0,-8)$

47.



$s(x)\to-\infty$ as $x\to-\infty$;
$s(x)\to\infty$ as $x\to\infty$
x-int: $(-2,0),(-1,0),(0,0)$;
y-int: $(0,0)$

49.



$g(x)\to-\infty$ as $x\to-\infty$;
$g(x)\to\infty$ as $x\to\infty$
x-int: $(3,0)$; y-int: $(0,-243)$

51. $p(x)=\left(\dfrac{1}{2}\right)(x+2)$
$\times(x+1)(x-1)(x-3)$

53. $p(x)=-(x-1)(x-2)(x-3)$

55. e **57.** a **59.** f

61. d **63.** f **65.** b

67. $(-\infty,-2)\cup(3,\infty)$

69. $(-\infty,-2)\cup(-1,0)$

71. $[-2,1]\cup[3,\infty)$

73. $[-5,-1]\cup[1,4]$

75. $\left(-\dfrac{1}{2},2\right)$

77. $(-\infty,-4)\cup(2,3)$

79. All integers between 5 and 27, inclusive

81. All integers between 11 and 23, inclusive

83. Between 3490 and 17,740 phones, inclusive.

85. About 141.4 weeks

5.2 Exercises

1. $3x^2-x+1+\dfrac{5x-1}{2x^2+2}$

3. $x-2+\dfrac{-2}{x^2-4x+4}$

5. $4x^2-14x+29+\dfrac{-65}{x+2}$

7. $x^3+6x^2-2x+5+\dfrac{2x+5}{3x^2-1}$

9. $2x^3-3x^2+2x-5$

11. $x^3+3x^2+10x+10+\dfrac{22}{x-3}$

13. $3x^2+5x+9+\dfrac{45}{3x-5}$

15. $2x-5+\dfrac{7}{x+3}$

17. $x^2-ix+6+\dfrac{1+i}{2x-i}$

19. x^2+3 **21.** $p(1)=4$

23. c is a zero **25.** c is a zero

27. $p(1)=12$ **29.** c is a zero

31. c is a zero **33.** c is a zero

35. c is a zero **37.** $p(5)=-2$

39. c is a zero

41. $x^2-4x+2+\dfrac{-1}{x+5}$

43. $x^7-3x^2+\dfrac{3}{x+1}$

45. $4x^2-4x+2$

47. $x^4-x^3-x^2-7x-14-\dfrac{10}{x-2}$

49. x^3-x^2+x

51. $2x^2-4ix+17+\dfrac{8+48i}{x-3i}$

53. $f(x)=-x^2-x+12$

55. $f(x)=-x^2+4x-13$

57. $f(x)=x^4-12x^3+54x^2-108x+81$

59. $f(x)=3x^4+9x^3-9x^2-21x+18$

61. $SA=(x+5)(x+2)$
$=x^2+7x+10$

5.3 Exercises

1. $\pm\left\{\frac{1}{3},\frac{2}{3},1,\frac{4}{3},2,\frac{8}{3},4,8\right\},\left\{-4,\frac{1}{3},2\right\}$

3. $\pm\{1,2,3,4,6,8,12,24\},\{\pm 2i,2,3\}$

5. $\pm\{1,2,7,14\},\{1,2,7\}$

7. $\pm\left\{\frac{1}{2},1,\frac{3}{2},\frac{5}{2},3,5,\frac{15}{2},15\right\},\left\{-1,\frac{5}{2},3\right\}$

9. $\pm\left\{\frac{1}{3},1,3\right\},\{-1,1,-i,i\}$

11. $\pm\{1,11\},\{-11,-1,1\}$

13. $\{-1,1,-i,i\}$

15. $\{-1,2-3i,2+3i\}$

17. $\{-2i,2i,2,3\}$

19. $\{4,1-2i,1+2i\}$

21. $\{-5i,5i,-2,1\}$

23. $\{-11,-1,1\}$

25. 0 pos., 3 or 1 neg.

27. 2 or 0 pos., 1 neg.

29. 3 or 1 pos., 1 neg.

31. 1 pos., 1 neg.

33. 3 or 1 pos., 0 neg.

35. 0 pos., 0 neg.

37. $[-5,1]$ **39.** $[-1,6]$

41. $[-3,6]$ **43.** $[-3,3]$

45. $[-3,6]$ **47.** $\{-4,-1,1\}$

49. $\{-1,2,5\}$ **51.** $\{2,3,\pm 2\sqrt{2}\}$

53. $\pm\{\sqrt{5},i\sqrt{5}\}$ **55.** $\left\{-2,-\frac{1}{2},6\right\}$

57. $f(-3)=-84,\ f(-1)=16$

59. $f(2)=-15,\ f(3)=24$

61. $f(2)=15,\ f(3)=-24$

63. $\{-1,2,5\}$ **65.** $\left\{-\frac{7}{3},\pm 1\right\}$

67. $\left\{-1,-\frac{2}{5},2,\frac{7}{3}\right\}$

69. $\left\{\pm 3,-2,-\frac{1}{3}\right\}$ **71.** $\left\{-5,\pm 1,\frac{5}{2},4\right\}$

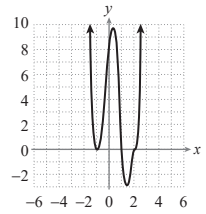
73. $\{-3,-2,-1\}$ **75.** $\{-9,\pm\sqrt{2}\}$

77. $\left\{-3,-\frac{1}{2},8\right\}$ **79.** $\left\{\pm 2,-\frac{3}{4},1\right\}$

81. $\left\{\sqrt[3]{-2},\pm\frac{\sqrt{6}}{2},\frac{5}{3}\right\}$

5.4 Exercises

1.



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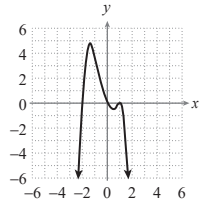

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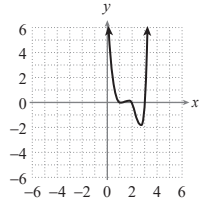

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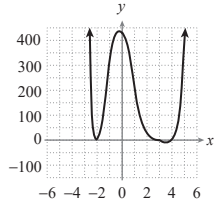
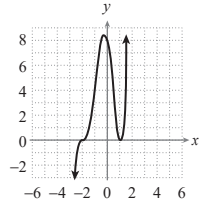
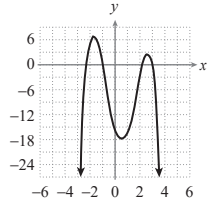


9.



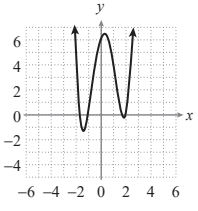
$f(x)=(x+2)^3(x-1)^2$

11.

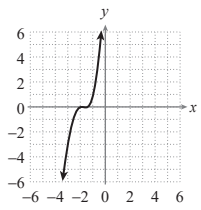


$s(x)=-(x-3)(x+1)$
$\times(x-\sqrt{5})(x+\sqrt{5})$

13.



$H(x)=(x-2)(x+1)$
$\times(x-\sqrt{3})(x+\sqrt{3})$

15. 

$f(x)=(x+2)^2(2x+3)$

17. $\{-2,1\}$ **19.** $\{-2,1,\pm i\sqrt{5}\}$

21. $\{-3,1,\pm 2i\}$ **23.** $\{\pm 1,\pm 4i\}$

25. $(x-3+2i)(x-3-2i)$
$\times(x-4)(x+1)$

27. $(x-1-3i)(x-2)(x+1)$

29. $\left(x+\sqrt{7}\right)\left(x-\sqrt{7}\right)(x-2+3i)$
$\times(x-2-3i)$

31. $(x-2)(x+1)(x-1+2i)$
$\times(x-1-2i)$

33. $x(x-3)(x-1)(x+11)$

35. $(x-2)(x-2+4i)(x-2-4i)$

37. $f(x)=-2x^3+18x^2-32x-52$

39. $f(x)=2x^5+2x^4-10x^3-2x^2$
$+16x-8$

41. $f(x)=3x^4-18x^3+12x^2-72x$

43. $f(x)=-x^3+2x^2+14x-40$

45. $f(x)=-x^3+4x^2+15x-68$

47. a. $V(x)=4x(5-x)(9-x)$

b. $x=0, x=5, x=9$

c. $x=0$ and $x=5$

49. a. $V(x)=x(17-2x)(9-2x)$

b. $x=0, x=\dfrac{9}{2}, x=\dfrac{17}{2}$

c. $x=0$ and $x=\dfrac{9}{2}$

5.5 Exercises

1. $x=1$

3. No vertical asymptote

5. $x=2$ **7.** $x=0$

9. $x=-\dfrac{1}{2}$

11. No vertical asymptote

13. $x=7$ **15.** $x=-2$

17. $x=-2, x=2$ **19.** $y=0$

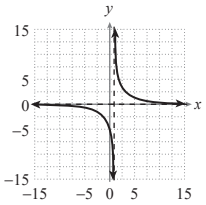
21. No horizontal or oblique asymptote

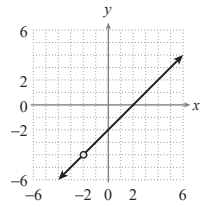
23. $y=0$ **25.** $y=2$

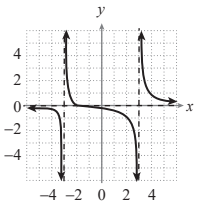
27. $y=3x+6$ **29.** $y=0$

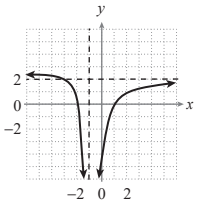
31. $y=0$ **33.** $y=x-11$

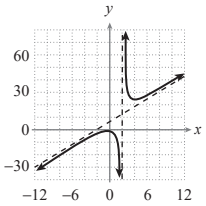
35. $y=5x+4$

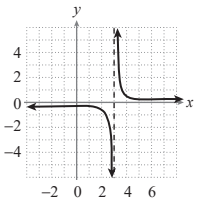
37. 

39. 

41. 

43. 

45. 

47. 

49. a. $x=-2$ **b.** $y=0$
c. None **d.** None **e.** $(0,5)$

51. a. $x=9$ **b.** $y=0$
c. None **d.** None
e. $\left(0,-\dfrac{1}{3}\right)$

53. a. $x=-1, x=1$ **b.** None
c. $y=x$ **d.** $\left(\sqrt[3]{3},0\right)$
e. $(0,3)$

55. a. $x=1$ **b.** None **c.** $y=3x$
d. None **e.** $(0,-3)$

57. $(-\infty,-2)\cup(-1,1)$

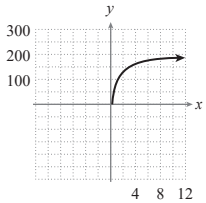
59. $(-8,-2)\cup(2,\infty)$

61. $(-\infty,-2)\cup(-2,3)$

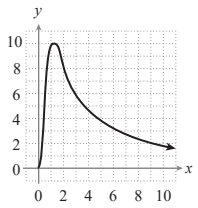
63. $(0,3)$

65. $(-2,-1)\cup(1,\infty)$

67. $(-\infty,-1)\cup\left[-\dfrac{1}{2},0\right)$

69. a. 

b. April's fish population approaches a maximum of 200 fish.

71. a. 

b. The concentration of the drug disappears in the long run.

Chapter 5 Project

1. $P(x)=-3x^2+5500x-729{,}000$

3. \$4933 for $x=1689$ to \$9568 for $x=144$

5. $[212,1621]$

Chapter 5 Review Exercises

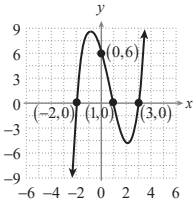
5. $\pm\sqrt{2},\pm\sqrt{5}$ **7.** $\pm\sqrt{2}$

9. $0,\dfrac{-1\pm\sqrt{5}}{2}$

11. x-int: -2, 1, 3; y-int: 6

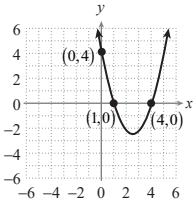
$f(x) \to -\infty$ as $x \to -\infty$;

$f(x) \to \infty$ as $x \to \infty$



13. x-int: 1, 4; y-int: 4

$g(x) \to \infty$ as $x \to \pm\infty$



15. $\left[\frac{5}{2}, 3\right]$

17. $[-2,-1] \cup [1,4]$

19. $(-\infty,0) \cup (0,1) \cup (2,\infty)$

21. All integers between 4 and 30, inclusive

23. $11x + 35 + \frac{100}{x-3}$

25. $2x^3 + 2x^2 - 2x - 3 + \frac{-2x-2}{x^2-x}$

27. $p(1) = 90$ **29.** $p\left(\frac{2}{3}\right) = -\frac{7}{3}$

31. $-x^3 + 2x^2 - 7x + 23$

33. $-x^3 + 7x^2 + x - 3 + \frac{-1}{x-1}$

35. $f(x) = x^2 - 4x - 12$

37. $f(x) = 2(x^2 - 4)(x - 3)$

39. $\pm\left\{\frac{1}{2}, 1, \frac{3}{2}, 3, \frac{9}{2}, 9\right\}, \left\{1, \frac{3}{2}, 3\right\}$

41. $\pm\{1,3,9\}, \{-3,-1\}$

43. $\left\{1, \frac{3}{2}, 3\right\}$ **45.** $\{-3,-1\}$

47. 4, 2 or 0 pos., 2 or 0 neg.

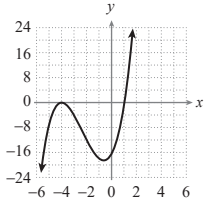
49. $[-3,7]$ **51.** $\left\{-\frac{5}{2}, -\frac{1}{2}, 7\right\}$

53. $f(2) = 3; f(4) = -15$

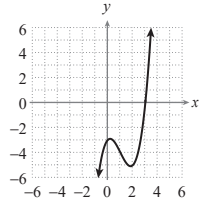
55. $\{\pm 3i, 4\}$ **57.** $\{3, 2 \pm \sqrt{3}\}$

59. $\left\{-\frac{1}{2}, 2\right\}$

61.



63. $(x^2+1)(x-3)$



65. $\left\{-1, \frac{5}{3}, \pm 2i\right\}$

67. $\{\pm 2i, -3, \pm 1\}$

69. $(x-5i)(x+5i)(x-6)(x+1)$

71. $(x+3)\left(x - \frac{1+i\sqrt{19}}{4}\right) \times \left(x - \frac{1-i\sqrt{19}}{4}\right)$

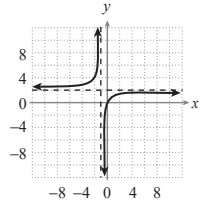
73. $f(x) = x^5 + 3x^4 - 3x^3 - 17x^2 - 18x - 6$

75. $x = \frac{5}{2}$ **77.** $x = 0$

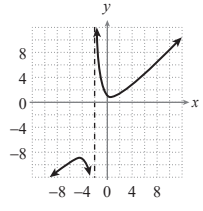
79. $y = 2x + 9$

81. No horizontal or oblique asymptote

83.



85.



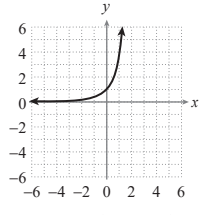
87. $\left(-3, \frac{7}{2}\right]$

89. $\left(-3, \frac{8}{9}\right) \cup (2,\infty)$

Chapter 6: Exponential and Logarithmic Functions

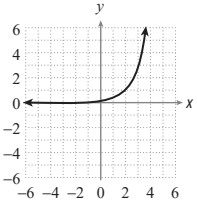
6.1 Exercises

1.



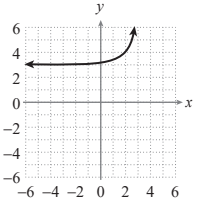
Dom $= (-\infty,\infty)$, Ran $= (0,\infty)$

3.



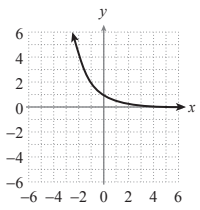
Dom $= (-\infty,\infty)$, Ran $= (0,\infty)$

5.



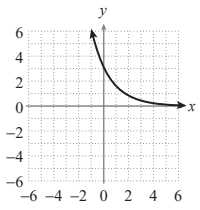
Dom $= (-\infty,\infty)$, Ran $= (3,\infty)$

7.



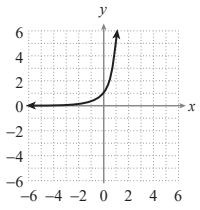
Dom $= (-\infty, \infty)$, Ran $= (0, \infty)$

9.



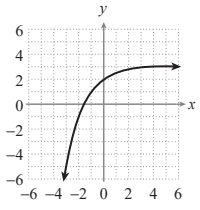
Dom $= (-\infty, \infty)$, Ran $= (0, \infty)$

11.



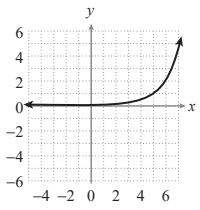
Dom $= (-\infty, \infty)$, Ran $= (0, \infty)$

13.



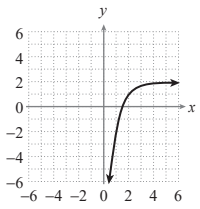
Dom $= (-\infty, \infty)$, Ran $= (-\infty, 3)$

15.



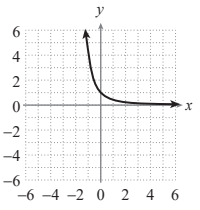
Dom $= (-\infty, \infty)$, Ran $= (0, \infty)$

17.



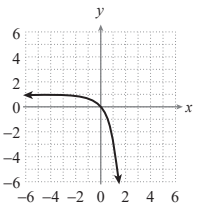
Dom $= (-\infty, \infty)$, Ran $= (-\infty, 2)$

19.



Dom $= (-\infty, \infty)$, Ran $= (0, \infty)$

21.



Dom $= (-\infty, \infty)$, Ran $= (-\infty, 1)$

23. $\{2\}$ **25.** $\{-2\}$ **27.** $\{-13\}$

29. $\{3\}$ **31.** $\{-2\}$

33. $\{-2, -1\}$

35. $\{7\}$ **37.** $\{3\}$ **39.** $\{9\}$

41. $\{-3\}$ **43.** $\{2\}$ **45.** $\{-1\}$

47. a **49.** i **51.** d

53. e **55.** h

6.2 Exercises

1. $V \approx 178$ people

3. $C \approx \$8526.20$

5. $W \approx 93$ computers

7. a. $a \approx 0.999567$

b. $A \approx 0.958$ grams

c. $A \approx 0.648$ grams

9. a. 3 years **b.** 9 years

11. 1118 rabbits

13. The bank offering 2.75% and monthly compounding.

15. Approximately 3.18%

17. \$134,392

19. a. 10 **b.** 7490 people

c. The function approaches 10,000 as time goes on.

21. a. $a \approx 0.965936$

b. $A \approx 0.707$ kg

c. $A \approx 7.628$ mg

23. a. \$1521.74 **b.** \$271.74

25. \$9459.48; \$9942.41

27. \$2835.71

29. \$20,000

31. \$7318.71

33. a. \$7647.95 **b.** \$7647.57

c. Yes; daily compounding is a frequency close enough to continuous compounding to make little difference at the hundredths place.

35. a. Linear:

$$y = 1.06098 \times 10^6 x - 1.94223 \times 10^9$$

Quadratic:

$$y = 8501.22x^2 - 3.08186 \times 10^7 x + 2.79424 \times 10^{10}$$

Exponential:

$$y = 2.172414 \times 10^{-12} (1.023986)^x$$

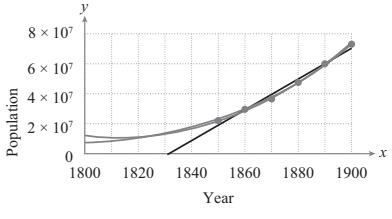
b. Linear: –32,466,000

Quadratic: 12,872,800

Exponential: 7,348,129

Actual: 5,308,483

None of these regression models appear to be very accurate this far from known data, though the population estimates given by the exponential model are closer to the actual population.



6.3 Exercises

1. $4 = \log_5 625$ **3.** $3 = \log_x 27$

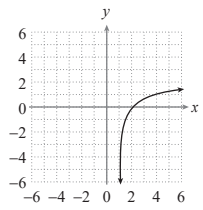
5. $3 = \log_{4.2} C$ **7.** $x = \log_4 31$

9. $\sqrt{3} = \log_{4x} 13$ **11.** $e^x = \log_2 11$

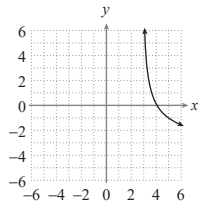
13. $81 = 3^4$ **15.** $4 = b^{\frac{1}{2}}$

17. $15 = 2^b$ **19.** $W = 5^{12}$

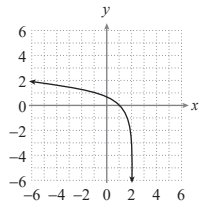
21. $2x = \pi^4$ **23.** $e^x = 2$

25. 

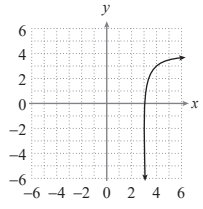
Dom: $(1,\infty)$, Ran: $(-\infty,\infty)$

27. 

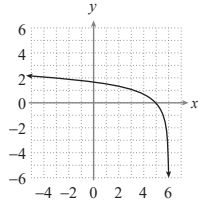
Dom: $(3,\infty)$, Ran: $(-\infty,\infty)$

29. 

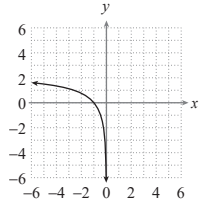
Dom: $(-\infty,2)$, Ran: $(-\infty,\infty)$

31. 

Dom: $(3,\infty)$, Ran: $(-\infty,\infty)$

33. 

Dom: $(-\infty,6)$, Ran: $(-\infty,\infty)$

35. 

