motion is $\|\mathbf{F}\|\cos\theta$, where θ is, as usual, the angle between the two vectors. The work done is then the product of $\|\mathbf{D}\|$ and $\|\mathbf{F}\|\cos\theta$, that is, $W = \|\mathbf{D}\|\,\|\mathbf{F}\|\cos\theta$. But this last expression should look familiar—it appears in the Dot Product Theorem, allowing us to write the simpler formula below.

$$W = \mathbf{F} \cdot \mathbf{D}$$

Example 7: Applying the Dot Product

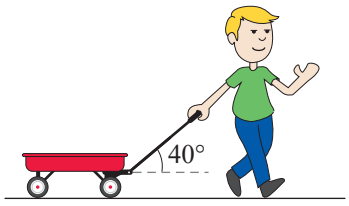

FIGURE 7

A child pulls a wagon along a sidewalk, exerting a force of 15 pounds on the handle of the wagon. The handle is at an angle of 40° to the horizontal. If the child pulls the wagon a distance of 50 feet, what work has been done?

Solution

We start by defining the force and distance vectors.

$$\mathbf{F} = 15\langle \cos 40°, \sin 40° \rangle \quad \text{and} \quad \mathbf{D} = \langle 50, 0 \rangle$$

The calculation is now straightforward.

$$\begin{aligned} W &= 15\langle \cos 40°, \sin 40° \rangle \cdot \langle 50, 0 \rangle \\ &\approx (15)(0.766)(50) + (15)(0.643)(0) \\ &= 574.5 \text{ foot-pounds} \end{aligned}$$

9.7 EXERCISES

PRACTICE

Calculate each of the following dot products. See Example 1.

1. $\langle 4, 3 \rangle \cdot \langle 5, -1 \rangle$
2. $\langle 2, 4 \rangle \cdot \langle -1, -1 \rangle$
3. $\langle 3, 5 \rangle \cdot \langle 2, 0 \rangle$
4. $\langle -1, 6 \rangle \cdot \langle 6, 1 \rangle$
5. $\langle 2, 2 \rangle \cdot \langle 2, 2 \rangle$
6. $\langle 1, 2 \rangle \cdot \langle 3, 4 \rangle$
7. $\langle -4, 3 \rangle \cdot \langle 2, 3 \rangle$
8. $\langle -2, -4 \rangle \cdot \langle 6, 2 \rangle$
9. $\mathbf{u} = 5\mathbf{i} + \mathbf{j}$, $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$
10. $\mathbf{u} = \mathbf{i} - 5\mathbf{j}$, $\mathbf{v} = -2\mathbf{i} - 4\mathbf{j}$

Find the indicated quantity given $\mathbf{u} = \langle -2, 3 \rangle$ and $\mathbf{v} = \langle 4, 4 \rangle$. See Example 1.

11. $\mathbf{v} \cdot \mathbf{v}$
12. $4\mathbf{u} \cdot \mathbf{v}$
13. $(\mathbf{u} \cdot \mathbf{u})\mathbf{u}$
14. $(\mathbf{u} \cdot \mathbf{v})2\mathbf{v}$

Find the magnitude of $\mathbf{u}$ using the dot product. See Example 2.

15. $\mathbf{u} = \langle 6, -1 \rangle$
16. $\mathbf{u} = \langle 10, 3 \rangle$
17. $\mathbf{u} = 2\mathbf{i} + 7\mathbf{j}$
18. $\mathbf{u} = -3\mathbf{i} + 4\mathbf{j}$

Find the angle between the given vectors. See Example 3.

19. $\mathbf{u} = \langle -2, 3 \rangle$, $\mathbf{v} = \langle 1, 0 \rangle$
20. $\mathbf{u} = \langle 5, 4 \rangle$, $\mathbf{v} = \langle 3, 2 \rangle$

21. $\mathbf{u} = \langle 3, 5 \rangle$, $\mathbf{v} = \langle 4, 4 \rangle$

22. $\mathbf{u} = \langle -4, 2 \rangle$, $\mathbf{v} = \langle 1, 5 \rangle$

23. $\mathbf{u} = -\mathbf{i} + 2\mathbf{j}$, $\mathbf{v} = 3\mathbf{i} - 3\mathbf{j}$

24. $\mathbf{u} = 5\mathbf{i} + 2\mathbf{j}$, $\mathbf{v} = 4\mathbf{i} + \mathbf{j}$

25. $\mathbf{u} = \cos\left(\frac{3\pi}{4}\right)\mathbf{i} + \sin\left(\frac{3\pi}{4}\right)\mathbf{j}$, $\mathbf{v} = \cos\left(\frac{\pi}{2}\right)\mathbf{i} + \sin\left(\frac{\pi}{2}\right)\mathbf{j}$

26. $\mathbf{u} = \cos\left(\frac{\pi}{4}\right)\mathbf{i} + \sin\left(\frac{\pi}{4}\right)\mathbf{j}$, $\mathbf{v} = \cos\left(\frac{5\pi}{6}\right)\mathbf{i} + \sin\left(\frac{5\pi}{6}\right)\mathbf{j}$

Use vectors to find the interior angles of the triangles given the following sets of vertices.

27. $(3, 3), (4, 2), (-1, -6)$

28. $(0, 0), (0, 5), (3, 6)$

29. $(-2, -1), (2, 4), (-4, 5)$

30. $(6, 3), (-5, 2), (-6, 1)$

Find $\mathbf{u} \cdot \mathbf{v}$ where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$. See Example 3.

31. $\|\mathbf{u}\| = 25, \|\mathbf{v}\| = 5, \theta = 120°$

32. $\|\mathbf{u}\| = 4, \|\mathbf{v}\| = 64, \theta = \frac{\pi}{6}$

33. $\|\mathbf{u}\| = 16, \|\mathbf{v}\| = 4, \theta = \frac{3\pi}{4}$

34. $\|\mathbf{u}\| = 9, \|\mathbf{v}\| = 10, \theta = \frac{2\pi}{3}$

Find two vectors orthogonal to the given vector. See Example 4. Answers may vary.

35. $\mathbf{u} = \langle 3, -3 \rangle$

36. $\mathbf{u} = \langle 4, 1 \rangle$

37. $\mathbf{u} = \langle 2, -6 \rangle$

38. $\mathbf{u} = \langle 5, 4 \rangle$

Determine whether $\mathbf{u}$ and $\mathbf{v}$ are orthogonal, parallel, or neither