Dom: $(-\infty,0)$, Ran: $(-\infty,\infty)$

37. e **39.** b **41.** h

43. d **45.** i **47.** -2

49. 3 **51.** $-\frac{1}{2}$ **53.** $\frac{3}{4}$

55. 2.89 **57.** $\frac{5}{3}$ **59.** 1

61. $\frac{1}{5}$ **63.** 2 **65.** $\{64\}$

67. $\{9\}$ **69.** $\left\{-\frac{1}{2}\right\}$ **71.** $\left\{\frac{1}{21}\right\}$

73. $\{10\}$ **75.** $\{36\}$

77. $\left\{\pm\frac{1}{10}\right\}$ **79.** $\{0.18\}$

81. $\left\{\pm\sqrt{e}\right\}$ or $\{\pm1.65\}$

83. $\{12.89\}$

85. $\{10,000,000,002\}$

6.4 Exercises

1. $3+3\log_5 x$

3. $2+\ln p-3\ln q$

5. $1+\log_9 x-3\log_9 y$

7. $\frac{3}{2}\ln x+\ln p+5\ln q-7$

9. $\log(2+3\log x)$

11. $1-\frac{1}{2}\log(x+y)$

13. $\log_2(y^2+z)-4\log_2 x-4$

15. $2\log_b x+\frac{1}{2}\log_b y-\log_b z$

17. $2+\log_b a+b\log_b c$

19. $\log\left(\frac{x}{y}\right)$

21. $\log_5(x+5)$

23. $\log_2\left(x^{\frac{4}{3}}+3x^{\frac{1}{3}}\right)$

25. $\ln\left(\frac{3p}{q^2}\right)$ **27.** $\log\left(\frac{x-10}{x}\right)$

29. $\ln\left(\frac{z^2}{x^3y^3}\right)$ **31.** $\log_5 4$

33. $\ln 45$ **35.** $\log_3 1=0$

37. $\ln 12$ **39.** $\log 11$

41. $\log_8(x^2-y)$ **43.** x^2

45. $\frac{e^2p}{x}$ **47.** $\frac{x^3}{y^4}$ **49.** x^2

51. 4 **53.** $12x^2$ **55.** 2.04

57. 0.95 **59.** 0.95 **61.** 2.45

63. 3.30 **65.** 0.74 **67.** 1.20

69. 1.86 **71.** -1 **73.** 3.85

75. 0.77 **77.** -1.76 **79.** 2

81. 7 **83.** 1

85. $4\sqrt{2}\approx5.66$ **87.** 9.05

89. 2.08 **91.** 12

93. 1,048,576 **95.** 10.25

97. 5,011,872 times stronger

99. 133 decibels

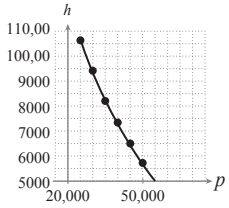
101. 7.62; yes

103. a. 15.05 minutes

b. 7:00 p.m.

c. 112 °F; no

105. $h(p)=81,751.7-7027.82\ln p$



6.5 Exercises

1. $x\approx0.26$ **3.** $x\approx3.12$

5. $x\approx3.89$ **7.** $x\approx-2.28$

9. $x\approx8.09$ **11.** $x\approx2.68$

13. $x\approx-1.12$ **15.** $x\approx52.77$

17. $x\approx\pm0.71$ **19.** $x=-12$

21. $x\approx1.32$ **23.** $x\approx3.27$

25. $x=125$ **27.** $x=5$

29. $x\approx40.17$ **31.** $x=35$

33. $x\approx9.38$ **35.** $x=4$

37. $x=\frac{1}{162}$ **39.** $x=1$

41. $x\approx100.04$ **43.** No solution

45. $x=5$ **47.** $x=8$

49. $x=\frac{37}{8}$ **51.** $x=5$

53. No solution **55.** $x=6$

57. $x=\sqrt{2}-1$ **59.** $x=2,3$

61. $x=1,2$ **63.** $f(x)=\log(2x^2)$

65. $f(x) = \ln 9x^2$

67. $f(x) = 256x$

69. $f(x) = \ln 1 = 0$

71. $f(x) = \ln 5^x$ **73.** $f(x) = \ln 5$

75. a. 17.36 years

b. 9.90 years

77. 4.98 hours **79.** 4.99 years

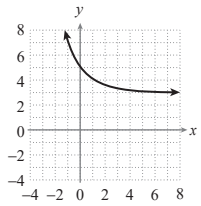
81. 0.271 years (about 99 days)

Chapter 6 Project

1. 242 **3.** 20,219

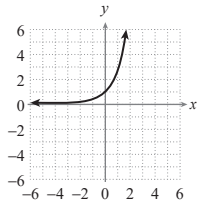
Chapter 6 Review Exercises

1.



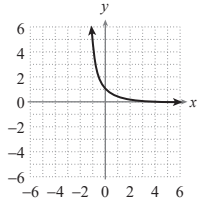
Dom $= (-\infty, \infty)$, Ran $= (3, \infty)$

3.



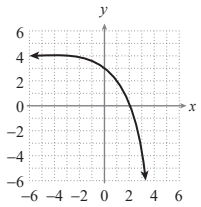
Dom $= (-\infty, \infty)$, Ran $= (0, \infty)$

5.



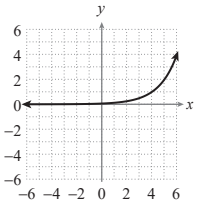
Dom $= (-\infty, \infty)$, Ran $= (0, \infty)$

7.



Dom $= (-\infty, \infty)$, Ran $= (-\infty, 4)$

9.



Dom $= (-\infty, \infty)$, Ran $= (0, \infty)$

11. $x = 5$ **13.** $x = 2$

15. $x = -1$ **17.** $x = \dfrac{8}{5}$

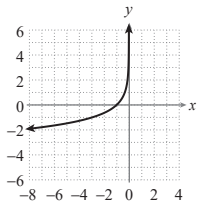
19. $x = 2$ **21.** 8 days

23. a. 173 flies **b.** 20 flies

25. $x = \log_3 8$ **27.** $\log_4 4096 = 3a$

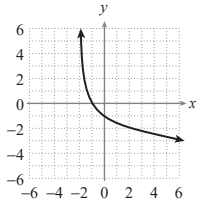
29. $3^{-3} = \dfrac{1}{27}$

31.



Dom $= (-\infty, 0)$, Ran $= (-\infty, \infty)$

33.



Dom $= (-2, \infty)$, Ran $= (-\infty, \infty)$

35. -2 **37.** -3 **39.** $\dfrac{2}{3}$

41. $x = 27$ **43.** $x = \dfrac{17}{2}$

45. $x \approx 155.41$

47. $\dfrac{3}{2}\log x - \dfrac{5}{2}\log \pi - \log 2$

49. $3 + 3\log_3 a$ **51.** $\log_2 \left(\dfrac{a^{\frac{5}{3}}}{b^{\frac{1}{3}} c} \right)$

53. $\log_2 (x - 3)$ **55.** $\log_3 \dfrac{x^2 - 2x}{x^2 + 4}$

57. $\dfrac{x}{y^2}$ **59.** 6.18 **61.** 0

63. a. 251,188,643

b. 7,079,458

c. 1,258,925,412

65. $\dfrac{6}{\log 321} \approx 2.39$

67. $\dfrac{\ln 5}{4 - 3\ln 5} \approx -1.94$

69. $\dfrac{\log 12}{\log 18} \approx 0.86$

71. -2 **73.** 73.5

75. $f(x) = \ln x^3$ **77.** $f(x) = \log x^{10}$

79. $f(x) = \log 7$

81. 20.4 months (1.7 years)

Chapter 7: Trigonometric Functions

7.1 Exercises

1. 225° **3.** $-67.5°$ **5.** 120°

7. 150° **9.** $-405°$ **11.** $\dfrac{47\pi}{180}$

13. $\dfrac{11\pi}{15}$ **15.** $\dfrac{37\pi}{45}$ **17.** $\dfrac{8\pi}{3}$

19. $\dfrac{25\pi}{36}$ **21.** 270° **23.** 540°

25. $-72°$ **27.** $\dfrac{\pi}{9}$ **29.** $-\dfrac{4\pi}{5}$

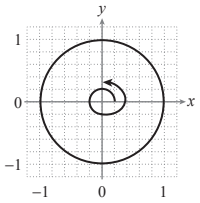
31. $\dfrac{\pi}{6}$ **33.** $\dfrac{83\pi}{90}$ **35.** 115°

37. 50° **39.** 345° **41.** 295°

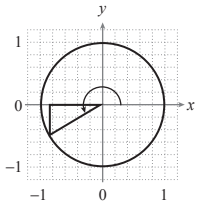
43. $\dfrac{5\pi}{4}$ **45.** $\dfrac{\pi}{4}$ **47.** $\dfrac{4\pi}{3}$

49. $\dfrac{13\pi}{4}$

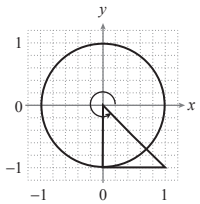
51.



53.



55.



57. 4 in. **59.** 11.78 ft

61. 8.64 m **63.** $\frac{9}{2}$

65. $\frac{21}{47}$ **67.** $\frac{45}{4}$

69. 1.48 inches **71.** 3.56 feet

73. 1.83 feet **75.** 1.45°

77. 445.79 miles **79.** 1335.24 km

81. 7853.58 km **83.** 1039.61 mph

85. 17.14 mph

87. a. 10π rad/s **b.** 122.84 in./s

89. $\frac{5\pi}{12} \approx 1.31$ **91.** 29.93 cm^2

93. 15.08 m^2 **95.** 314.16 ft^2

97. 9.82 $in.^2$ **99.** 26.18 cm

101. 11.7 in.

7.2 Exercises

1. $\sin\theta = \cos\theta = \frac{\sqrt{2}}{2}$,
$\tan\theta = \cot\theta = 1$,
$\csc\theta = \sec\theta = \sqrt{2}$

3. $\sin\theta = \frac{1}{3}, \cos\theta = \frac{2\sqrt{2}}{3}$,
$\tan\theta = \frac{\sqrt{2}}{4}, \csc\theta = 3$,
$\sec\theta = \frac{3\sqrt{2}}{4}, \cot\theta = 2\sqrt{2}$

5. $\sin\theta = \cos\theta = \frac{\sqrt{2}}{2}$,
$\tan\theta = \cot\theta = 1$,
$\csc\theta = \sec\theta = \sqrt{2}$

7. $\sin\theta = \frac{5}{7}, \cos\theta = \frac{2\sqrt{6}}{7}$,
$\tan\theta = \frac{5\sqrt{6}}{12}, \csc\theta = \frac{7}{5}$,
$\sec\theta = \frac{7\sqrt{6}}{12}, \cot\theta = \frac{2\sqrt{6}}{5}$

9. $\sin\theta = \frac{12}{13}, \cos\theta = \frac{5}{13}$,
$\tan\theta = \frac{12}{5}, \csc\theta = \frac{13}{12}$,
$\sec\theta = \frac{13}{5}, \cot\theta = \frac{5}{12}$

11. $\sin\theta = \frac{33}{65}, \cos\theta = \frac{56}{65}$,
$\tan\theta = \frac{33}{56}, \csc\theta = \frac{65}{33}$,
$\sec\theta = \frac{65}{56}, \cot\theta = \frac{56}{33}$

13. $\sin\theta = \frac{1}{2}, \cos\theta = \frac{\sqrt{3}}{2}$,
$\tan\theta = \frac{\sqrt{3}}{3}, \csc\theta = 2$,
$\sec\theta = \frac{2\sqrt{3}}{3}, \cot\theta = \sqrt{3}$

15. $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \csc\left(\frac{\pi}{4}\right) = \sqrt{2}$

17. $\sec 60° = 2$

19. $\csc\left(\frac{\pi}{6}\right) = 2$

21. $\sec 5° \approx 1.0038, \tan 5° \approx 0.0875$

23. $\cot\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{3}$

25. $\tan(87.2°) \approx 20.4465$

27. 0.9945 **29.** 1.0355

31. 28.6537 **33.** 3.0777

35. 0.3827 **37.** 2

39. 38.9053° **41.** 25.325°

43. 21.6656° **45.** 0.1149

47. 0.7746 **49.** 5

51. True **53.** True

55. False **57.** 751.19 feet

59. 17.47 feet **61.** 48.54 yards

63. 12.04 m **65.** 314.57 feet

67. 13.86 feet **69.** 6.86 feet

71. 20 m **73.** 3196.80 feet

75. The two triangles are similar, meaning that the lengths of the sides of Belinda's triangle are scaled by some common factor k relative to the corresponding sides of Adam's triangle. So, for instance, Adam would find that $\sin\theta = \frac{a}{c}$ and Belinda would find that $\sin\theta = \frac{ka}{kc} = \frac{a}{c}$.

7.3 Exercises

1. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ **3.** $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

5. $(-1, 0)$ **7.** $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

9. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

11. $\frac{5\pi}{3} + 2n\pi, n \in \mathbb{Z}$

13. $\frac{7\pi}{6} + 2n\pi, n \in \mathbb{Z}$

15. $\sin\left(\frac{\pi}{2}\right) = \csc\left(\frac{\pi}{2}\right) = 1$,
$\cos\left(\frac{\pi}{2}\right) = \cot\left(\frac{\pi}{2}\right) = 0$,
$\tan\left(\frac{\pi}{2}\right) = \sec\left(\frac{\pi}{2}\right) =$ undefined

17.
$\sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}, \cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$,
$\tan\left(\frac{3\pi}{4}\right) = \cot\left(\frac{3\pi}{4}\right) = -1$,
$\csc\left(\frac{3\pi}{4}\right) = \sqrt{2}, \sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}$

19. $\sin(-520°) \approx -0.3420$,
$\cos(-520°) \approx -0.9397$,
$\tan(-520°) \approx -0.3640$,
$\csc(-520°) \approx -2.9238$,
$\sec(-520°) \approx -1.0642$,
$\cot(-520°) \approx -2.7475$

21. $\sin(-1105°) \approx -0.4226$,
$\cos(-1105°) \approx 0.9063$,
$\tan(-1105°) \approx -0.4663$,
$\csc(-1105°) \approx -2.3662$,
$\sec(-1105°) \approx 1.1034$,
$\cot(-1105°) \approx -2.1445$

23. $\sin 670° \approx -0.7660$,
$\cos 670° \approx 0.6428$,
$\tan 670° \approx -1.1918$,
$\csc 670° \approx -1.3054$,
$\sec 670° \approx 1.5557$,
$\cot 670° \approx -0.8391$

25. $\sin(-215°) \approx 0.5736$,
$\cos(-215°) \approx -0.8192$,
$\tan(-215°) \approx -0.7002$,
$\csc(-215°) \approx 1.7434$,
$\sec(-215°) \approx -1.2208$,
$\cot(-215°) \approx -1.4281$

27. $\sin 780° = \frac{\sqrt{3}}{2}$, $\cos 780° = \frac{1}{2}$,
$\tan 780° = \sqrt{3}$, $\csc 780° = \frac{2\sqrt{3}}{3}$,
$\sec 780° = 2$, $\cot 780° = \frac{\sqrt{3}}{3}$

29. $\theta' = 82°$ **31.** $\theta' = 60°$

33. $\theta' = \frac{\pi}{2}$ **35.** $\theta' = \frac{\pi}{6}$

37. $\theta' = \frac{\pi}{5}$ **39.** $\theta' = \frac{\pi}{2}$

41. $\theta' = \frac{\pi}{4}$ **43.** $\theta' = 75°$

45. IV **47.** IV **49.** I

51. II **53.** a **55.** a

57. b **59.** c **61.** c

63. $\sin\left(\frac{9\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

65. $\tan\left(\frac{5\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

67. $\sin 313° = -\sin 47° \approx -0.7314$

69. $\tan(-168°) = \tan 12° \approx 0.2126$

71. $\sin 216° = -\sin 36° \approx -0.5878$

73. $\cos(-330°) = \cos 30° = \frac{\sqrt{3}}{2}$

75. $\tan 718° = -\tan 2° \approx -0.0349$

77. b **79.** e

81. $\cot 135° = \cos(-45°) = -1$

83. $\sin(-60°) = \cos 150° = -\frac{\sqrt{3}}{2}$

85. $\csc\left(\frac{5\pi}{6}\right) = \sec\left(-\frac{\pi}{3}\right) = 2$

87. $\cos\left(-\frac{3\pi}{6}\right) = \sin\pi = 0$

89. $\sin\left(-\frac{4\pi}{5}\right) = \cos\left(\frac{13\pi}{10}\right)$
≈ -0.5878

91. $\csc\left(\frac{3\pi}{2}\right) = \sec(-\pi) = -1$

93. $\cot\left(\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

95. $\sec 105° = \csc(-15°) \approx -3.8637$

97. $\tan\theta \approx -0.727$, $\cot\theta \approx -1.376$

99. $\tan\theta = 1.732$, $\cot\theta \approx 0.577$

101. $\tan\theta \approx 9.476$, $\cot\theta \approx 0.106$

103. $\csc\theta = \frac{125}{100} = \frac{5}{4}$, $\cot\theta = -\frac{3}{4}$

105. No such angle exists, as this would require the opposite side to be longer than the hypotenuse.

107. $\theta = -\frac{2\pi}{3}$, $\tan\theta = -\sqrt{3}$

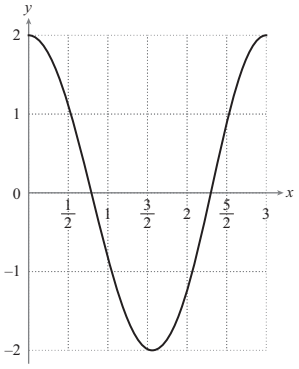
109. $\cos\left(-\frac{5\pi}{4}\right), -0.7071$

111. $\tan\left(-\frac{\pi}{4}\right), -1$

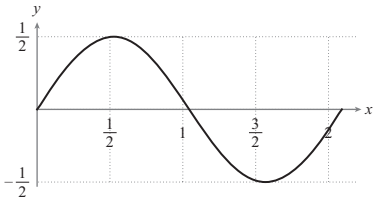
113. cot 75°, 0.2679

7.4 Exercises

1. Amplitude = 2,
frequency = $\frac{1}{\pi}$



3. Amplitude = $\frac{1}{2}$,
frequency = $\frac{3}{2\pi}$



5. A = 2, P = 2π, no phase shift

7. A = 4, P = 2π, no phase shift

9. A = 1, P = 2π, no phase shift

11. A = 3, P = 2π, shifted left 7 units

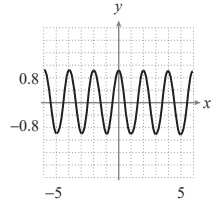
13. A = 2, P = π, no phase shift

15. A = $\frac{3}{2}$, P = 2, no phase shift

17. A = 0.5, P = $\frac{\pi}{4}$, shifted left $\frac{1}{8}$ units

19. A = $\frac{8}{5}$, P = 1, shifted left $\frac{2}{\pi}$ units

21.



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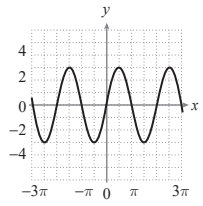

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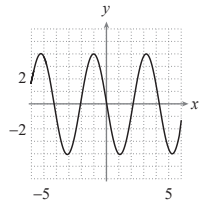

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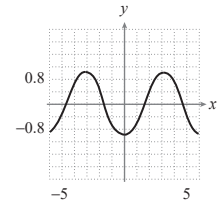

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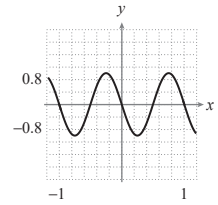

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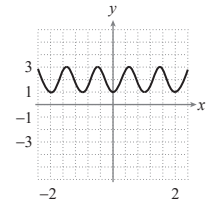

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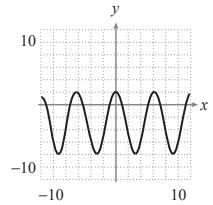

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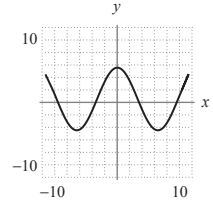

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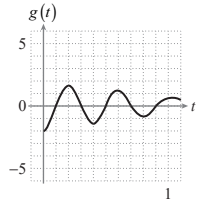

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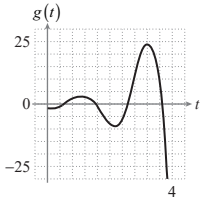

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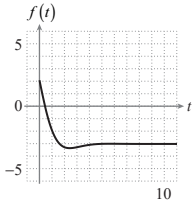

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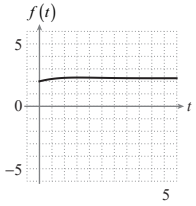


45. Frequency = 0.5 Hz

47. A = 3 inches, P = 2 seconds,
$g(t) = 3\cos(\pi t)$

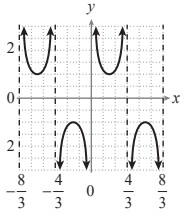
49. A = 1.5 ft, P = $\frac{3}{5}$ seconds,
$g(t) = 1.5\cos\left(\frac{10\pi}{3}t\right)$

51. a. $\cos\left(x + \frac{3\pi}{2}\right)$

b. $\cos\left(x - \frac{(4n+1)\pi}{2}\right)$

7.5 Exercises

1.



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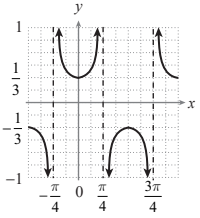

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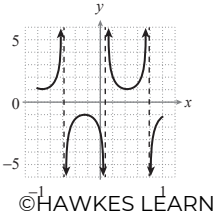

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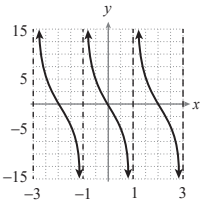

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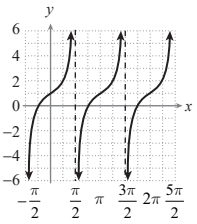

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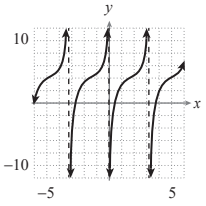

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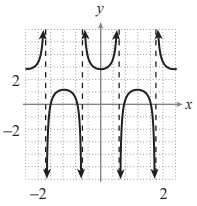


7.6 Exercises

1. $-\frac{\pi}{2}$ **3.** $\frac{\pi}{4}$ **5.** $\frac{\pi}{6}$

7. 0 **9.** $-\frac{\pi}{3}$ **11.** $\frac{\pi}{3}$

13. $-\frac{\pi}{4}$ **15.** $\frac{2\pi}{3}$ **17.** $-\frac{\pi}{6}$

19. $\frac{\pi}{2}$ **21.** $\frac{\pi}{4}$ **23.** $\frac{2\pi}{3}$

25. −0.2014 **27.** −1.1198

29. 1.9823 **31.** 0.2014

33. 0.0067 **35.** −0.1126

37. $\frac{\pi}{2}$ **39.** $\frac{1}{2}$ **41.** −0.8

43. 0.3714

45. 0.8944 **47.** 0.75

49. $\frac{\sqrt{3}}{2}$ **51.** 1 **53.** −1

55. $\frac{\sqrt{2}}{2}$ **57.** 1 **59.** $\frac{\sqrt{2}}{2}$

61. $-\sqrt{2}$ **63.** $\frac{\sqrt{2}}{2}$ **65.** -1

67. $\frac{\sqrt{1-x^2}}{x}$

69. $\sqrt{9x^2+1}$ **71.** $\left|\frac{1}{x}\right|\sqrt{x^2-1}$

73. $\theta = 47.88848865$

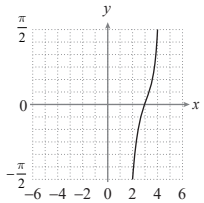
75. $\theta = 32.76975166$

77. $\theta = 60.10239082$

79. $\theta = 0.5736213262$

81. $\theta = -0.8480623625$

83.



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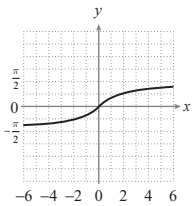


87. a. 0.2450

b. 0.7854

c. 0.9505

Chapter 7 Project

1. United Nations, NY: approx. 235.0°;
California Academy of Sciences: approx. 220.5°;
Smithsonian, Washington, DC: approx. 226.0°;
St. Isaac's Cathedral, Russia: approx. 311.4°;
Paris Observatory, France: approx. 270.9°

3. Approx. 14.92 ft

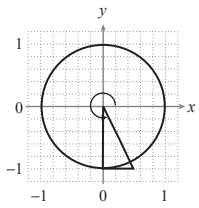
5. Approx. 12.6°

Chapter 7 Review Exercises

1. 4° **3.** –315°

5. $\frac{7\pi}{30}$ **7.** $-\frac{79\pi}{180}$

9.



11. 15.71 ft **13.** 30.72 meters

15. 1296.35 km **17.** 3 rad

19. $108\pi\,\text{ft}^2$ or $339.29\,\text{ft}^2$

21. $\sin\theta = \frac{2\sqrt{5}}{5}, \cos\theta = \frac{\sqrt{5}}{5}$,

$\tan\theta = 2, \csc\theta = \frac{\sqrt{5}}{2}$,

$\sec\theta = \sqrt{5}, \cot\theta = \frac{1}{2}$

23. 4.0108 **25.** 0.2225

27. 15.2203° **29.** False

31. 129.13 feet

33. $\sin(-460°) \approx -0.9848$,
$\cos(-460°) \approx -0.1736$,
$\tan(-460°) \approx 5.6713$,
$\csc(-460°) \approx -1.0154$,
$\sec(-460°) \approx -5.7588$,
$\cot(-460°) \approx 0.1763$

35. $\sin\left(\frac{7\pi}{3}\right) = \frac{\sqrt{3}}{2}, \cos\left(\frac{7\pi}{3}\right) = \frac{1}{2}$,

$\tan\left(\frac{7\pi}{3}\right) = \sqrt{3}, \csc\left(\frac{7\pi}{3}\right) = \frac{2\sqrt{3}}{3}$,

$\sec\left(\frac{7\pi}{3}\right) = 2, \cot\left(\frac{7\pi}{3}\right) = \frac{\sqrt{3}}{3}$

37. $\theta' = 37°$ **39.** $\theta' = \frac{\pi}{4}$

41. III

43. $\tan\left(\frac{4\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

45. $\sin(-42°) = \cos 132° \approx -0.6691$

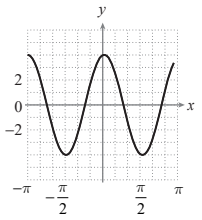
47. $\cos\left(\frac{5\pi}{4}\right) = \sin\left(-\frac{3\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

49. $\sin\theta = \frac{13}{5}$

51. $A = 6,\ P = 2\pi,$ no phase shift

53. $A = 9,\ P = \pi,$ shifted left π units

55.



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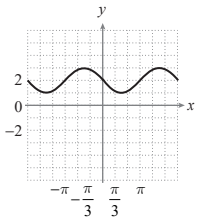

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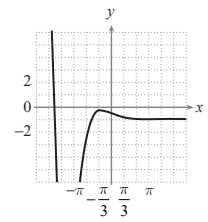

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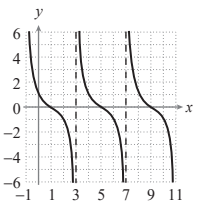
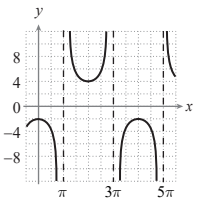


63.



65. $\frac{\pi}{2}$ **67.** $\frac{\pi}{3}$

69. 0.4636 rad or 26.5651 deg

71. 0.9 **73.** $-\frac{\pi}{4}$

75. $\frac{\sqrt{3}}{3}$ **77.** 2

79. –1.336110366

81. 55.32339906

Chapter 8: Trigonometric Identities and Equations

8.1 Exercises

1. $\sec x$ **3.** $\sin t$

5. $\cos x - \sec x$ **7.** $\tan \alpha$

9. $\sec y$ **11.** $\tan^2 x$

13. $\sec \theta$ **15.** $3 \sin \theta$

17. $8 \cos \theta$ **19.** $5 \sec \theta$

8.2 Exercises

1. $\dfrac{-\sqrt{6}+\sqrt{2}}{4}$ **3.** $-\sqrt{3}-2$

5. $-\sqrt{3}-2$ **7.** $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

9. $\dfrac{\sqrt{2}+\sqrt{6}}{4}$ **11.** $\sqrt{3}+2$

13. $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ **15.** $\sqrt{3}+2$

17. $\dfrac{-\sqrt{6}-\sqrt{2}}{4}$ **19.** $\sqrt{3}+2$

21. $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ **23.** $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

25. $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ **27.** $\dfrac{56}{65}$

29. $\dfrac{36}{85}$ **31.** $\dfrac{4\sqrt{6}-\sqrt{21}}{25}$

33. $\sin 45° = \dfrac{\sqrt{2}}{2}$

35. $\tan 135° = -1$

37. $\tan\left(\dfrac{\pi}{4}\right) = 1$

39. $\sin 150° = \dfrac{1}{2}$

41. $\cos 135° = -\dfrac{\sqrt{2}}{2}$

43. $\tan 60° = \sqrt{3}$

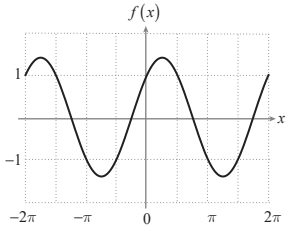
45. $-\dfrac{56}{33}$ **47.** $\dfrac{4+3\sqrt{3}}{10}$

49. 1

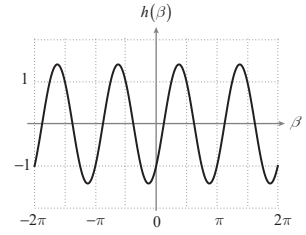
51. $\dfrac{2x^2+\sqrt{1-x^2}}{\sqrt{1+4x^2}}$

53. $x\left(\sqrt{1-4x^2} - 2\sqrt{1-x^2}\right)$

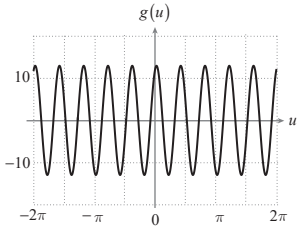
55. $\sqrt{2}\sin\left(x+\dfrac{\pi}{4}\right)$



57. $\sqrt{2}\sin\left(2\beta-\dfrac{\pi}{4}\right)$



59. $13\sin(5u+1.1760)$



73. True **75.** False

79. True **81.** True

83. False

8.3 Exercises

1. $\cos(2x) = \dfrac{7}{25}$;
$\sin(2x) = \dfrac{24}{25}$;
$\tan(2x) = \dfrac{24}{7}$

3. $\cos(2x) = \dfrac{1}{3}$;
$\sin(2x) = -\dfrac{2\sqrt{2}}{3}$;
$\tan(2x) = -2\sqrt{2}$

5. $\cos(2x) = \dfrac{1}{2}$;
$\sin(2x) = \dfrac{\sqrt{3}}{2}$;
$\tan(2x) = \sqrt{3}$

7. $\dfrac{\sin x - \sin x \cos(2x)}{2}$

9. $\dfrac{1-\cos(2x)-\cos(4x)}{16} + \dfrac{\cos(2x)\cos(4x)}{16}$

11. $\dfrac{3\sin x - 4\sin x\cos(2x)}{3+4\cos(2x)+\cos(4x)} + \dfrac{\sin x\cos(4x)}{3+4\cos(2x)+\cos(4x)}$

13. $\dfrac{\sqrt{2+\sqrt{2}}}{2}$ **15.** $\dfrac{\sqrt{2+\sqrt{3}}}{2}$

17. $\dfrac{\sqrt{2+\sqrt{3}}}{2}$

19. $\dfrac{\sin(6x)+\sin 0}{2}$

21. $\dfrac{5(\sin 120° - \sin 90°)}{2}$

23. $\dfrac{\cos(2y)-\cos(2x)}{2}$

25. $\dfrac{\cos\left(\dfrac{7\pi}{12}\right) - \cos\left(\dfrac{23\pi}{12}\right)}{2}$

27. $\sin\left(\dfrac{\pi}{2}\right) - \sin\left(\dfrac{\pi}{6}\right)$

29. $2\sin(4x)\cos(2x)$

31. $-2\sin(2\beta)\sin\beta$

33. $2\cos 75°\sin 60°$

35. $-2\sin\left(\dfrac{17\pi}{24}\right)\sin\left(\dfrac{11\pi}{24}\right)$

47. $\cos(2x) = 2\cos^2 x - 1$

$\cos(3x) = 4\cos^3 x - 3\cos x$

$\cos(4x) = 8\cos^4 x - 8\cos^2 x + 1$

$\sin(2x) = 2\sin x \cos x$

$\sin(3x) = \sin x(4\cos^2 x - 1)$

$\sin(4x) = \sin x(8\cos^3 x - 4\cos x)$

8.4 Exercises

1. $x = \frac{7\pi}{6} + 2n\pi, x = \frac{11\pi}{6} + 2n\pi$

3. $x = \frac{\pi}{4} + 2n\pi, x = \frac{7\pi}{4} + 2n\pi$

5. $x = \frac{\pi}{6} + 2n\pi, x = \frac{11\pi}{6} + 2n\pi$

7. No solution

9. $x = \frac{2\pi}{3} + n\pi$

11. $x = n\pi$ **13.** $x = 2n\pi$

15. No solution

17. $x = \frac{2\pi}{3} + 2n\pi, x = \frac{4\pi}{3} + 2n\pi$

19. $x = \frac{5\pi}{12} + \frac{n\pi}{2}$

21. $x = \frac{\pi}{6}, \frac{5\pi}{6}$

23. $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

25. $x = 0, \pi, 2\pi$ **27.** $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

29. $x = \tan^{-1}\left(-4 + \sqrt{59}\right)$,

$\tan^{-1}\left(-4 + \sqrt{59}\right) + \pi$,

$\tan^{-1}\left(-4 - \sqrt{59}\right) + \pi$,

$\tan^{-1}\left(-4 - \sqrt{59}\right) + 2\pi$

41. True **43.** True

45. True **47.** $x = \frac{2\pi}{3} + n\pi$

49. True

51. $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

53. $x = \frac{\pi}{2}, \pi$ **55.** $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

57. $x = \frac{\pi}{4}, \frac{5\pi}{4}$

59. $x = 45°, 135°, 225°, 315°$

61. $x = 90°, 210°, 330°$

63. No solution

65. $x = 30°, 210°, 240°, 300°$

67. $s = -4, 3$; t has no solution.

69. $s = \frac{1 \pm \sqrt{2}}{2}$

$t = 1.7794, 4.5038$

71. 17.3°

73. 2.6779 and 5.8195

75. 0.6662, 2.4754, 3.8078, and 5.6169

77. 1.9948 and 5.8592

79. 0.6993, 2.6078, 3.6754, and 5.5839

81. $x = 10°, 50°, 130°, 170°, 250°, 290°$

83. They can have at most two real solutions.

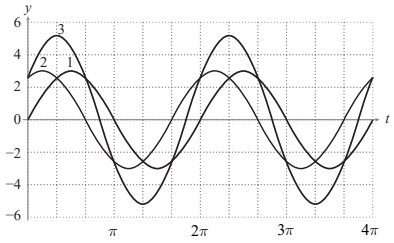
85. They may have as many as $2a$ solutions from 0° to 360°.

Chapter 8 Project

1. $y_1 + y_2 = 2\sqrt{2}\sin\left(kx - \omega t + \frac{\pi}{4}\right)$,

Amplitude : $2\sqrt{2}$

3.



We can suggest that the addition of two waves with equal amplitudes averages their displacements.
(Answers will vary.)

Chapter 8 Review Exercises

1. $\csc x$ **3.** $-\tan^2 y$

5. $-1 - \sin\theta$ **13.** $4\sec\theta$

15. $10\cot\theta$ **17.** $\frac{\sqrt{3}}{2}$

19. $\frac{\sqrt{2} - \sqrt{6}}{4}$ **21.** $\sqrt{3}$

23. $\frac{13}{84}$ **25.** $\sin 210°; -\frac{1}{2}$

29. $\frac{\sqrt{1 - x^2} - x^2}{\sqrt{1 + x^2}}$

31. $2\sin\left(x + \frac{7\pi}{4}\right)$

33. $\cos(2x) = -\frac{7}{25}, \sin(2x) = \frac{24}{25}$,

$\tan(2x) = -\frac{24}{7}$

37. $\frac{\sin x - \sin x \cos(4x)}{8}$

39. $2 + \sqrt{3}$

41. $2 - \sqrt{3}$

43. $\frac{\sin(2x) - \sin(2y)}{2}$

45. $\frac{1}{2}(\sin 180° + \sin 150°)$

47. $2\cos(4\alpha)\sin\alpha$

49. $-2\sin\left(\frac{17\pi}{24}\right)\sin\left(\frac{\pi}{24}\right)$

51. $x = \frac{\pi}{6} + 2n\pi, x = -\frac{\pi}{6} + 2n\pi$

53. $x = \frac{\pi}{2} + 2n\pi, x = -\frac{\pi}{2} + 2n\pi$

55. $x = \frac{\pi}{3} + 2n\pi, x = \frac{5\pi}{3} + 2n\pi$,

$x = \pi + 2n\pi$

57. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

59. True

61. $x = \frac{\pi}{3} + 2n\pi, x = \frac{2\pi}{3} + 2n\pi$

63. $x = 0°, 180°$

Chapter 9: Additional Topics in Trigonometry

9.1 Exercises

1. $C = 105°$, $b \approx 4.2426$, $c \approx 5.7956$

3. $C = 60°$, $a \approx 4.9067$, $c \approx 4.5221$

5. $A = 80°$, $a \approx 3.9392$, $b \approx 3.7588$

7. $C = 150°$, $b \approx 1.0154$, $c \approx 2.9238$

9. No triangle

11. $A \approx 20.7048°$, $B \approx 114.2952°$, $b \approx 5.1559$

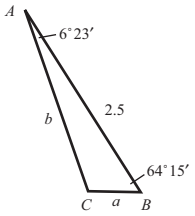
13. $A \approx 35.9044°$, $B \approx 4.0956°$, $a \approx 8.2110$

15. $A \approx 19.1638°$, $C \approx 80.8362°$, $c \approx 6.0148$

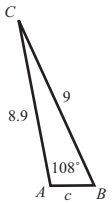
17. $A \approx 6.7456°$, $C \approx 63.2544°$, $c \approx 7.6026$

19. $h \approx 6.02$, $c < h$, so a triangle cannot be created.

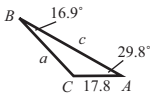
21. $C = 109°22'$, $a \approx 0.29$, $b \approx 2.39$



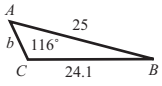
23. $c \approx 0.31$, $B \approx 70.13°$, $C \approx 1.87°$



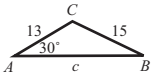
25. $C = 133.4°$, $a \approx 30.34$, $c \approx 44.49$



27. $b \approx 1.92$, $A \approx 60.05°$, $B \approx 3.95°$



29. $c \approx 24.78$, $C \approx 124.32°$, $B \approx 25.68°$



31. 94.34 **33.** 9.74

35. 18.47 **37.** 335.18

39. 21 feet **41.** 442.3 feet

43. 9.4 feet **45.** 26.3 feet

47. 32.6 feet, 26.5 feet

49. 105.5 miles **51.** 3.5 feet

53. 178.3882 ft^2

9.2 Exercises

1. $a = \sqrt{37}$, $B \approx 25.2850°$, $C \approx 94.7150°$

3. $b \approx 4.5985$, $A \approx 41.7854°$, $C \approx 88.2146°$

5. $c \approx 4.1063$, $A \approx 103.0643°$, $B \approx 46.9357°$

7. $c \approx 7.0752$, $A \approx 41.6113°$, $B \approx 68.3887°$

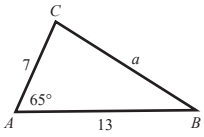
9. $A \approx 46.5675°$, $B \approx 104.4775°$, $C \approx 28.9550°$

11. $A \approx 121.8554°$, $B \approx 39.5712°$, $C \approx 18.5734°$

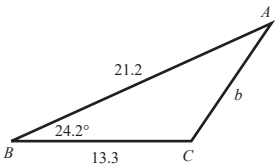
13. $A = B = C = 60°$

15. $A = 90°$, $B \approx 36.8699°$, $C \approx 53.1301°$

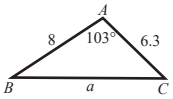
17. $a \approx 11.88$, $B \approx 32.28°$, $C \approx 82.72°$



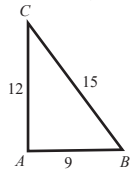
19. $b \approx 10.58$, $A \approx 31.01°$, $C \approx 124.79°$



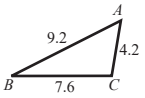
21. $a \approx 11.24$, $B \approx 33.1°$, $C \approx 43.9°$



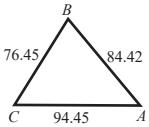
23. $A \approx 90°$, $B \approx 53.13°$, $C \approx 36.87°$



25. $A \approx 54.82°$, $B \approx 26.85°$, $C \approx 98.33°$



27. $A \approx 50.22°$, $B \approx 71.71°$, $C \approx 58.07°$



29. 89.29 **31.** 11.15

33. 65.5744 feet

35. 2.4×10^7 miles

37. 21.4413 feet

39. 22.6199°, 67.3801°

41. 160.1188°

43. 337,940.9589 square nautical miles

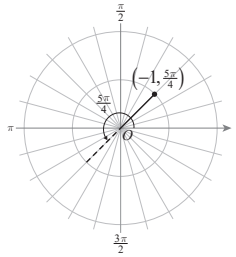
45. **a.** 61.9372 in.^{2}

b. 584.2397 in.^{2}

c. 136.1041 in.^{2}

9.3 Exercises

1.



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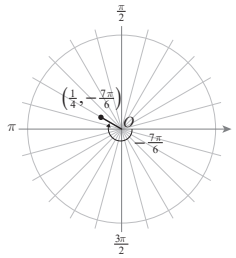

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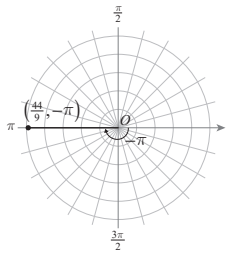


7. $(3.54, -3.54)$

9. $(-4.42, -4.42)$

11. $(-2.60, -1.50)$

13. $(-3, 0)$ and $(3, \pi)$

15. $\left(\sqrt{145}, -0.08\right)$ and $\left(-\sqrt{145}, 3.06\right)$

17. $\left(2\sqrt{21}, 1.76\right)$ and $\left(-2\sqrt{21}, -1.38\right)$

19. $r^2 = 25$

21. $r\cos\theta = 12$

23. $\sin\theta = \cos\theta$

25. $r\cos\theta = 16a$

27. $r^2 - 4ar\cos\theta = 0$

29. $r^2\sin^2\theta - 4r\cos\theta - 4 = 0$

31. $x^2 + y^2 = 5x$

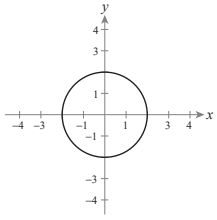
33. $x^2 + y^2 = 49$

35. $y = \dfrac{1}{2}$

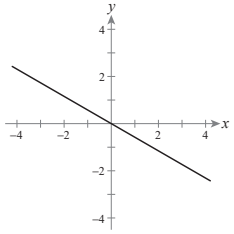
37. $x^4 + y^4 + 2x^2y^2 = 2xy$

39. $4y + 7x = 12$

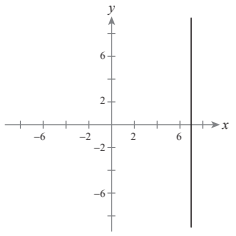
41. $x^2 + y^2 = 4$



43. $y = -\dfrac{x}{\sqrt{3}}$



45. $x = 7$



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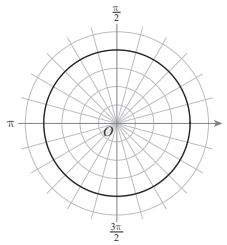

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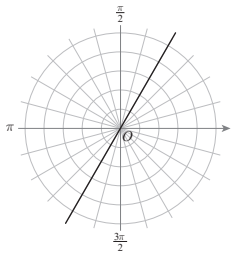

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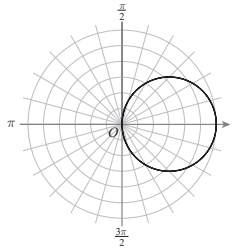

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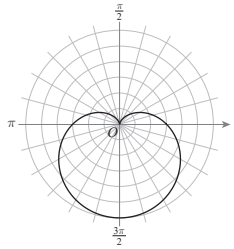

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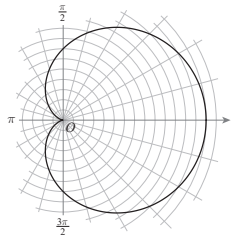

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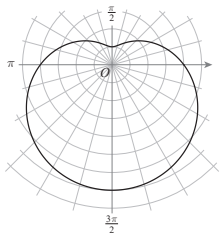
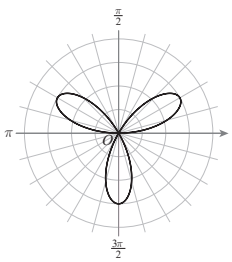
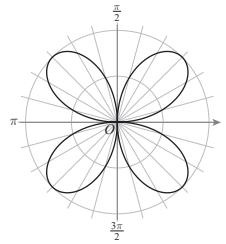


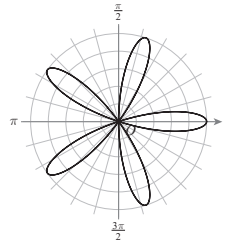
59.



61.



63.



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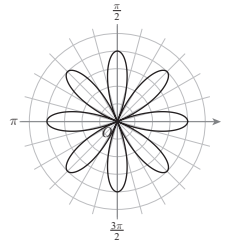

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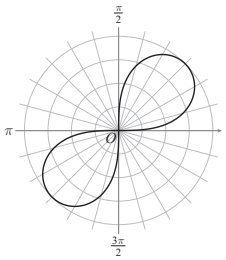


69. $(0,0), \left(\dfrac{\sqrt{2}}{2}, \dfrac{\pi}{4}\right)$

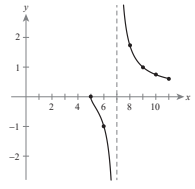
71. $(0,0), \left(1+\dfrac{\sqrt{2}}{2}, \dfrac{3\pi}{4}\right), \left(1-\dfrac{\sqrt{2}}{2}, -\dfrac{\pi}{4}\right)$

73. The graph of $f(\theta - a)$ is that of $f(\theta)$ rotated about the origin by a radians.

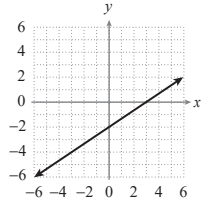
9.4 Exercises

1.

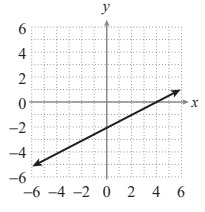
t	x	y
0	5	0
1	6	1
2	7	undefined
3	8	$\sqrt{3}$
4	9	1
5	10	$\frac{\sqrt{5}}{3}$
6	11	$\frac{\sqrt{6}}{4}$



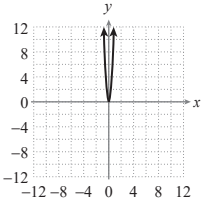
3. $2x = 3y + 6$



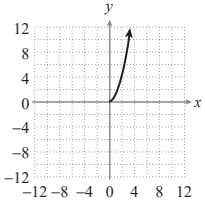
5. $x = 2y + 4$



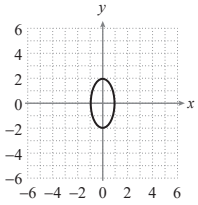
7. $y = 16x^2$



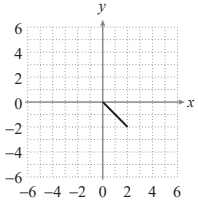
9. $x = \sqrt{y}$



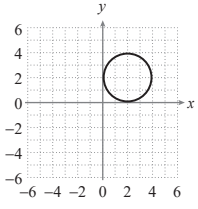
11. $y = \pm 2\sqrt{1 - x^2}$



13. $y = -x, 0 \le x \le 2$



15. $y = \pm\sqrt{4x - x^2} + 2$



17. $x = t, y = t^2 + 2t + 1$

19. $x = t, y = -t^2 - 5$

21. $x = t, y = \pm\sqrt{t - 4}$

23. $x = t, y = \frac{1}{t}$

25. $x = t, y = |t - 1|$

27. $x = t, y = \pm\sqrt{1 - t^2}$

29. $x = t, y = t^2 - t - 6$

31. $x = t, y = \frac{1}{4}t + \frac{19}{2}$

33. $x = t, y = \frac{4}{7}t$

35. $x = t, y = \frac{1}{2}t - 3$

37. $x = t - 5, y = 2 - |t - 5|$

39. $x = 4t, y = -2t + \frac{5}{2}$

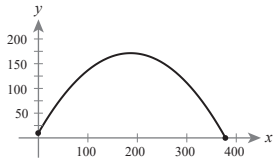
41. $x = 5t + 1, y = 25t^2 - 10t + 1$

43. $x = -4 + 3\cos\theta, y = 2 + 3\sin\theta$

45. $x = 6\cos\theta, y = -2 + 6\sin\theta$

47. a. $x \approx 58.67t$,
$y \approx -16t^2 + 101.61t + 10$

b.



c. Approx. 126.42 ft

d. Approx. 378.42 ft

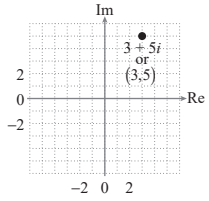
e. $t \approx 6.45$ s

f. Yes

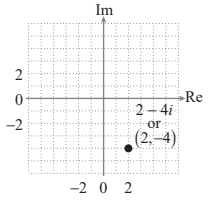
49. $x = 12(\theta - \sin\theta)$,
$y = 12(1 - \cos\theta)$

9.5 Exercises

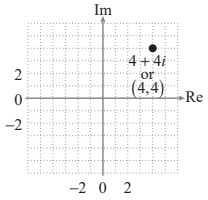
1. $\sqrt{34}$



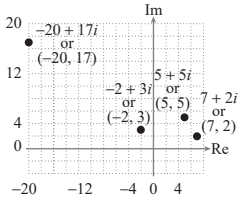
3. $\sqrt{20} = 2\sqrt{5}$



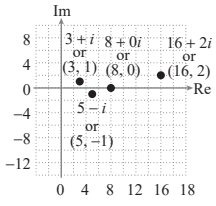
5. $\sqrt{32} = 4\sqrt{2}$



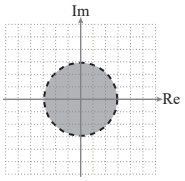
7.



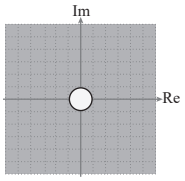
9.



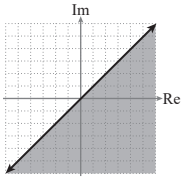
11.



13.



15.



17. $\sqrt{10}\left(\cos(3.46)+i\sin(3.46)\right)$

19. $\sqrt{5}\left(\cos(1.11)+i\sin(1.11)\right)$

21. $2\sqrt{5}\left(\cos(0.46)+i\sin(0.46)\right)$

23. $2\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)$

25. $5\left(\cos(0.93)+i\sin(0.93)\right)$

27. $8\left(\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right)$

29. $\frac{-3\sqrt{3}}{2}+\frac{3i}{2}$

31. $-1-i\sqrt{3}$ **33.** $-\frac{5}{\sqrt{2}}+\frac{5i}{\sqrt{2}}$

35. $\frac{-3\sqrt{3}}{4}+\frac{3i}{4}$ **37.** $1.01+4.9i$

39. $16\left(\cos 30°+i\sin 30°\right)=8\sqrt{3}+8i$

41. $3\sqrt{6}\left(\cos\left(\frac{17\pi}{12}\right)+i\sin\left(\frac{17\pi}{12}\right)\right)$
$=-1.9-7.1i$

43. $2\sqrt{10}\left(\cos(2.42)+i\sin(2.42)\right)$
$=\left(-3-\sqrt{3}\right)+\left(3\sqrt{3}-1\right)i$

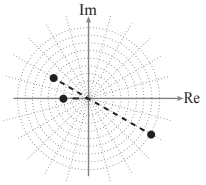
45. $2\left(\cos 180°+i\sin 180°\right)=-2$

47. $\frac{10}{3}\left(\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)\right)=\frac{10i}{3}$

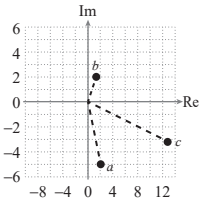
49. $\frac{1}{\sqrt{2}}\left(\cos\left(-\frac{3\pi}{4}\right)+i\sin\left(-\frac{3\pi}{4}\right)\right)$
$=-\frac{1}{2}-\frac{i}{2}$

51. $2\left(\cos\left(\frac{5\pi}{12}\right)+i\sin\left(\frac{5\pi}{12}\right)\right)$
$=0.52+1.93i$

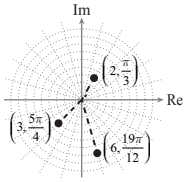
53. $a=4\left(\cos\left(\frac{5\pi}{6}\right)+i\sin\left(\frac{5\pi}{6}\right)\right)$,
$b=2\left(\cos\pi+i\sin\pi\right)$,
$c=8\left(\cos\left(\frac{11\pi}{6}\right)+i\sin\left(\frac{11\pi}{6}\right)\right)$



55. $a=2-5i, b=\sqrt{2}+2i$,
$c=10+2\sqrt{2}+i\left(4-5\sqrt{2}\right)$



57. $a=2e^{\frac{\pi}{3}i}, b=3e^{\frac{5\pi}{4}i}, c=6e^{\frac{19\pi}{12}i}$

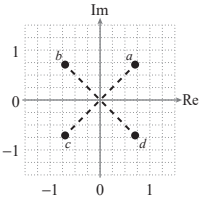


59. $32e^{\frac{\pi}{3}i}$

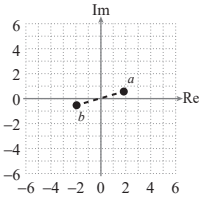
61. $1.04\times10^{13}e^{2.9i}$

63. $e^{2\pi i}$

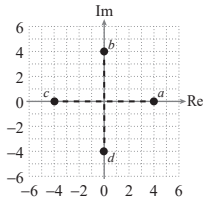
65. $a=e^{\frac{\pi i}{4}}, b=e^{\frac{3\pi i}{4}}, c=e^{\frac{5\pi i}{4}}, d=e^{\frac{7\pi i}{4}}$



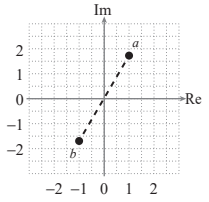
67. $a=2e^{\frac{\pi i}{12}}, b=2e^{\frac{13\pi i}{12}}$



69. $a=4, b=4e^{\frac{\pi i}{2}}, c=4e^{\pi i}, d=4e^{\frac{3\pi i}{2}}$



71. $a=2e^{60°i}, b=2e^{240°i}$



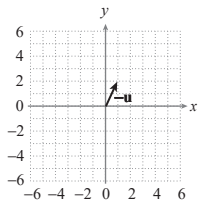
73. $2\sqrt{2}e^{\frac{\pi i}{12}}, 2\sqrt{2}e^{\frac{13\pi i}{12}}$

75. $2e^{\frac{\pi i}{5}}, 2e^{\frac{3\pi i}{5}}, 2e^{\pi i}, 2e^{\frac{7\pi i}{5}}, 2e^{\frac{9\pi i}{5}}$

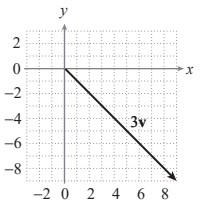
77. $5e^{\frac{\pi i}{4}}, 5e^{\frac{3\pi i}{4}}$

9.6 Exercises

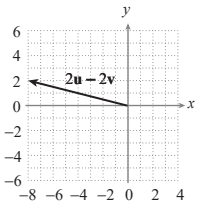
1.



3.



5.



7. $\mathbf{v}=\langle 3,-3\rangle, \|\mathbf{v}\|=3\sqrt{2}$

9. $\mathbf{v}=\langle 5,3\rangle, \|\mathbf{v}\|=\sqrt{34}$

11. $\mathbf{v}=\langle 5,-1\rangle, \|\mathbf{v}\|=\sqrt{26}$

13. $\mathbf{v}=\langle -7,7\rangle, \|\mathbf{v}\|=7\sqrt{2}$

15. $\mathbf{v}=\langle -4,-6\rangle, \|\mathbf{v}\|=2\sqrt{13}$

17. a. $\langle -2,8\rangle$ **b.** $\langle 8,-4\rangle$
c. $\langle -4,0\rangle$

19. a. $\langle 1,4\rangle$ **b.** $\langle -11,12\rangle$
c. $\langle 6,-8\rangle$

21. a. $\langle -5,-10\rangle$ **b.** $\langle -8,-2\rangle$
c. $\langle 6,4\rangle$

23. $-\mathbf{u} = \langle -1,-1\rangle$,
$2\mathbf{u} - \mathbf{v} = \langle -1,5\rangle$,
$\mathbf{u} + \mathbf{v} = \langle 4,-2\rangle$,
$\|\mathbf{u}\| = \sqrt{2}, \|\mathbf{v}\| = 3\sqrt{2}$

25. $-\mathbf{u} = \langle 4,-4\rangle$,
$2\mathbf{u} - \mathbf{v} = \langle -12,12\rangle$,
$\mathbf{u} + \mathbf{v} = \langle 0,0\rangle$,
$\|\mathbf{u}\| = 4\sqrt{2}, \|\mathbf{v}\| = 4\sqrt{2}$

27. a. $\left\langle \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right\rangle$
b. $\mathbf{u} = 6\mathbf{i} - 3\mathbf{j}$

29. a. $\left\langle \frac{-5}{\sqrt{26}}, \frac{1}{\sqrt{26}} \right\rangle$
b. $\mathbf{u} = 5\mathbf{i} - \mathbf{j}$

31. a. $\left\langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle$
b. $\mathbf{u} = 3\mathbf{i} + 3\mathbf{j}$

33. $\|\mathbf{v}\| = 5, \theta = 30°$

35. $\|\mathbf{v}\| = 5, \theta = 36.9°$

37. $\langle 3\sqrt{3}, 3\rangle$

39. $\langle -9\sqrt{2}, 9\sqrt{2}\rangle$

41. $\left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$

43. $\left\langle \frac{8}{\sqrt{13}}, \frac{12}{\sqrt{13}} \right\rangle$

45. $\langle 2\sqrt{3}, 2\rangle$

47. 38.67 mph, N 77.76° W

49. $\mathbf{F} = \langle 150, -1235\rangle$,
$|\mathbf{F}| = 1244.08$ pounds

9.7 Exercises

1. 17 **3.** 6 **5.** 8
7. 1 **9.** −7 **11.** 32
13. $\langle -26,39\rangle$ **15.** $\sqrt{37}$
17. $\sqrt{53}$ **19.** 123.7°
21. 14.0° **23.** 161.6°
25. $\frac{\pi}{4}$ **27.** 8°, 69°, 103°
29. 57.1°, 60.8°, 62.1°
31. −62.5 **33.** $-32\sqrt{2}$
35. $\langle 1,1\rangle, \langle 5,5\rangle$
37. $\langle 3,1\rangle, \langle -6,-2\rangle$
39. Neither
41. Orthogonal
43. $\text{proj}_{\mathbf{v}}\mathbf{u} = \langle 2,1\rangle$,
$\text{perp}_{\mathbf{v}}\mathbf{u} = \langle -1,2\rangle$
45. $\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{6}{5}, \frac{2}{5} \right\rangle$,
$\text{perp}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{9}{5}, -\frac{27}{5} \right\rangle$
47. $\text{proj}_{\mathbf{v}}\mathbf{u} = \left\langle -\frac{60}{17}, -\frac{15}{17} \right\rangle$,
$\text{perp}_{\mathbf{v}}\mathbf{u} = \left\langle \frac{9}{17}, -\frac{36}{17} \right\rangle$
49. 14 **51.** 3
53. 3479.3 pounds
55. 109.6 pounds
57. 1638.3 ft-lb
59. 11,431,535.3 ft-lb

9.8 Exercises

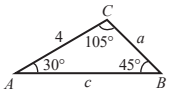
1. 0 **3.** 1 **5.** $\frac{3}{5}$
7. −1.04 **9.** 74.21 **11.** Even
13. Odd **15.** Odd **37.** 0
45. −0.33 **47.** 0.69 **49.** 0.35

Chapter 9 Project

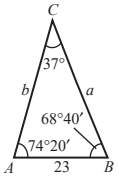
1. Approx. 921,479 ft²
3. a. Approx. 8414 pounds
b. Approx. 1298.4 pounds
c. Approx. 2045.6 feet
d. 2,822,928 foot-pounds

Chapter 9 Review Exercises

1. $C = 105°$, $c \approx 5.46$, $a \approx 2.83$

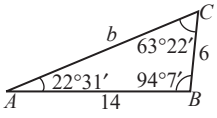


3. $B = 68°40'$, $a \approx 36.80$, $b \approx 35.60$

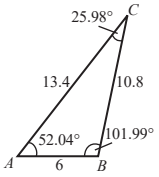


5. 10.14

7. $A \approx 22°31'$, $C \approx 63°22'$, $b \approx 15.62$



9. $A \approx 52.04°$, $B \approx 101.99°$, $C \approx 25.98°$



11. 22.19

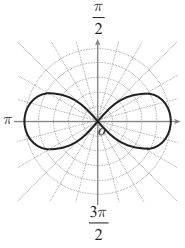
13. $(-6.06, -3.5)$

15. $(15.62, 0.88)$ and $(-15.62, -2.26)$

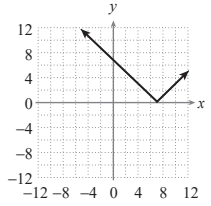
17. $r^2 - 9ar\cos\theta = 0$

19. $x + y = 4$

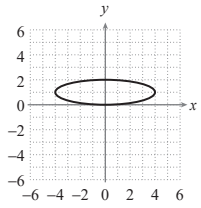
21.



23. $y = |x - 7|$

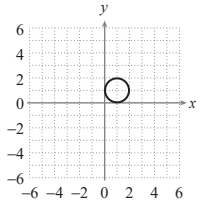


25. $\dfrac{x^2}{16}+(y-1)^2=1$

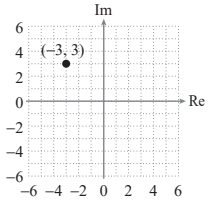


27. $x=t,\ y=2-6t$

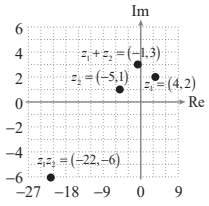
29. $x=1+\cos\theta,\ y=1+\sin\theta$



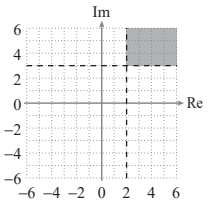
31. $3\sqrt{2}$



33.



35.



37. $\sqrt{17}\left(\cos(1.33)+i\sin(1.33)\right)$

39. $\dfrac{3}{2}+\dfrac{3i\sqrt{3}}{2}$

41. $5(\cos 120°+i\sin 120°)$,

$-\dfrac{5}{2}+\dfrac{5i\sqrt{3}}{2}$

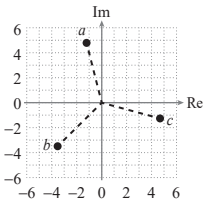
43. $24(\cos 315°+i\sin 315°)$,

$12\sqrt{2}-12i\sqrt{2}$

45. $177{,}147e^{120°i}$

47. $a=5e^{i\left(\frac{7\pi}{12}\right)},b=5e^{i\left(\frac{5\pi}{4}\right)}$,

$c=5e^{i\left(\frac{23\pi}{12}\right)}$

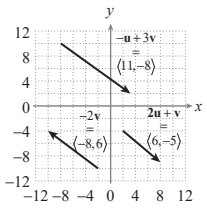


$i\left(\frac{\pi}{4}\right)$ $i\left(\frac{11\pi}{12}\right)$ $i\left(\frac{19\pi}{12}\right)$

49. $2e$, $2e$, $2e$

51. $\mathbf{v}=\langle -10,-6\rangle,\ \|\mathbf{v}\|=2\sqrt{34}$

53.



55. $-\mathbf{u}=\langle -5,-1\rangle, 2\mathbf{u}-\mathbf{v}=\langle 7,1\rangle$,

$\mathbf{u}+\mathbf{v}=\langle 8,2\rangle, \|\mathbf{u}\|=\sqrt{26}$,

$\|\mathbf{v}\|=\sqrt{10}$

57. a. $\left\langle \dfrac{2}{\sqrt{5}},\dfrac{1}{\sqrt{5}}\right\rangle$

b. $6\mathbf{i}+3\mathbf{j}$

59. $\|\mathbf{v}\|=\sqrt{26},\ \theta=-11.3°$

61. $\langle 3.6,-4.8\rangle$

63. 40.01 mph, N 40.23° W

65. $\langle -108,-270\rangle$

67. $\sqrt{10}$ **69.** $\dfrac{5\pi}{12}$ **71.** -36

73. $\text{proj}_{\mathbf{v}}\mathbf{u}=\left\langle \dfrac{3}{2},\dfrac{3}{2}\right\rangle$,

$\text{proj}_{\mathbf{v}}\mathbf{u}=\left\langle \dfrac{5}{2},\dfrac{-5}{2}\right\rangle$

75. 31

77. 417,558.5 ft-lb

79. $\dfrac{15}{8}$ **81.** 1.31

Chapter 10: Conic Sections

10.1 Exercises

1. Center: $(5,2)$

Foci: $\left(5,2\pm\sqrt{21}\right)$

Vertices: $(5,7),(5,-3)$

3. Center: $(-2,-5)$

Foci: $\left(-2\pm\sqrt{6},-5\right)$

Vertices: $(1,-5),(-5,-5)$

5. Center: $(-3,2)$

Foci: $\left(-3\pm\sqrt{2},2\right)$

Vertices: $(-1,2),(-5,2)$

7. Center: $(-5,1)$

Foci: $\left(-5,1\pm2\sqrt{3}\right)$

Vertices: $(-5,5),(-5,-3)$

9. Center: $(-4,2)$

Foci: $\left(-4\pm3\sqrt{2},2\right)$

Vertices: $\left(-4\pm3\sqrt{3},2\right)$

11. Center: $(2,0)$

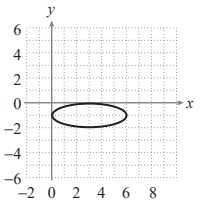
Foci: $(4,0),(0,0)$

Vertices: $\left(2\pm\sqrt{5},0\right)$

13. e **15.** f

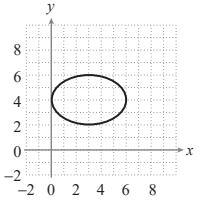
17. c **19.** b

21.

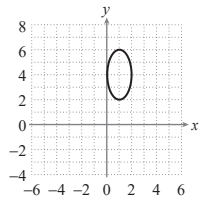


$\left(3\pm2\sqrt{2},-1\right)$

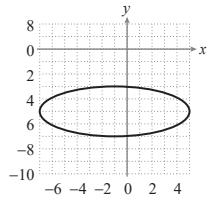
23.



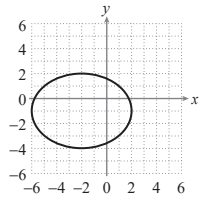
$\left(3\pm\sqrt{5},4\right)$

25. 

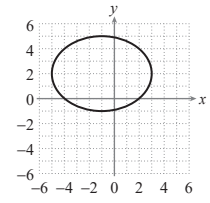
$(1, 4 \pm \sqrt{3})$

27. 

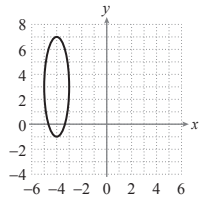
$(-1 \pm \sqrt{21}, -5)$

29. 

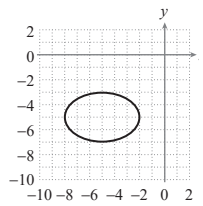
$(-2 \pm \sqrt{7}, -1)$

31. 

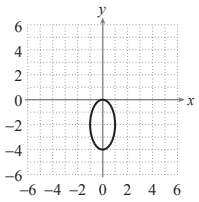
$(-1 \pm \sqrt{7}, 2)$

33. 

$(-5, 3 \pm \sqrt{15})$

35. 

$(-5 \pm \sqrt{5}, -5)$

37. 

$(0, -2 \pm \sqrt{3})$

39. $\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$

41. $(x-1)^2 + \dfrac{(y-1)^2}{9} = 1$

43. $\dfrac{(x-3)^2}{36} + \dfrac{y^2}{27} = 1$

45. $(x+2)^2 + \dfrac{(y+3)^2}{4} = 1$

47. $\dfrac{(x-5)^2}{16} + \dfrac{(y-3)^2}{15} = 1$

49. $\dfrac{(x-2)^2}{4} + \dfrac{(y+2)^2}{9} = 1$

51. $\dfrac{(x-1)^2}{9} + \dfrac{y^2}{16} = 1$

53. $e = \dfrac{\sqrt{11}}{6}$; major $= 24$; minor $= 20$

55. $e = \dfrac{2\sqrt{2}}{3}$; major $= 12$; minor $= 4$

57. $e = \dfrac{\sqrt{3}}{2}$; major $= 4$; minor $= 2$

59. $e = \dfrac{\sqrt{2}}{2}$; major $= 4$; minor $= 2\sqrt{2}$

61. $e = \dfrac{\sqrt{42}}{7}$; major $= 14$

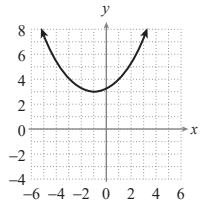
minor $= 2\sqrt{7}$

63. $e \approx 0.249$

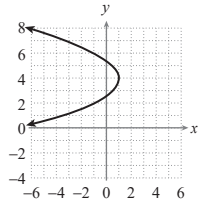
65. Yes, just barely, if the boat is centered on the river.

67. The string should be 5 cm long, and the tacks should be 4 cm apart.

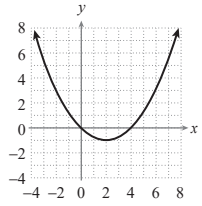
10.2 Exercises

1. 

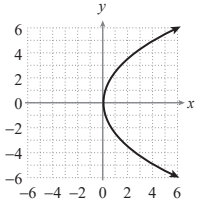
$(-1, 4), y = 2$

3. 

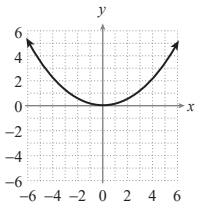
$\left(\dfrac{1}{2}, 4\right), x = \dfrac{3}{2}$

5. 

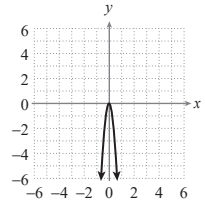
$(2, 0), y = -2$

7. 

$\left(\dfrac{3}{2}, 0\right), x = -\dfrac{3}{2}$

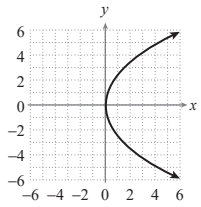
9. 

$\left(0, \dfrac{7}{4}\right), y = -\dfrac{7}{4}$

11. 

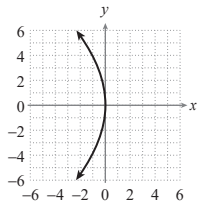
$\left(0, -\dfrac{1}{48}\right), y = \dfrac{1}{48}$

13.



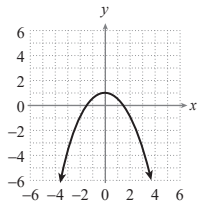
$\left(\frac{3}{2},0\right), x=-\frac{3}{2}$

15.



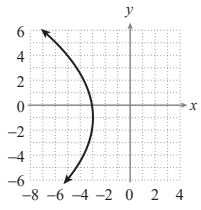
$(-4,0), x=4$

17.



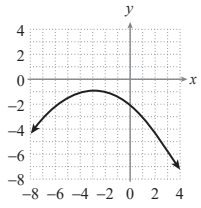
$\left(0,\frac{1}{2}\right), y=\frac{3}{2}$

19.



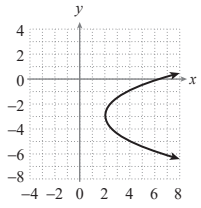
$(-6,-1), x=0$

21.



$(-3,-3), y=1$

23.



$\left(\frac{5}{2},-3\right), x=\frac{3}{2}$

25. g **27.** b

29. e **31.** d

33. $(y-1)^2=-4(x+1)$

35. $(x-3)^2=8(y+1)$

37. $(y+2)^2=24(x-3)$

39. $(x+3)^2=-2(y+1)$

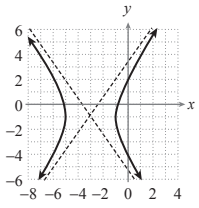
41. $(y-3)^2=10(x+4)$

43. $(y+1)^2=-8(x-2)$

45. 2 feet **47.** 1.5 inches

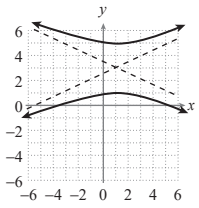
10.3 Exercises

1.



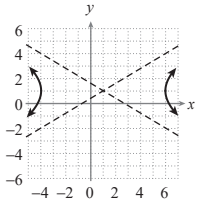
$\left(-3\pm\sqrt{13},-1\right)$

3.



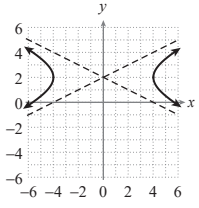
$\left(1,3\pm2\sqrt{5}\right)$

5.



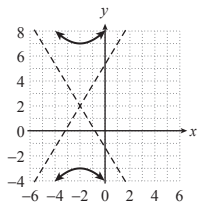
$\left(1\pm\sqrt{34},1\right)$

7.



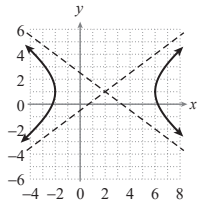
$\left(\pm2\sqrt{5},2\right)$

9.



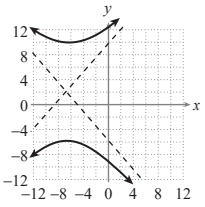
$\left(-2,2\pm\sqrt{34}\right)$

11.



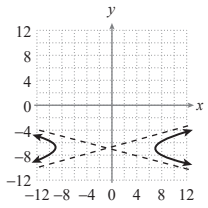
$(7,1),(-3,1)$

13.



$\left(-7,2\pm\sqrt{113}\right)$

15.



$\left(-1\pm2\sqrt{17},-7\right)$

17. Center: $(-3,2)$

Foci: $\left(-3\pm\sqrt{13},2\right)$

Vertices: $(-1,2),(-5,2)$

19. Center: $(1,-4)$

Foci: $\left(1\pm2\sqrt{3},-4\right)$

Vertices: $\left(1\pm\sqrt{3},-4\right)$

21. Center: $(-2,1)$

Foci: $\left(-2\pm\sqrt{30},1\right)$

Vertices: $(3,1),(-7,1)$

23. Center: $(-3,-1)$

Foci: $\left(-3\pm2\sqrt{3},-1\right)$

Vertices: $(-1,-1),(-5,-1)$

25. Center: $(1,0)$

Foci: $\left(1\pm\frac{\sqrt{5}}{2},0\right)$

Vertices: $(2,0),(0,0)$

27. Center: $(8,5)$

Foci: $\left(8\pm4\sqrt{5},5\right)$

Vertices: $(12,5),(4,5)$

29. a **31.** b

33. g **35.** h

37. $\frac{x^2}{4}-\frac{y^2}{5}=1$

39. $y^2-4x^2=1$

41. $\frac{5x^2}{2}-\frac{5(y-4)^2}{18}=1$

43. $\frac{(x-6)^2}{9}-\frac{(y-5)^2}{7}=1$

45. $\frac{(x+4)^2}{4}-\frac{(y-3)^2}{16}=1$

47. $\frac{(x-1)^2}{9}-\frac{(y+1)^2}{4}=1$

49. $\frac{(y+4)^2}{16}-\frac{(x-3)^2}{25}=1$

51. $\frac{x^2}{\left(6\times10^7\right)^2}-\frac{y^2}{\left(7.2\times10^7\right)^2}=1$

10.4 Exercises

1. $\left(4\sqrt{3}+3,-4+3\sqrt{3}\right)$

3. $\left(\frac{-5\sqrt{2}}{16},\frac{3\sqrt{2}}{16}\right)$

5. $(-1.2097,-13.5476)$

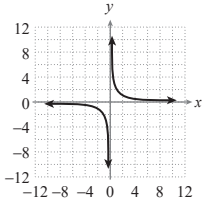
7. $(3.3485,-1.5002)$

9. Ellipse **11.** Hyperbola

13. Hyperbola **15.** Ellipse

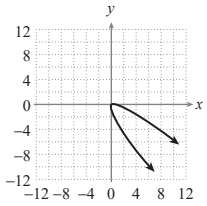
17. Hyperbola

$\theta=\frac{\pi}{4},x'^2-y'^2=4$



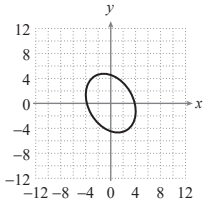
19. Parabola

$\theta=\frac{\pi}{4},y'=-\sqrt{2}x'^2$



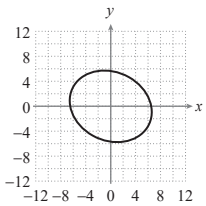
21. Ellipse

$\theta=\frac{\pi}{6},\frac{x'^2}{13}+\frac{y'^2}{25}=1$

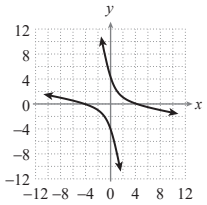


23. Ellipse

$\theta=\frac{\pi}{3},\frac{x'^2}{30}+\frac{y'^2}{46}=1$



25.



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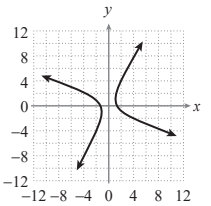

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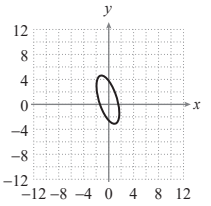
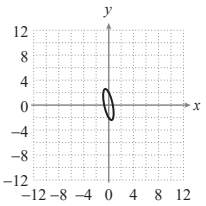


31.



33. c **35.** a

37. g **39.** e

41. The objective of the rotation of axes is to eliminate the $x'y'$-term. If your final equation contains an $x'y'$-term, you know that a mistake has occurred.

43. a. Use the rotation of axes procedure to obtain the equation $4x'^2+16y'^2-16=0$. Now we know $F=-16$ and $F'=-16$. We can plug these values in $F=F'$ and obtain $-16=-16$, which is true.

b. Use the rotation of axes procedure to obtain the equation $4x'^2+16y'^2-16=0$. Now we know $A=7$, $C=13$, $A'=4$, and $C'=16$. We can plug these values in $A+C=A'+C'$ and obtain $7+13=4+16$, or $20=20$, which is true.

c. Use the rotation of axes procedure to obtain the equation $4x'^2+16y'^2-16=0$. Now we know $A=7,B=-6\sqrt{3},C=13$, $A'=4,B'=0,$ and $C'=16$. We can plug these values in $B^2-4AC=B'^2-4A'C'$ and obtain

$$\left(-6\sqrt{3}\right)^2-4(7)(13)=(0)^2-4(4)(16)$$

or $-256=-256$, which is true.

10.5 Exercises

1. c **3.** f **5.** b

7. Hyperbola, $y=\frac{7}{6}$

9. Ellipse, $x=-3$

11. Hyperbola, $x = \frac{1}{3}$

13. Ellipse, $x = 5$

15. Hyperbola, $x = -\frac{6}{5}$

17. Parabola, $y = \frac{3}{2}$

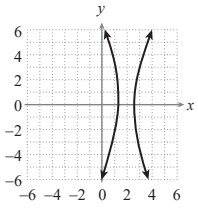
19. Hyperbola, $x = -\frac{4}{7}$

21. $r = \frac{2}{1-\cos\theta}$

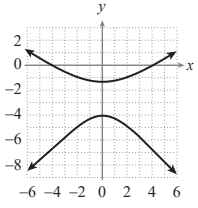
23. $r = \frac{3}{1-4\sin\theta}$

25. $r = \frac{3}{1+\frac{1}{4}\cos\theta}$

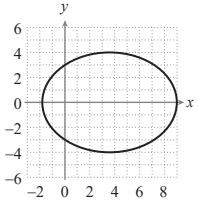
27.



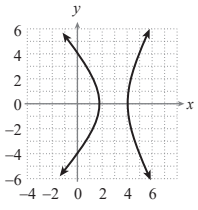
29.



31.



33.



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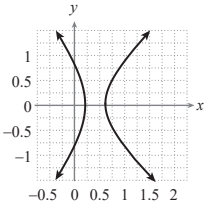

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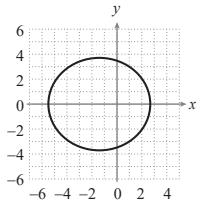
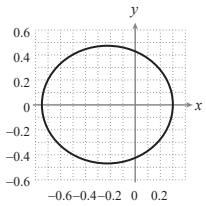
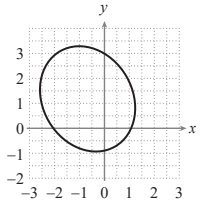


39.



41.



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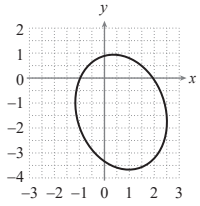
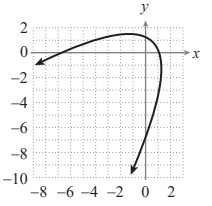


45.



Chapter 10 Project

1. $x^2 = -\frac{1125}{4}(y-80)$

3. $\frac{x^2}{22,500} + \frac{y^2}{6400} = 1, y \geq 0$

5. Answers may vary.
The semiellipse design, since it gives more space for ships to pass through.

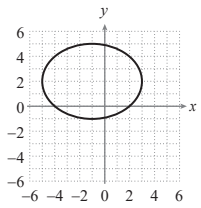
Chapter 10 Review Exercises

1. Center : $(3,-1)$

Vertices : $(7,-1),(-1,-1)$

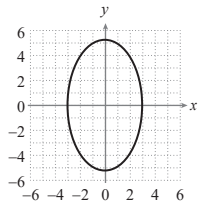
Foci : $\left(3 \pm 2\sqrt{3},-1\right)$

3.



$\left(-1 \pm \sqrt{7},2\right)$

5.



$\left(0,\pm 3\sqrt{2}\right)$

7. $\frac{(x+1)^2}{9} + \frac{(y-4)^2}{16} = 1$

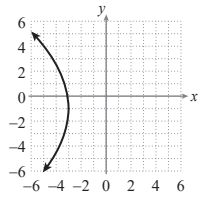
9. $\frac{4(x-2)^2}{9} + \frac{4(y+1)^2}{9} = 1$

11. $\frac{(x-2)^2}{16} + \frac{y^2}{12} = 1$

13. $\frac{(x+1)^2}{4} + \frac{(y-1)^2}{25} = 1$

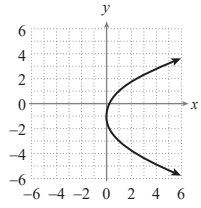
15. $30\pi a - \pi a^2$

17.



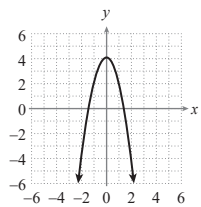
$(-6,-1), x = 0$

19.



$(1,-1),\ x = -1$

21.



$\left(0,\frac{31}{8}\right),\ y = \frac{33}{8}$

23. $(x+2)^2 = 4(y-3)$

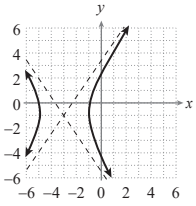
25. $(y+1)^2 = 2\left(x-\frac{5}{2}\right)$

27. $(y+1)^2 = 16(x-2)$

29. $(y+1)^2 = -16(x-5)$

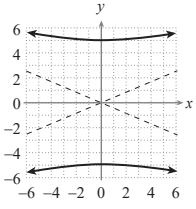
31. $\frac{5}{4}$ inches

33.



$\left(-3\pm\sqrt{13},-1\right)$

35.



$(0,\pm 13)$

37. Center: $(0,2)$
Foci: $\left(\pm 2\sqrt{10},2\right)$
Vertices: $(\pm 6,2)$

39. Center: $(3,3)$
Foci: $\left(3,3\pm\sqrt{53}\right)$
Vertices: $(3,1),(3,5)$

41. $\frac{(x+1)^2}{4} - \frac{(y+2)^2}{25} = 1$

43. $\frac{(y-2)^2}{9} - \frac{(x-2)^2}{1} = 1$

45. $\frac{(y-7)^2}{4} - \frac{(x+1)^2}{9} = 1$

47. $\frac{(y+2)^2}{9} - \frac{x^2}{4} = 1$

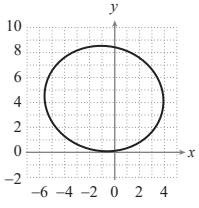
49. $\left(11+43\sqrt{3},-11\sqrt{3}+43\right)$

51. $\left(3+3\sqrt{3},9-\sqrt{3}\right)$

53. Ellipse,

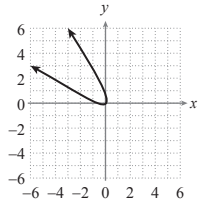
$\theta = \frac{\pi}{3}$,

$$\frac{\left(x'-\frac{25\sqrt{3}}{13}\right)^2}{\frac{25,000}{1521}} + \frac{\left(y'-\frac{25}{9}\right)^2}{\frac{25,000}{1053}} = 1$$



55. Parabola,

$\theta = \frac{\pi}{4}, y' = \sqrt{2}x'^2$



57. Parabola, $y = \frac{7}{4}$

59. Ellipse, $x = \frac{7}{2}$

61. $r = \frac{4}{1+\frac{1}{4}\cos\theta}$

63. $r = \frac{3}{1+9\cos\theta}$

Chapter 11: Systems of Equations and Inequalities

11.1 Exercises

1. $(-5,2)$ **3.** $(5,3)$

5. $\varnothing$

7. $\left\{\left(\frac{y-3}{2},y\right)\middle|\, y\in\mathbb{R}\right\}$

9. $(-1,7)$ **11.** $(3,11)$

13. $\{(x,4x+1)\,|\,x\in\mathbb{R}\}$

15. $(2,19)$ **17.** $(-5,1)$

19. $(5,6)$

21. $\{(-y-2,y)\,|\,y\in\mathbb{R}\}$

23. $(-5,4)$ **25.** $(-1,1)$

27. $(3,-5)$ **29.** $\varnothing$

31. $(-1,3,0)$ **33.** $(2,2,-1)$

35. $\left\{\left(\frac{y-z+2}{3},y,z\right)\middle|\, y\in\mathbb{R},z\in\mathbb{R}\right\}$

37. $\varnothing$ **39.** $(1,1,0)$

41. $(9,1,1)$ **43.** $(3,1,-2)$

45. $(4,5,5)$ **47.** $\left(\frac{49}{3},\frac{-16}{3},\frac{5}{4}\right)$

49. $(0,3,2)$

51. 22 pennies, 23 nickels

53. 25 people

55. Eliza is 15 years old.

57. 7 shirts and 4 pairs of shorts

59. 3 quarters, 11 dimes, and 28 pennies

61. Jim is 28 years old.

63. 3 thumb screws

65. Apples: \$0.78, Oranges: \$0.93, Mangoes: \$1.05

67. $(0.43,1.28,3.64)$

69. $(-3.42,2.98,2.76)$

71. $(6,8,7)$

11.2 Exercises

1. a. 3×2 **b.** -1 **c.** None

3. a. 5×2 **b.** None **c.** 10

5. a. 3×4 **b.** None **c.** 286

7. a. 3×2 **b.** 1 **c.** None

9. a. 2×5 **b.** 5 **c.** 2

11. $\left[\begin{array}{ccc|c} -3 & 1 & -2 & -4 \\ \frac{1}{2} & -4 & -1 & 1 \\ 0 & -3 & 3 & 1 \end{array}\right]$

13. $\left[\begin{array}{ccc|c} -\frac{3}{2} & -1 & 0 & -1 \\ 2 & 2 & 3 & 0 \\ 0 & -1 & 6 & 0 \end{array}\right]$

15. $\left[\begin{array}{ccc|c} \frac{12}{5} & \frac{1}{2} & -\frac{3}{2} & \frac{1}{5} \\ 1 & 0 & 3 & 1 \\ 5 & 2 & 1 & -2 \end{array}\right]$

17. $\left[\begin{array}{ccc|c} \frac{2}{3} & -\frac{4}{3} & -2 & 0 \\ 8 & -2 & 6 & 7 \\ 3 & -2 & 0 & 0 \end{array}\right]$

19. $\left[\begin{array}{ccc|c} \frac{1}{2} & -14 & -\frac{1}{4} & -8 \\ \frac{1}{5} & -\frac{7}{6} & \frac{1}{4} & -3 \\ 5 & -5 & \frac{8}{3} & -5 \end{array}\right]$

21. $\begin{cases} x = 8 \\ y = 3 \end{cases}$

23. $\begin{cases} x + 3y + 6z = 16 \\ y + 2z = 9 \\ z = 4 \end{cases}$

25. $\begin{cases} 9y + 13z = 27 \\ 2x + 21z = 19 \\ 7x + 18y = 32 \end{cases}$

27. $\left[\begin{array}{cc|c} 2 & -5 & 3 \\ 0 & -7 & 5 \end{array}\right]$

29. $\left[\begin{array}{cc|c} 1 & 3 & -2 \\ 9 & -2 & 7 \end{array}\right]$

31. $\left[\begin{array}{cc|c} 8 & -2 & -4 \\ -6 & 2 & -14 \end{array}\right]$

33. $\left[\begin{array}{cc|c} 4 & 12 & -6 \\ 9 & 9 & 6 \end{array}\right]$

35. $\left[\begin{array}{cc|c} 4 & -1 & 5 \\ -6 & 2 & 0 \end{array}\right]$

37. $\left[\begin{array}{ccc|c} 18 & -6 & 15 & 42 \\ -7 & 19 & 2 & 3 \\ -4.5 & 5.5 & -2 & 3.5 \end{array}\right]$

39. $\left[\begin{array}{ccc|c} 5 & 18 & 22 & 5 \\ 32 & -9 & -27 & -23 \\ -9 & 21 & 12 & 9 \end{array}\right]$

41. $\left[\begin{array}{ccc|c} 0 & 1 & -9 & -3 \\ 1 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$

43. $\left[\begin{array}{cc|c} -1 & 4 & -3 \\ 1 & -6 & \frac{5}{2} \end{array}\right]$

45. $\left[\begin{array}{ccc|c} 1 & 5 & -9 & 11 \\ 0 & -1 & 8 & -7 \\ 0 & -17 & 41 & 1 \end{array}\right]$

47. Neither **49.** Neither

51. Neither **53.** $(3,-1)$

55. $(1,3)$ **57.** $(-7,3)$

59. $\varnothing$ **61.** $(3,2)$

63. $\{(-2y-4, y) \mid y \in \mathbb{R}\}$

65. $\varnothing$ **67.** $(4,0,3)$

69. $(15,-21,8)$ **71.** $(3,-5)$

73. $\{(x,-3x-2) \mid x \in \mathbb{R}\}$

75. $(-4,1)$ **77.** $(6,4)$

79. $(-11,-5)$ **81.** $(7,3,3)$

83. $(2,2,-1)$ **85.** $\{(1,y,0) \mid y \in \mathbb{R}\}$

87. $(3,-2,3)$ **89.** $(2,3,4)$

91. $(9,-19,7)$ **93.** $(1,-2,-1,3)$

95. 42, 26, 87

97. Small: 10, Medium: 24, Large: 48

11.3 Exercises

1. 11 **3.** 15

5. $ab - x^2$ **7.** 8

9. -10 **11.** -39

13. $\{-2,3\}$ **15.** $\{-5,1\}$

17. $\{-5,-4\}$ **19.** $\{-6,4\}$

21. $\{2,5\}$ **23.** 3

25. -9 **27.** 2

29. -2 **31.** 159

33. 78 **35.** -254

36. -84 **37.** 404

39. 4 **41.** 120

43. 10 **45.** x^4

47. x^8 **49.** $(76,-53)$

51. $\{(-y-2, y) \mid y \in \mathbb{R}\}$

53. $\varnothing$ **54.** $(3,-2,3)$

55. $\{(-3z-5,-6z-10,z) \mid z \in \mathbb{R}\}$

57. $\left\{\left(\frac{-5y-z-5}{2}, \frac{-5y+3z-19}{2}, y, z\right) \middle| \begin{array}{l} y \in \mathbb{R}, \\ z \in \mathbb{R} \end{array}\right\}$

59. $\left\{\begin{pmatrix} -z+8, \\ -5z+31, \\ -2z+37, z \end{pmatrix} \middle| z \in \mathbb{R}\right\}$

61. $(1647, 2071)$

63. $\varnothing$

65. $(-3,-1,0,-4)$

67. Candy bars: 5, Ice cream: 6

69. 0.012 **71.** 0.564

73. 1194 **75.** $(1,-1,2)$

77. $(2,1,0,3)$

11.4 Exercises

1. $\begin{bmatrix} 5 & -1 \\ 0 & 0 \\ 2 & 13 \end{bmatrix}$ **3.** $\begin{bmatrix} 6 & -3 \\ 18 & 30 \\ -9 & 21 \end{bmatrix}$

5. Not possible **7.** $\begin{bmatrix} 14 & -14 \\ 8 & 0 \\ -4 & 14 \end{bmatrix}$

9. $\begin{bmatrix} -7 & 5 \\ 3 & 10 \\ -3 & -8 \end{bmatrix}$ **11.** Not possible

13. $a = 3, b = -1, c = 10$

15. $a = 2, b = -2, c = -1$

17. Not possible **19.** $x = 10, y = 5$

21. $x = 3, y = 1$ **23.** Not possible

25. $a = 8, b = 5$ **27.** $\begin{bmatrix} 24 & -5 \end{bmatrix}$

29. $\begin{bmatrix}35 & 18\end{bmatrix}$ **31.** Not possible

33. $\begin{bmatrix}-30 & -3\end{bmatrix}$

35. $\begin{bmatrix}15 & -3 & -24\\25 & -5 & -40\\30 & -6 & -48\end{bmatrix}$

37. $\begin{bmatrix}-34 & -7\end{bmatrix}$

39. $\begin{bmatrix}11 & 0\\0 & 11\end{bmatrix}$

41. $\begin{bmatrix}32 & -20\\56 & -35\\-16 & 10\end{bmatrix}$

43. Not possible

45. $\begin{bmatrix}14 & -13\\-13 & 5\end{bmatrix}$

47. $\begin{bmatrix}179 & 76\end{bmatrix}$

49. $\frac{2}{3}$ for store A; $\frac{1}{3}$ for store B

51. Solution is incorrect.
Explanations may vary.

53. $\begin{bmatrix}23.94 & -7.56 & 28.98\\21.66 & -6.84 & 26.22\end{bmatrix}$

55. $\begin{bmatrix}-23.94 & -26.72\end{bmatrix}$

57. $\begin{bmatrix}-79.59\\39.21\\10.08\end{bmatrix}$

11.5 Exercises

1. $\begin{bmatrix}14 & -5\\1 & 9\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}7\\2\end{bmatrix}$

3. $\begin{bmatrix}1 & 2\\9 & -3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-6\\-14\end{bmatrix}$

5. $\begin{bmatrix}3 & -7 & 1\\1 & -1 & 0\\0 & 8 & 5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}-4\\2\\-3\end{bmatrix}$

7. $\begin{bmatrix}\frac{3}{5} & -\frac{8}{5}\\0 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\\2\end{bmatrix}$

9. $\begin{bmatrix}4 & -3\\2 & -4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-9\\13\end{bmatrix}$

11. $\begin{bmatrix}2 & -1 & 3\\-1 & 1 & 0\\4 & -5 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\17\\-2\end{bmatrix}$

13. $\begin{bmatrix}-\frac{1}{20} & -\frac{1}{5}\\\frac{1}{4} & 0\end{bmatrix}$

15. $\begin{bmatrix}-5 & -4\\4 & 3\end{bmatrix}$

17. $\begin{bmatrix}-5 & 0\\2 & 2\end{bmatrix}$

19. Not invertible

21. $\begin{bmatrix}2 & 1 & -4\\-4 & -2 & -3\\-1 & -1 & -4\end{bmatrix}$

23. $\begin{bmatrix}-1 & 2 & -1\\0 & -1 & 1\\0 & -4 & 3\end{bmatrix}$

25. $\begin{bmatrix}-1 & -2 & 1\\-2 & 1 & -3\\1 & 2 & 0\end{bmatrix}$

27. $\begin{bmatrix}-2 & 1 & 1\\2 & 0 & -1\\-1 & 0 & 1\end{bmatrix}$

29. $\begin{bmatrix}2 & -1 & 2\\0 & 1 & -1\\-3 & -2 & -4\end{bmatrix}$

31. No **33.** Yes

35. No

37. $\left(-2, -\frac{5}{2}\right)$

39. $\left\{\left(\frac{3y-1}{2}, y\right)\middle| y\in\mathbb{R}\right\}$

41. $(-2,0)$ **43.** $(8,-19)$

45. $(0,5)$ **47.** $(-4,5,-1)$

49. $(-13,19,23)$; $(0,0,-1)$; $(1,-1,-1)$

51. $(1,-8,7)$; $(3,1,1)$; $(4,2,0)$

53. $\begin{bmatrix}\frac{-1}{4} & \frac{1}{10}\\\frac{-1}{4} & \frac{3}{10}\end{bmatrix}$

55. $\begin{bmatrix}0.053 & -0.258\\0.113 & 0.076\end{bmatrix}$

57. $\begin{bmatrix}0.004 & -0.003 & 0.009\\0 & 0.020 & 0.029\\0.012 & 0.014 & 0.013\end{bmatrix}$

11.6 Exercises

1. $\frac{A_1}{x-3}+\frac{A_2}{x+3}$

3. $\frac{A_1}{x+3}+\frac{A_2}{x+4}+\frac{A_3}{(x+4)^2}$

5. $\frac{A_1}{x+3}+\frac{A_2}{x-2}+\frac{A_3}{x+2}$

7. d **9.** h

11. a **13.** c

15. $-\frac{1}{x}+\frac{2}{x-2}+\frac{2}{x+2}$

17. $\frac{5}{72(x-2)}-\frac{1}{32x}-\frac{7}{288(x+4)}-\frac{x+17}{72(x^2+8)}$

19. $\frac{10}{x-4}-\frac{5}{x-2}$

21. $\frac{3}{2(x+3)}+\frac{3}{14(x-1)}-\frac{12}{7(x+6)}$

23. $\frac{1}{2(x-1)}-\frac{1}{2(x+1)}$

25. $\frac{1}{48(x-2)}-\frac{x}{80(x^2+4)}-\frac{3}{40(x^2+4)}-\frac{1}{120(x+4)}$

27. $\frac{5}{4(x-2)}-\frac{1}{4(x+2)}$

29. $\frac{1}{16(x+2)}+\frac{1}{16(x-2)}-\frac{x}{8(x^2+4)}$

31. $\frac{1}{24(x+6)}+\frac{1}{30(x+3)}-\frac{7}{40(x-2)}+\frac{1}{6(x-3)}$

33. $\frac{15}{16(x+3)}+\frac{1}{16(x-1)}-\frac{9}{4(x+3)^2}$

35. $\dfrac{1}{x+3}+\dfrac{1}{x-3}$

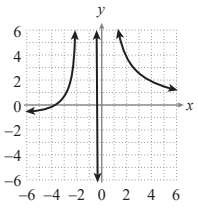
37. $\dfrac{1}{4(x-2)}+\dfrac{3}{4(x+6)}$

39. $\dfrac{1}{a}\left(\dfrac{1}{x}-\dfrac{1}{x+a}\right)$

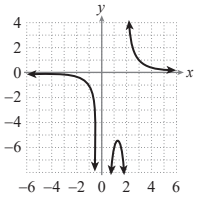
41. $\dfrac{1}{2a}\left(\dfrac{1}{a+x}+\dfrac{1}{a-x}\right)$

43. $\dfrac{1}{a-1}\left(\dfrac{1}{x+1}+\dfrac{1}{x+a}\right)$

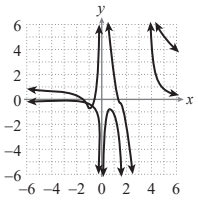
45. True



47. True

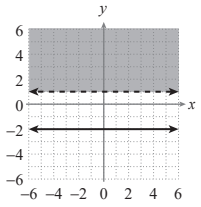


49. False

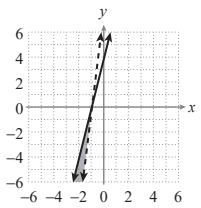


11.7 Exercises

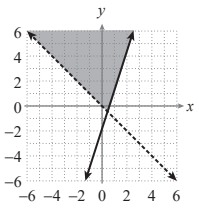
1.



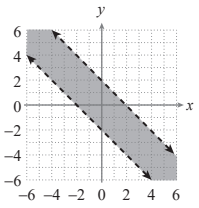
3.



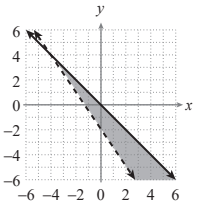
5.



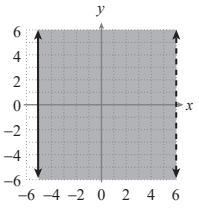
7.



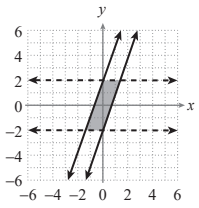
9.



11.



13.



15. Min $= 0$ at $(0,0)$;
Max $= 12$ at $(3,0)$

17. Min $= 0$ at $(0,0)$;
Max $= 12$ at $(0,3)$

19. Min $= 100$ at $\left(\dfrac{10}{3},10\right)$;
Max $= 250$ at $(20,10)$

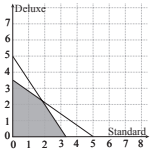
21. Min $= \dfrac{150}{7}$ at $\left(\dfrac{50}{7},0\right)$;
Max $= 66$ at $(8,6)$

23. Min $= 0$ at $(0,0)$;
Max $= \dfrac{165}{8}$ at $\left(\dfrac{111}{8},\dfrac{27}{8}\right)$

25. Min $= 210$ at $(14,0)$;
Max $= 600$ at $(0,20)$

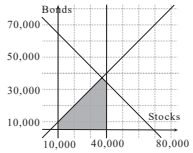
27. Min $= \dfrac{680}{7}$ at $\left(\dfrac{88}{7},\dfrac{-36}{7}\right)$;
Max $= \dfrac{1980}{7}$ at $\left(\dfrac{108}{7},\dfrac{-6}{7}\right)$

29.



$40x+60y \le 200$;
$8x+5y \le 25$;
$x \ge 0$; $y \ge 0$

31.



$x+y \le 75{,}000$;
$x-y \ge 0$;
$x \le 40{,}000$;
$x \ge 10{,}000$;
$y \ge 5000$

33. Type X: 75 units;
Type Y: 100 units;
Maximum profit: \$712.50

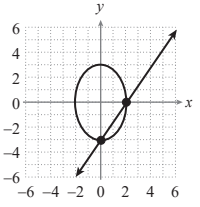
35. The volunteer could choose any of the following points: $(9,9),(11,6),(13,3),(15,0)$. In each of these points, the first coordinate represents the number of packages from Company A and the second coordinate represents the number of packages from Company B. The minimum cost is \$22.50.

37. **a.** 120 flip phones;
80 smart phones;
Minimum cost: \$1160

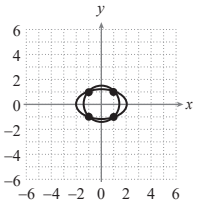
b. 100 flip phones;
170 smart phones;
Maximum profit: \$650

11.8 Exercises

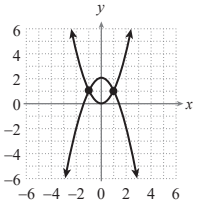
1. $\{(0,-3),(2,0)\}$



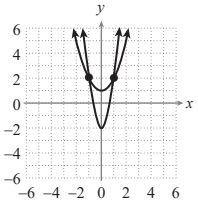
3. $\left\{\begin{matrix}(-1,-1),(1,1)\\(-1,1),(1,-1)\end{matrix}\right\}$



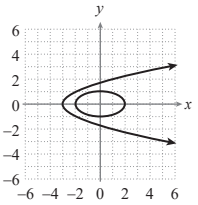
5. $\{(-1,1),(1,1)\}$



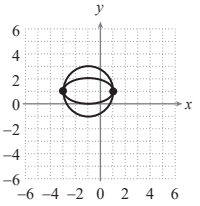
7. $\{(-1,2),(1,2)\}$



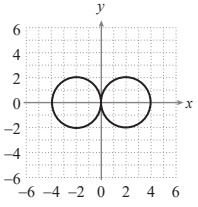
9. No solution



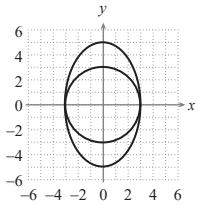
11. $\{(-3,1),(1,1)\}$



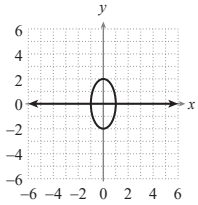
13. $\{(0,0)\}$



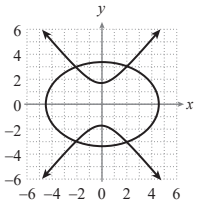
15. $\{(\pm 3,0)\}$



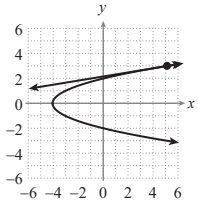
17. $\{(\pm 1,0)\}$



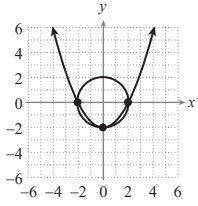
19. $\{(2,\pm 3),(-2,\pm 3)\}$



21. $\{(5,3)\}$



23. $\{(\pm 2,0),(0,-2)\}$



25. $\left\{\begin{matrix}\left(-i\sqrt{6},-6\right),\left(i\sqrt{6},-6\right),\\\left(-\sqrt{5},5\right),\left(\sqrt{5},5\right)\end{matrix}\right\}$

27. $\{(-2,3)\}$

29. $\left\{\begin{matrix}(-1,-4),(1,4),\left(-2\sqrt{2},-\sqrt{2}\right),\\\left(2\sqrt{2},\sqrt{2}\right)\end{matrix}\right\}$

31. $\{(-3i,-5),(3i,-5),(0,4)\}$

33. $\left\{\left(-\sqrt{3},0\right),\left(\sqrt{3},0\right)\right\}$

35. $\left\{(-i,-6),(i,-6),\left(-\sqrt{2},3\right),\left(\sqrt{2},3\right)\right\}$

37. $\left\{\begin{matrix}(1,1),\left(-\frac{3}{2},\frac{\sqrt{14}}{2}\right),\\(1,-1),\left(-\frac{3}{2},-\frac{\sqrt{14}}{2}\right)\end{matrix}\right\}$

39. $\left\{\left(\frac{7}{2},\frac{17}{4}\right)\right\}$

41. $\left\{\begin{matrix}\left(-6,-i\sqrt{15}\right),(2,1),\\\left(-6,i\sqrt{15}\right),(2,-1)\end{matrix}\right\}$

43. $\left\{(0,1),\left(-\frac{100}{101},-\frac{99}{101}\right)\right\}$

45. $\left\{\begin{matrix}\left(-\frac{\sqrt{42}}{6},-\frac{\sqrt{66}}{6}\right),\\\left(-\frac{\sqrt{42}}{6},\frac{\sqrt{66}}{6}\right),\\\left(\frac{\sqrt{42}}{6},-\frac{\sqrt{66}}{6}\right),\\\left(\frac{\sqrt{42}}{6},\frac{\sqrt{66}}{6}\right)\end{matrix}\right\}$

47. $\{(-5,0),(-2,0),(-1,0)\}$

49. $\left\{\begin{matrix}(3,2),(2,3),\\\left(\frac{-1\pm i\sqrt{23}}{2},\frac{-1\pm i\sqrt{23}}{2}\right)\end{matrix}\right\}$

51. $\left\{(0,-1),\left(\frac{\sqrt{6}}{3},\frac{2\sqrt{6}}{9}-1\right)\right\}$

53. $\left\{\begin{matrix}\left(\sqrt{2}+1,2\sqrt{2}\right),\left(-\sqrt{2}+1,-2\sqrt{2}\right),\\\left(2\sqrt{2}+1,\sqrt{2}\right),\left(-2\sqrt{2}+1,-\sqrt{2}\right)\end{matrix}\right\}$

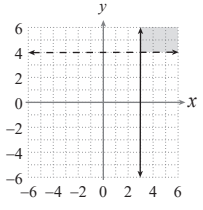
55. $\left\{(4,1),\left(1,1+i\sqrt{3}\right)\right\}$

57. $\{(0,4),(0,-3),(3,-2),(3,3)\}$

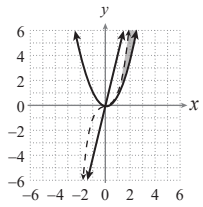
59. $\{(4,0)\}$

61. $\left\{\left(\pm\frac{1}{2},\pm\frac{1}{4}\right)\right\}$

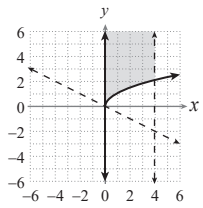
63. b



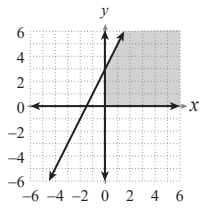
65. b, d



67. a, d



69.



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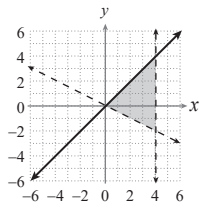

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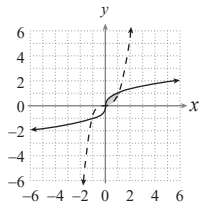
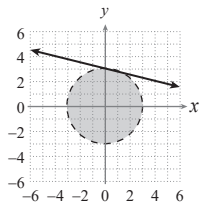


75.



77. 9 inches by 5 inches

79. 60 mph and 70 mph

81. -12 and 7

83. $h = 6$ cm, $r = 3$ cm

Chapter 11 Project

1.

The number of months passed	Shop Name		
	Joe's Java	Buck's Café	Tweak's Coffee
1	26.85%	43.45%	29.70%
2	28.47%	42.10%	29.43%
3	29.88%	40.92%	29.20%
4	31.12%	39.89%	28.98%
5	32.21%	39.00%	28.80%
6	33.16%	38.21%	28.63%

3. As our market share increases, the shares of both Buck's Café and Tweak's Coffee decrease. As such, they will have to adjust their marketing strategies to stop the decrease and potentially increase their shares.

Chapter 11 Review Exercises

1. $\varnothing$ **3.** $(3,0,2)$

5. $\varnothing$

7. $\{(3-3y, y) \mid y \in \mathbb{R}\}$

9. $(3,2)$ **11.** $(8,12,10)$

13. $y = \frac{4}{15}x^2 - x + \frac{11}{15}$

15. a. 1×4 **b.** 8 **c.** None

17. $\left[\begin{array}{ccc|c} 4 & 5 & -1 & 0 \\ 1 & 3 & 2 & 3 \\ 10 & -1 & -6 & 0 \end{array}\right]$

19. $\begin{cases} 8x + 7z = 5 \\ -3y + 4z = 16 \\ 16x - 2y + z = 2 \end{cases}$

21. $\left[\begin{array}{cc|c} 0 & -5 & -11 \\ 1 & 2 & 3 \end{array}\right]$

23. $\left[\begin{array}{cc|c} 1 & -4 & -4 \\ 1 & 7 & 11 \end{array}\right]$

25. $(2,-1)$ **27.** $(3,-5)$

29. $2x^4$ **31.** 7

33. 9, -9 **35.** $(-4,1)$

37. $\varnothing$

39. $\begin{bmatrix} 4 & -16 & 4 \\ -5 & 8 & 12 \end{bmatrix}$

41. Not possible **43.** $\begin{bmatrix} 9 & -23 & 3 \\ 5 & -3 & 8 \end{bmatrix}$

45. $w = -2, x = 1, y = 3, z = -4$

47. $x = 2, y = -3$

49. $\begin{bmatrix} 12 & 46 \end{bmatrix}$

51. $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -3 & -1 \\ -3 & 0 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \\ 5 \end{bmatrix}$

53. $\begin{bmatrix} \frac{3}{16} & \frac{1}{8} \\ -\frac{1}{8} & \frac{1}{4} \end{bmatrix}$

55. Not possible

57. No **59.** Yes

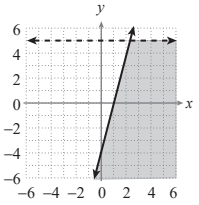
61. $\left(1, -\frac{1}{3}\right)$

63. $(-15,20,-33), (-2,3,-4), (-9,11,-17)$

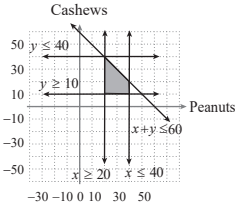
65. $\frac{A_1}{x+4} + \frac{A_2}{x-1}$

67. $-\frac{3}{2(2x-5)^2} + \frac{1}{2(2x-5)}$

69.



71. $x \geq 20$, $y \geq 10$, $x \leq 40$, $y \leq 40$, $x + y \leq 60$

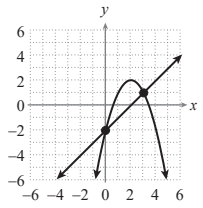


73. Min = 0 at $(0,0)$
Max = 30 at $(5,0)$

75. Min = 8 at $(0,2)$
Max = 24 at $\left(\frac{24}{7},\frac{12}{7}\right)$

77. 12 vases should be produced,
12 pitchers should be produced,
Max profit: \$660

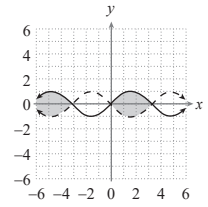
79. $\{(0,-2),(3,1)\}$



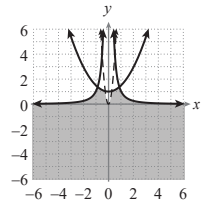
81. $\left\{(i,-1),(-i,-1),\left(\frac{\sqrt{2}}{2},\frac{1}{2}\right),\left(-\frac{\sqrt{2}}{2},\frac{1}{2}\right)\right\}$

83. $\{(2,1)\}$

85.



87.



89. 36 mph and 24 mph

Chapter 12: Sequences, Series, Combinatorics, and Probability

12.1 Exercises

1. Infinite

3. Finite **5.** Finite

7. Infinite **9.** Infinite

11. $2,-1,-4,-7,-10$

13. $1,\frac{3}{2},\frac{9}{5},2,\frac{15}{7}$

15. $\frac{2}{3},-\frac{4}{9},\frac{8}{27},-\frac{16}{81},\frac{32}{243}$

17. $\frac{1}{2},\frac{4}{3},\frac{9}{4},\frac{16}{5},\frac{25}{6}$

19. $-1,3,-6,10,-15$

21. $26,7,0,-1,-2$

23. $-1,\sqrt{2},-\sqrt{3},2,-\sqrt{5}$

25. $1,5,9,13,17$

27. $\frac{1}{2},1,2,4,8$

29. $3,36,729,20,736,759,375$

31. $\frac{5}{4},2,\frac{5}{2},\frac{20}{7},\frac{25}{8}$

33. $1,3,6,10,15$

35. Undefined, 9, 4, $\frac{25}{9},\frac{9}{4}$

37. $\frac{1}{3},\frac{1}{2},\frac{5}{9},\frac{7}{12},\frac{3}{5}$

39. $0,-1,-4,-9,-16$

41. $2,4,16,256,65,536$

43. $1,2,6,24,120$

45. $2,\sqrt{5},\sqrt{6},\sqrt{7},2\sqrt{2}$

47. 1, 0, −27, 0, 125

49. $a_n = 7n-2$

51. $a_1 = -1, a_n = -na_{n-1}, n \geq 2$

53. $a_n = \left(\frac{1}{n}\right)^2$

55. $a_n = 9n-43$

57. $a_n = 2^{n-3}$

59. $a_n = \frac{n}{2^n}$

61. $-2+1+4+7+10+13+16 = 49$

63. $\sum_{i=1}^{6} i^3 = 441$

65. $45+80+125+180+245$
$+320+405+500 = 1900$

67. $-6-12-24-48-96-192$
$= -378$

69. $\sum_{i=2}^{9} 3^i = 29,520$

71. $S_n = \frac{n}{4(n+4)}, S = \frac{1}{4}$

73. $S_n = 2^n - 1, S_{15} = 32,767$

75. $S_n = \frac{n}{2(n+1)}, S = \frac{1}{2}$

77. $S_n = -\ln(n+1)$, series diverges

79. $S_n = \frac{3n}{4(3n+4)}, S = \frac{1}{4}$

81. $4,7,11,18,29$

83. $10,20,30,50,80$

85. $13,-17,-4,-21,-25$

87. $1,-3,-3,9,-27$

89. 987

91. 1, 2, 4, 8, 16, 32, 64, 128

12.2 Exercises

1. $a_n = 3n-5$

3. $a_n = -2n+9$

5. $a_n = 9n-4$

7. $a_n = -6n+9$

9. $a_n = 19n+5$

11. $a_n = n+\frac{5}{2}$

13. $a_n = -\frac{19}{2}n+\frac{43}{2}$

15. $a_n = -2n+1$

17. $a_n = -4n+33$

19. $d = 2$ **21.** $d = 1$ **23.** No

25. No **27.** 13 **29.** 2

31. 2

33. $d = 2.5$; 5, 7.5, 10, 12.5, 15

35. $d = 7$; 7, 14, 21, 28, 35

37. $d = 9$; −62, −53, −44, −35, −26

39. $d = 5$ **41.** $d = 1$ **43.** $d = -2$

45. 195 **47.** a_{73} **49.** 117

51. 26 **53.** -8 **55.** 13

57. 55 **59.** 14,350

61. 17,114 **63.** -1475

65. $-\frac{3219}{5}$ **67.** 902

69. -1316 **71.** 6 years

73. 1620 pounds

75. \$625; \$8100

77. 266.4

79. -152

81. 10,382.05

12.3 Exercises

1. $a_n = -3(2)^{n-1}$

3. $a_n = 2\left(-\frac{1}{3}\right)^{n-1}$

5. $a_n = \left(-\frac{1}{4}\right)^{n-1}$

7. $a_n = \left(\frac{1}{7}\right)^{n-1}$

9. $a_n = (-3)^{n-1}$

11. $a_n = 3\left(\frac{2}{3}\right)^{n-1}$

13. $a_n = 7(-2)^{n-1}$

15. $a_n = \frac{1}{16}(2)^{n-1}$

17. $a_n = \frac{39}{68}\left(\frac{4}{3}\right)^{n-1}$

19. No **21.** $r = \frac{1}{2}$ **23.** $r = 2$

25. $r = 7$

27. $r = 3$; 8, 24, 72, 216, 648

29. $r = 2$; $\frac{1}{4}, \frac{1}{2}, 1, 2, 4$

31. $r = \frac{1}{5}$;
62,500, 12,500, 2500, 500, 100

33. $\frac{5}{16,384}$

35. $-2,147,483,648$

37. $r = \pm 2$ **39.** $r = \pm\frac{1}{5}$

41. $\frac{52,222,139,775}{1,048,576} \approx 49,802.9$

43. $\frac{10,923}{16,384} \approx 0.666687$

45. $\frac{73,810}{19,683} \approx 3.749936$

47. $-109,200$

49. $-\frac{3}{2}$ **51.** Series diverges

53. $\frac{2,476,099}{160,000} \approx 15.475619$

55. Series diverges

57. $\frac{28,561}{152,064} \approx 0.187822$

59. $\frac{123}{999}$ **61.** $-\frac{35}{9}$ **63.** $\frac{989}{99}$

65. \$14,802.44 **67.** 1.845×10^{19}

69. Approximately 13,778 students

71. $S_{30} = 1.1 \times 10^{30}$; $r = 10$

73. Yes; explanations will vary (any example such that $r = 1$ and $d = 0$).

75. 9.98

77. 2.137×10^{-18}

79. 6.54×10^{51}

12.4 Exercises

1. $S_{k+1} = \frac{1}{3k+9}$

3. $S_{k+1} = \frac{(k+1)(k+2)(2k+3)}{4}$

5. Basic Step:
$n = 1, 1 = 1$ and $\frac{1(1+1)}{2} = 1$;
Induction Step:
If $1+2+3+\cdots+k = \frac{k(k+1)}{2}$,
then $(1+2+3+\cdots+k)+(k+1)$
$= \frac{k(k+1)}{2} + (k+1)$
$= \frac{k^2+k+2k+2}{2}$
$= \frac{(k+1)(k+2)}{2}$

7. Basic Step:
$n = 1, 2(1) = 2$ and $1(1+1) = 2$;
Induction Step:
If $2+4+6+\cdots+2k = k(k+1)$,
then
$(2+4+6+\cdots+2k)+2(k+1)$
$= k^2+k+2k+2$
$= (k+1)(k+2)$

9. Basic Step:
$n = 1, 4^{1-1} = 1$ and $\frac{4^1-1}{3} = 1$;
Induction Step:
If $4^0+4^1+4^2+\cdots+4^{k-1} = \frac{4^k-1}{3}$,
then $4^0+4^1+4^2+\cdots+4^{k-1}+4^{k+1-1}$
$= \frac{4^k-1}{3} + 4^k = \frac{4^k-1+3\cdot 4^k}{3}$
$= \frac{4\cdot 4^k-1}{3} = \frac{4^{k+1}-1}{3}$

11. Basic Step:
$n = 1, \frac{1}{(3(1)-2)(3(1)+1)} = \frac{1}{4}$
and $\frac{1}{3(1)+1} = \frac{1}{4}$;
Induction Step:
If $\frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \frac{1}{7\cdot 10}$
$+\cdots+\frac{1}{(3k-2)(3k+1)} = \frac{k}{3k+1}$,
then $\frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \frac{1}{7\cdot 10}$
$+\cdots+\frac{1}{(3k-2)(3k+1)}$
$+\frac{1}{(3(k+1)-2)(3(k+1)+1)}$
$= \left[\begin{array}{l}\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\frac{1}{7\cdot 10} \\ +\cdots+\frac{1}{(3k-2)(3k+1)}\end{array}\right]$
$+\frac{1}{(3k+1)(3k+4)}$
$= \frac{k}{3k+1} + \frac{1}{(3k+1)(3k+4)}$
$= \frac{3k^2+4k+1}{(3k+1)(3k+4)}$
$= \frac{(3k+1)(k+1)}{(3k+1)(3k+4)} = \frac{(k+1)}{(3(k+1)+1)}$

13.

Basic Step:

$n=1, 5(1)=5$ and $\dfrac{5(1)(1+1)}{2}=5;$

Induction Step:

If $5+10+15+\cdots+5k=\dfrac{5k(k+1)}{2},$

then $5+10+15+\cdots+5k+5(k+1)$

$=(5+10+15+\cdots+5k)+5k+5$

$=\dfrac{5k(k+1)}{2}+5k+5$

$=\dfrac{5k^2+15k+10}{2}$

$=\dfrac{5(k+1)(k+2)}{2}$

$=\dfrac{5(k+1)[(k+1)+1]}{2}$

15.

Basic Step:

$n=1,\ 1+\dfrac{1}{1}=2$ and $1+1=2$

Induction Step:

If $\left(1+\dfrac{1}{1}\right)\left(1+\dfrac{1}{2}\right)\left(1+\dfrac{1}{3}\right)\cdots\left(1+\dfrac{1}{k}\right)=k+1,$

then

$\left(1+\dfrac{1}{1}\right)\left(1+\dfrac{1}{2}\right)\left(1+\dfrac{1}{3}\right)\cdots\left(1+\dfrac{1}{k}\right)\left(1+\dfrac{1}{k+1}\right)$

$=(k+1)\left(1+\dfrac{1}{k+1}\right)$

$=k+1+\dfrac{k+1}{k+1}=(k+1)+1$

17.

Basic Step:

$n=1, 3(1)-2=1$ and $\dfrac{1}{2}(3(1)-1)=1;$

Induction Step:

If $1+4+7+10+\cdots+(3k-2)=\dfrac{k}{2}(3k-1),$

then

$[1+4+7+10+\cdots+(3k-2)]$

$+[3(k+1)-2]$

$=\dfrac{k}{2}(3k-1)+(3k+1)$

$=\dfrac{k(3k-1)+2(3k+1)}{2}$

$=\dfrac{3k^2+5k+2}{2}$

$=\dfrac{(k+1)(3k+2)}{2}$

$=\dfrac{k+1}{2}(3(k+1)-1)$

19. Basic Step:

$n=2, 3^2=9$ and $2(2)+1=5,$

so $3^2>2(2)+1;$

Induction Step:

If $3^k>2k+1,$ then

$3^{k+1}=3^1\cdot 3^k>3(2k+1)$

$=6k+3>2k+3$

$=2k+2+1=2(k+1)+1$

21.

Basic Step:

$n=1, 1^3=1$ and $\dfrac{1^2(1+1)^2}{4}=1;$

Induction Step:

If $1^3+2^3+3^3+4^3+\cdots+k^3=\dfrac{k^2(k+1)^2}{4}$

then

$(1^3+2^3+3^3+4^3+\cdots+k^3)+(k+1)^3$

$=\dfrac{k^2(k+1)^2}{4}+(k+1)^3$

$=\dfrac{k^2(k+1)^2+4(k+1)^3}{4}$

$=\dfrac{(k+1)^2(k+2)^2}{4}=\dfrac{(k+1)^2((k+1)+1)^2}{4}$

23. Basic Step:

$n=1, a^1=a$ so $a^1>1,$ when $a>1;$

Induction Step:

If $a^k>1,$ then

$a^{k+1}=a^k\cdot a^1>1\cdot a=a>1$

25.

Basic Step:

$n=1, 1^4=1$ and

$\dfrac{1(1+1)(2(1)+1)(3(1)^2+3(1)-1)}{30}=1;$

Induction Step:

If $1^4+2^4+3^4+\cdots+k^4$

$=\dfrac{k(k+1)(2k+1)(3k^2+3k-1)}{30},$

then

$(1^4+2^4+3^4+\cdots+k^4)+(k+1)^4$

$=\dfrac{k(k+1)(2k+1)(3k^2+3k-1)}{30}+(k+1)^4$

$=\dfrac{6k^5+45k^4+130k^3+180k^2+119k+30}{30}$

$=\dfrac{(k+1)(k+2)(2k+3)(3k^2+9k+5)}{30}$

$=\dfrac{\left[\begin{array}{l}(k+1)(k+2)(2(k+1)+1)\\ \times(3(k+1)^2+3(k+1)-1)\end{array}\right]}{30}$

27.

Basic Step:

$n=2,\ \dfrac{1}{\sqrt{1}}+\dfrac{1}{\sqrt{2}}=1+\dfrac{\sqrt{2}}{2}$

and $1+\dfrac{\sqrt{2}}{2}>\sqrt{2};$

Induction Step:

If $\dfrac{1}{\sqrt{1}}+\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{3}}+\cdots+\dfrac{1}{\sqrt{k}}>\sqrt{k},$

then

$\left[\dfrac{1}{\sqrt{1}}+\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{3}}+\cdots+\dfrac{1}{\sqrt{k}}\right]+\dfrac{1}{\sqrt{k+1}}$

$>\sqrt{k}+\dfrac{1}{\sqrt{k+1}}=\dfrac{\sqrt{k}\left(\sqrt{k+1}\right)+1}{\sqrt{k+1}}$

$=\dfrac{\sqrt{k^2+k+1}}{\sqrt{k+1}}>\dfrac{\sqrt{k^2+1}}{\sqrt{k+1}}$

$=\dfrac{k+1}{\sqrt{k+1}}=\sqrt{k+1}$

29. Basic Step:

$n=1, (ab)^1=ab$ and $a^1b^1=ab;$

Induction Step:

If $(ab)^k=a^kb^k,$ then

$(ab)^{k+1}=(ab)^k\cdot(ab)$

$=a^kb^k\cdot ab=(a\cdot a^k)(b\cdot b^k)$

$=a^{k+1}b^{k+1}$

31.

Basic Step:

$n=1, \ln(x_1)=\ln x_1;$

Induction Step:

If $\ln(x_1\cdot x_2\cdot x_3\cdot\cdots\cdot x_k)$

$=\ln x_1+\ln x_2+\ln x_3+\cdots+\ln x_k$

when $x_1>0, x_2>0, \ldots, x_n>0,$

then $\ln(x_1\cdot x_2\cdot x_3\cdot\cdots\cdot x_k\cdot x_{k+1})$

$=\ln(x_1\cdot x_2\cdot x_3\cdot\cdots\cdot x_k)+\ln(x_{k+1})$

$=(\ln x_1+\ln x_2+\ln x_3+\cdots+\ln x_k)+\ln x_{k+1}$

33.

Basic Step: $n=2,$

$(9^2-8(2)-1)=64$ of which 64 is a factor;

Induction Step:

If $(9^k-8k-1)=64p$ for some integer $p,$

then $(9^{k+1}-8(k+1)-1)$

$=9\cdot 9^k-8k-9$

$=9\cdot 9^k-9\cdot 8k+8\cdot 8k-9$

$=9(9^k-8k-1)+64k$

$=9(64p)+64k=64(9p+k)$

35.
Basic Step: $n = 1$,
$(1^3 - 1 + 3) = 3$, which is divisible by 3;
Induction Step:
If $\frac{k^3 - k + 3}{3} = p$
or $k^3 - k + 3 = 3p$ for some integer p,
then $(k+1)^3 - (k+1) + 3$
$= k^3 + 3k^2 + 2k + 3$
$= (k^3 - k + 3) + (3k^2 + 3k)$
$= 3p + 3(k^2 + k) = 3(p + k^2 + k)$

37.
Basic Step: $n = 1$,
$1(1+1)(1+2) = 6$, which is divisible by 6;
Induction Step:
If $\frac{k(k+1)(k+2)}{6} = p$
or $k(k+1)(k+2) = 6p$
for some integer p, then
$(k+1)(k+2)(k+3)$
$= k^3 + 6k^2 + 11k + 6$
$= (k^3 + 3k^2 + 2k) + (3k^2 + 9k + 6)$
$= k(k+1)(k+2) + 3(k+1)(k+2)$
$= 6p + 3(k+1)(k+2)$.
$6p$ is clearly divisible by 6.
In order for $3(k+1)(k+2)$ to be divisible by 6, it must be divisible by 2 and 3. It is clearly divisible by 3.
If k is odd, then the term $(k+1)$ must be even, making it divisible by 2.
If k is even, then the term $(k+2)$ is even, making it divisible by 2.
Therefore, $3(k+1)(k+2)$ is divisible by 6.

39.
$0 + 1 + 2 + 3 + \cdots + (n-1) = \frac{n(n-1)}{2}$;
Basic Step:
$n = 1, (1-1) = 0$ and $\frac{1(1-1)}{2} = 0$;
Induction Step:
If $0 + 1 + 2 + \cdots + (k-1) = \frac{k(k-1)}{2}$,
then
$[0 + 1 + 2 + \cdots + (k-1)] + (k+1-1)$
$= \frac{k(k-1)}{2} + k = \frac{k^2 - k + 2k}{2}$
$= \frac{k(k+1)}{2} = \frac{(k+1)((k+1)-1)}{2}$

41. The induction step does not work for $n = 1$. In the case of $n = 1$, $n + 1 = 2$ and the groups formed by removing the first horse and then the last horse do not overlap.

12.5 Exercises

1. combination

3. combination

5. 12 **7.** 720

9. 15 **11.** 792

13. $\frac{5!}{2!} = 60$

15. $\frac{7!}{2!2!} = 1260$

17. $\frac{11!}{2!2!2!} = 4,989,600$

19. $243x^5 + 405x^4y + 270x^3y^2 + 90x^2y^3 + 15xy^4 + y^5$

21. $x^4 - 12x^3 + 54x^2 - 108x + 81$

23. $7776x^{10} + 6480x^8y + 2160x^6y^2 + 360x^4y^3 + 30x^2y^4 + y^5$

25. $2401x^8 + 10,976x^6y^2 + 18,816x^4y^4 + 14,336x^2y^6 + 4096y^8$

27. $x^2 + 2xy + 2xz + y^2 + 2yz + z^2$

29. $64x^6 + 960x^5 + 6000x^4 + 20,000x^3 + 37,500x^2 + 37,500x + 15,625$

31. 32

33. $x^{16} + 48x^{15}y + 1080x^{14}y^2 + 15,120x^{13}y^3$

35. $129,140,163x^{\frac{17}{4}} + 3,658,971,285x^4y$

37. $651,168x^5$

39. $10^3 = 1000$

41. $9^7 = 4,782,969$

43. $15! \approx 1.308 \times 10^{12}$

45. $3! = 6$

47. $5^{10} = 9,765,625$

49. $36^6 = 2,176,782,336$

51. $26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 = 11,232,000$

53. ${}_{30}P_{12} \approx 4.143 \times 10^{16}$

55. ${}_{36}P_8 \approx 1.220 \times 10^{12}$

57. ${}_7P_6 = 5040$
${}_7P_7 = 5040$ as well. (Having a child remain standing is numerically equivalent to putting a seventh chair in the room.)

59. ${}_{26}P_3 = 15,600$

61. ${}_7C_3 = 35$

63. ${}_9C_2 = 36$

65. ${}_{75}C_5 = 17,259,390$

67. ${}_{10}C_4 \cdot {}_8C_4 \cdot {}_{13}C_4 = 10,510,500$

69. 112 cones

71. 96 outfits

73. 288 schedules

75. 120 5-letter strings

77. 303,600 ways

79. 495 pizzas

81. 752,538,150 groups

83. 420 ways

85. $\binom{n}{n-k} = \frac{n!}{(n-k)!(n-(n-k))!}$
$= \frac{n!}{(n-k)!(n-n+k)!}$
$= \frac{n!}{(n-k)!k!} = \binom{n}{k}$

87. $2^n = (1+1)^n$
$= \sum_{k=0}^{n} \binom{n}{k} (1)^k (1)^{n-k} = \sum_{k=0}^{n} \binom{n}{k}$
$= \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n}$

12.6 Exercises

1. $\frac{3}{5}$ **3.** $\frac{9}{13}$

5. $\frac{1}{3}$

7. a. 0 **b.** $\frac{5}{8}$

9. a. 0 **b.** $\frac{3}{5}$

11. a. 0 **b.** 1

13. a. $\frac{1}{8}$ **b.** $\frac{9}{16}$

15. The set of all ordered 4-tuples made up of H's and T's. There are 16 such 4-tuples.

17. The set of all ordered pairs that have either an H or a T in the first slot and one of the 13 hearts in the second slot. There are 26 such ordered pairs.

19. The set of all ordered triples with any of the 6 values in each slot. There are 216 such triples.

21. The set of the 38 pockets.

23. a. $\frac{2}{3}$ **b.** $\frac{1}{3}$

25. a. $\frac{3}{8}$ **b.** $\frac{1}{8}$ **c.** $\frac{1}{2}$

27. $\frac{3}{10}$

29. $\frac{387,420,489}{1,000,000,000} \approx 0.3874$

31. $\frac{3}{8}$

33. a. $\frac{11}{26}$ **b.** $\frac{9}{52}$ **c.** $\frac{2}{13}$

35. a. $\frac{1}{169}$ **b.** $\frac{1}{221}$

37. 18.75%

39. a. $\frac{1}{6}$ **b.** $\frac{5}{18}$ **c.** $\frac{1}{9}$

41. $\frac{1}{20}$ **43.** $\frac{2}{9}$ **45.** $\frac{3}{10}$

47. 27 tickets

Chapter 12 Project

1. a. $\frac{9}{19}$ **b.** $\frac{9}{19}$ **c.** $\frac{1}{38}$

d. $\frac{3}{38}$ **e.** $\frac{1}{38}$

3. Approximately –\$0.05 (or the person betting will lose about 5 cents per play on average)

Chapter 12 Review Exercises

1. –3, 9, –27, 81, –243

3. –3, –4, –5, –6, –7

5. $a_n = 6n - 13$ **7.** $a_n = n^2 - 1$

9. $a_1 = -2, a_n = n(a_{n-1})$ for $n \geq 2$

11. $-3 - 5 - 7 - 9 - 11 - 13 = -48$

13. $\sum_{i=2}^{7} i^3 = 783$

15. –8 – 16 – 32 – 64 – 128 = –248

17. $S_n = \frac{n}{2(n+2)}$, $S_{80} = \frac{20}{41}$

19. $S_n = 3 - 3^{n+1}$, series diverges

21. –10, –12, –22, –34, –56

23. $a_n = \frac{5}{2}n + 9$

25. $a_n = 3n - 1$ **27.** $a_n = 9n - 14$

29. 275 **31.** a_{36} **33.** 8827

35. 66

37. $a_n = 3\left(\frac{1}{5}\right)^{n-1}$

39. $a_n = 6(4)^{n-1}$

41. $a_n = 8\left(\frac{1}{4}\right)^{n-1}$

43. $r = \pm\frac{2}{3}; \pm\frac{45}{8}, \frac{15}{4}, \pm\frac{5}{2}, \frac{5}{3}, \pm\frac{10}{9}$

45. –2 **47.** $\frac{381}{512}$ **49.** –12

51. 1

53.

Basic Step: $n = 1, (3(1)+2) = 5$

and $\frac{1(3(1)+7)}{2} = 5$;

Induction Step:

If $5 + 8 + 11 + \cdots + (3k+2) = \frac{k(3k+7)}{2}$,

then

$$5 + 8 + 11 + \cdots + (3k+2) + (3(k+1)+2)$$
$$= \frac{k(3k+7)}{2} + (3k+5)$$
$$= \frac{3k^2 + 7k + 6k + 10}{2}$$
$$= \frac{(k+1)(3k+10)}{2}$$
$$= \frac{(k+1)(3(k+1)+7)}{2}$$

55. Basic Step: $n = 1$, $11^1 - 7^1 = 4$, which is divisible by 4;
Induction Step:
If $\frac{11^k - 7^k}{4} = p$ or $11^k - 7^k = 4p$
for some integer p, then
$11^{k+1} - 7^{k+1} = 11 \cdot 11^k - 7 \cdot 7^k$
$= 4 \cdot 11^k + 7 \cdot 11^k - 7 \cdot 7^k$
$= 4 \cdot 11^k + 7(11^k - 7^k)$
$= 4 \cdot 11^k + 7(4p) = 4(11^k + 7p)$

57. $8 \cdot 10^3 \cdot 26^3 = 140,608,000$

59. $\frac{8!}{3!2!} = 3360$

61. ${}_{21}P_5 = 2,441,880$

63. $-32y^5 + 80y^4 - 80y^3 + 40y^2 - 10y + 1$

65. $3125x^{10} - 6250x^8y + 5000x^6y^2 - 2000x^4y^3 + 400x^2y^4 - 32y^5$

67. a. $\frac{1}{9}$ **b.** $\frac{2}{3}$

69. a. $\frac{2}{7}$ **b.** $\frac{4}{7}$

71. a. $\frac{1}{10}$ **b.** $\frac{4}{5}$

73. $\frac{33}{108,290}$

Chapter 13: An Introduction to Limits, Continuity, and the Derivative

13.1 Exercises

1. -7 **3.** $2c-5+h$

5. $\dfrac{1}{c^2-12c+ch-6h+36}$

7. $\dfrac{\ln(c+h)-\ln c}{h}$

9. $\dfrac{3\left(\sqrt{c-2+h}-\sqrt{c-2}\right)}{h}$ or $\dfrac{3}{\sqrt{c-2}+\sqrt{c-2+h}}$

11. $c=-1$

a. 2 **b.** 2 **c.** 2

13. $c=3$

a. 0.0833 **b.** 0.1149

c. 0.1111

15. 3 **17.** 0 **19.** $-\dfrac{1}{4}$

21. The exact answer is –2.

23. The exact answer is 2.

25. The exact answer is 2.

27. The exact answer is $\dfrac{1}{e}$ or approx. 0.3679.

29. The exact answer is $\dfrac{1}{5\ln 10}$ or approx. 0.0869.

31. The exact answer is –4.

33. a. 96 ft **b.** 16 ft/s

c. 16 **d.** 2.5 s

35. a. 176 ft **b.** 176 ft/s

c. 192 ft/s **d.** 64 ft/s **e.** 6 s

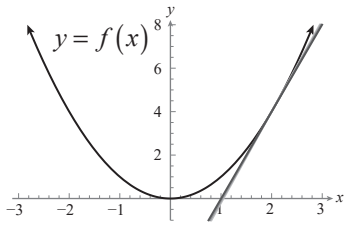
37. a. 3 m/s **b.** 15 m/s

c. $6t_0+3$

39. a. 5 s; 45 m/s **b.** 10 s

c. $-1.83g$

41.



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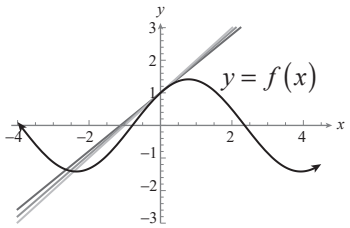

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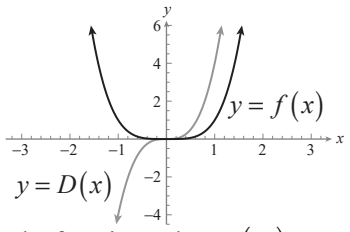
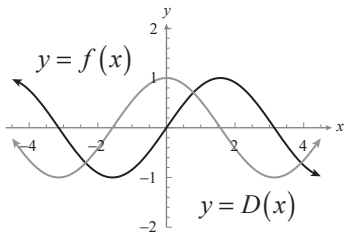


The function value $D(x_0)$ at any given x_0 is approximately equal to the slope of the graph of f at x_0.

47.



The function value $D(x_0)$ at any given x_0 is approximately equal to the slope of the graph of f at x_0.

49. $x=1$ **51.** $x=1$

13.2 Exercises

1. 4 **3.** 0

5.

x	y
1	2.414
1.4	2.814
1.41	2.824
1.414	2.828

The table points to a limit of $2\sqrt{2}\approx 2.828$.

7.

x	y
1.5	113.33
1.1	15.94
1.01	10.46
1.001	10.045

The table points to a limit of 10.

9.

x	y
3	–3.4696
3.14	–3.0048
3.141	–3.0018
3.1415	–3.0003

The table points to a limit of –3.

11.

x	y
7.5	210.5
7.1	994.1
7.01	9814.0
7.001	98014.0

The table points to an undefined limit.

13. $\lim\limits_{x\to 1^-} f(x)=-\infty$; $\lim\limits_{x\to 1^+} f(x)=\infty$

15. $\lim\limits_{x\to 3^-} h(x)=\infty$; $\lim\limits_{x\to 3^+} h(x)=\infty$

17. $\lim\limits_{x\to -1^-} q(x)=\infty$; $\lim\limits_{x\to -1^+} q(x)=-\infty$

19. $\lim\limits_{x\to 1.5^-} v(x)=-\infty$; $\lim\limits_{x\to 1.5^+} v(x)=\infty$

21. $\lim\limits_{x\to\left(\frac{\pi}{2}+k\pi\right)^-} \tan x=\infty$;

$\lim\limits_{x\to\left(\frac{\pi}{2}+k\pi\right)^+} \tan x=-\infty \quad (k\in\mathbb{Z})$

23. $\lim\limits_{x\to 6^-} s(x)=-\infty$; $\lim\limits_{x\to 6^+} s(x)=\infty$

25. $\lim\limits_{x\to\infty} f(x)=0$; $\lim\limits_{x\to-\infty} f(x)=0$

27. $\lim\limits_{x\to\infty} h(x)=0$; $\lim\limits_{x\to-\infty} h(x)=0$

29. $\lim\limits_{x\to\infty} q(x)=-1$; $\lim\limits_{x\to-\infty} q(x)=-1$

31. $\lim\limits_{x\to\infty} v(x)=\infty$; $\lim\limits_{x\to-\infty} v(x)=-\infty$

33. $\lim\limits_{x\to\infty} \tan x$ does not exist;

$\lim\limits_{x\to-\infty} \tan x$ does not exist

35. $\lim\limits_{x\to\infty} s(x)=1$; $\lim\limits_{x\to-\infty} s(x)=-1$

37. a. –2 **b.** 1

39. a. 1 **b.** 4

41. a. 1 **b.** $-\infty$

43. a. 1 **b.** 1

45. a. Does not exist **b.** 0

47. $\lim\limits_{x\to\infty} f(x)=\infty$; $\lim\limits_{x\to-\infty} f(x)=-\infty$

49. $\lim\limits_{x\to\infty} h(x)=-\infty$; $\lim\limits_{x\to-\infty} h(x)=-\infty$

51. $\lim_{x\to\infty} F(x) = \infty$

53. $\lim_{x\to\infty} H(x) = \infty$; $\lim_{x\to-\infty} H(x) = \infty$

55. $\lim_{x\to\infty} u(x) = \infty$; $\lim_{x\to-\infty} u(x) = \infty$

57. $\lim_{x\to-\infty} s(x) = -\infty$

59. False; see $h(x)$ of Example 5 at $x = 2$.

61. True

63. True

65. Does not exist

67. 2 **69.** $\frac{3}{2}$

71. 0

13.3 Exercises

1. $\delta \approx 0.23$ or smaller

3. $\delta \approx 0.2$ or smaller

5. $\delta = 0.02$ or smaller

7. $\delta = 0.1$ or smaller

9. $\delta = \sqrt{0.1}$ or smaller

11. $\delta = 0.0\overline{9}$ or smaller

13. $\delta = 1 - e^{-0.1} \approx 0.0952$ or smaller

15. $N = 10$ or larger

17. $N = -10$ or smaller

19. $N = \ln 0.1$ or smaller

21. $\delta = 0.1$ or smaller

23. $\delta = 0.1$ or smaller

25. $\delta = \frac{\pi}{2} - \arctan 100 \approx 0.0099997$ or smaller

57. The limit does not exist.

59. The limit does not exist.

61. The limit is 0.

63. **a.** 1256.64 N

b. Approx. 0.16 mm

65. L is incorrectly quantified and switched with c.

67. ε is incorrectly quantified.

69. The inequality $0 \le |x - c|$ is incorrect.

71. False; the function value and limit at c need not be equal.

73. False; consider $f(x) = -x^2$ and $g(x) = x^2$ at $c = 0$.

85. Vert. asym.: $x \approx -1.3340$, $x \approx 1.1759$

87. Vert. asym.: $x = (2n+1)\pi - 6$, $n \in \mathbb{Z}$

89. Vert. asym.: $x = \frac{(2n-1)\pi}{2}$, $n \in \mathbb{Z}$, $n \neq 0$

13.4 Exercises

1. **a.** -4 **b.** 21

3. 5 **5.** 7 **7.** 9

9. 22 **11.** -4 **13.** -2

15. $\sqrt[3]{2}$ **17.** $\sqrt[3]{100}$ **19.** 16

21. 12 **23.** 11 **25.** $\frac{33}{5}$

27. $\frac{1}{6}$ **29.** $\frac{1}{2\sqrt{5}}$ **31.** $\frac{1}{9}$

33. 6 **35.** 4 **37.** $\frac{1}{2\sqrt{x}}$

39. $\frac{32}{7}$ **41.** $-\frac{1}{6}$ **43.** -1

45. 8 **47.** 11

49. $9 - 2(c-2)$

51. 1 **53.** 0 **55.** 0

57. $-2e^2$

71. The limit is 0.

79. $\frac{r}{2}$

13.5 Exercises

1. Points of continuity: $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$
Points of discontinuity: $c = 0$, $\lim_{x\to 0} f(x)$ does not exist; $c = 3$, $\lim_{x\to 3} f(x) \neq f(3)$

5. $c = 0$, nonremovable

7. $c = 3$, removable

9. $c = 2$, nonremovable

11. None

13. $c = -\frac{(2n+1)\pi}{2}$ for $n \in \{-1, 0, 1, 2, \ldots\}$, nonremovable

15. $c = 4$, removable

17. None

19. None

21. $c = \pm 1$, nonremovable

23. None

25. All integers, nonremovable

27. $c = \pm\sqrt{n}$, n a positive integer, nonremovable

29. $c = \frac{1}{n}$, n a nonzero integer, nonremovable

35. Continuous on $[-3, -1] \cup (0, 2] \cup [3, \infty)$

37. Continuous on $(-\sqrt{3}, \sqrt{3})$

39. Continuous except on $\left\{\frac{n-1}{\pi} \middle| n \in \mathbb{Z}\right\} \cup \{\ln k - 2 | k \in \mathbb{N}\}$

45. $g(1) = 2$ and $g(2) = 3$ will make g continuous.

47. $F(3) = \frac{1}{4}$ will make F continuous.

49. $H(0) = 0$ will make H continuous.

51. Not continuous at 3

53. Continuous on $\left[0, \frac{1}{\pi}\right]$

55. $a = 3$

57. $a = 0.5, b = 1.25$

59. Yes; $c = 1$

61. No; discontinuity at 1

63. Yes; $c = \frac{\pi}{9} - \frac{2}{3}$ or $c = \frac{5\pi}{9} - \frac{2}{3}$

73. **a.** Because speeds of everyday objects are smaller than c by orders of magnitude, the denominator of ΔT is approximately 1.

b. No moving object can reach the speed of light.

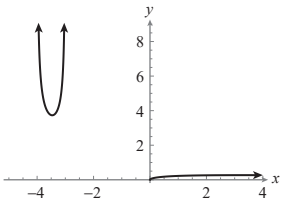
85. False; consider the Dirichlet function.

87. True

89. −0.2895 **91.** 0.8241

93. −1.7321

95.



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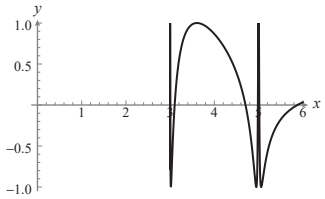

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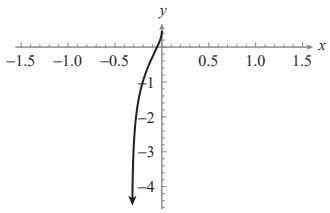


13.6 Exercises

1. a. $f'(-1)=2$ **b.** $f'(1)=-2$

3. $y=4x-6$ **5.** $y=\frac{1}{2}x+4$

7. $y=12x-16$ **9.** $y=\frac{1}{2}x+1$

11. $y=-4x+4$

13. $y=-\frac{1}{16}x+\frac{3}{4}$

15. $f'(x)=0$; tangent is horizontal for all x

17. $f'(x)=4$; tangent is never horizontal

19. $f'(x)=6x$; tangent is horizontal at $x=0$

21. $f'(x)=x+5$; tangent is horizontal at $x=-5$

23. $f'(x)=3x^2+1$; tangent is never horizontal

25. $f'(x)=4x^3$; tangent is horizontal at $x=0$

27. $f'(x)=-\frac{10}{(2x-4)^2}$; tangent is never horizontal

29. $f'(x)=-\frac{7}{(x-3)^2}$; tangent is never horizontal

31. $f'(x)=-\frac{2x}{(x^2+1)^2}$; tangent is horizontal at $x=0$

33. $f'(x)=\frac{5}{2\sqrt{5x}}$; tangent is never horizontal

35. $f'(x)=\frac{1}{\sqrt{2x+1}}$; tangent is never horizontal

37. $f'(x)=\frac{x}{\sqrt{x^2+1}}$; tangent is horizontal at $x=0$

39. $y=6x-6$

41. $y=-\frac{1}{2}x+1$ or $y=-\frac{1}{2}x-1$

43. $54y+x=27$

45. $f'(3.6)=-\frac{1}{4}$

47. $h'(3)=10$ **49.** $G'(7)=\frac{1}{2}$

51. $u'(-3)=-\frac{1}{2}$ **53.** $w'(5)=-\frac{1}{54}$

55. $G'(-2)=-32$

57. c **59.** d

73. $t=1$, $t=5$; the particle stops at 2 s and at 4 s.

75. a. The respective velocities are $16\,\text{ft/s}$ and $-16\,\text{ft/s}$; the speed is $16\,\text{ft/s}$ in both cases.

b. It rises 36 ft above the cliff and reaches the bottom at approx. 4.91 s.

c. Approx. $-109.12\,\text{ft/s}$

77. She seems to have stopped for an extended time period (probably a red light); soon after, she turned around and sped back home (perhaps forgot something at home). Then she sped back to class, this time managing to do so without a red light, and also driving faster.

79. a. $P(x)=0.1x^2+40x-190$

b. Approx. 5 suitcases

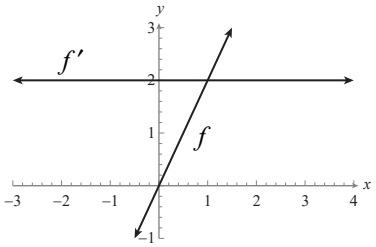
c. $P(25)=872.50$, $P(30)=1100$, $P(40)=1570$

d. $P'(x)=0.2x+40$

e. $P'(25)=45$, $P'(30)=46$, $P'(40)=48$

81. $C(x)=20+3x$; $C'(x)=3$

83.



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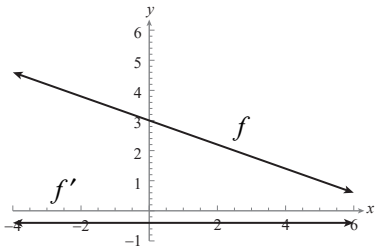

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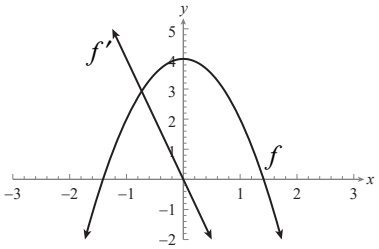

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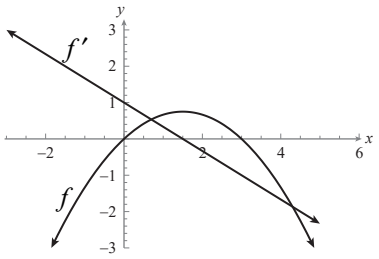

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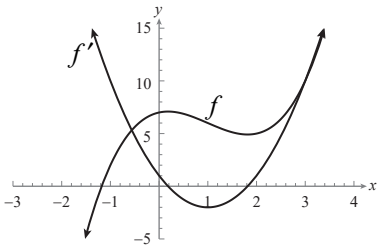

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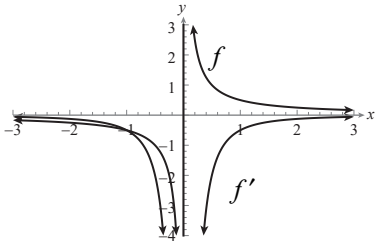

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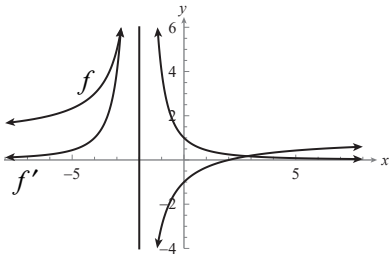

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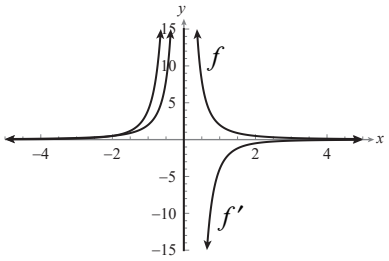

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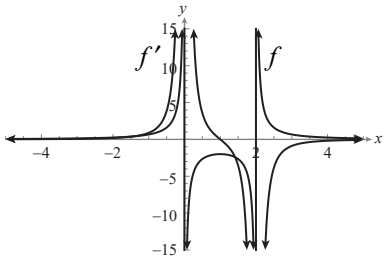

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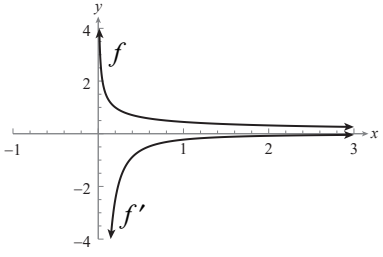

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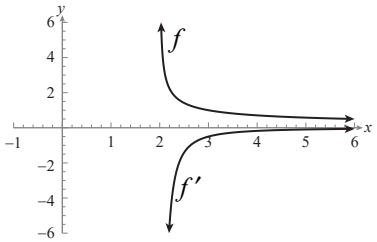

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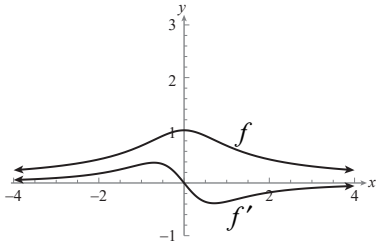
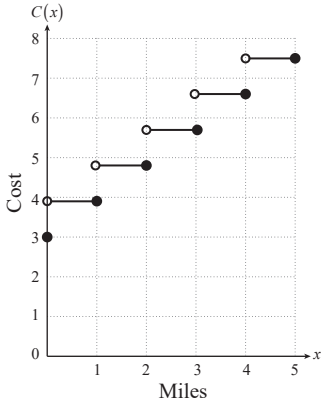


Chapter 13 Project

1.



3. $\lim_{x \to 1.7} C(x) = 4.8$

5. 0

7. $(9,10]$, \$12

Chapter 13 Review Exercises

1. 5

3. $-\dfrac{1}{2(1+c)(1+c+h)}$

5. $c = 1, h = 0.1$; average rate of change $= 5$

7. $c = 0, h = -0.01$; average rate of change $= -0.51$

9. The exact answer is 3.

11. The exact answer is $\dfrac{1}{2}$.

13. **a.** 598 ft **b.** 576 ft/s
c. 608 ft/s **d.** 544 ft/s
e. At 19 seconds

15. 3

17. Does not exist (or $-\infty$)

19.

x	y
0.5	0.7071
0.1	0.7943
0.01	0.9550
0.001	0.9931

The table points to a limit of 1.

21.

x	y
0.5	0.47943
0.1	0.11969
0.01	−0.00264
0.001	−0.00194

The table points to a limit of 0.

23. $\delta = 0.005$ or smaller

29. 14 **31.** 7 **33.** 8

35. $-\dfrac{1}{10}$ **37.** ∞ **39.** $\dfrac{1}{4}$

41. −2 **43.** 5 **45.** $2x$

51. $c = 4$, nonremovable

53. All integers, nonremovable

55. No discontinuities

61. $y = 3x - 1$

63. $f'(x) = 2 - 2x$; tangent is horizontal at $x = 1$

69. **a.** $P(x) = 0.1x^2 + 15x - 247.5$
b. 15 toys **c.** $P'(x) = 0.2x + 15$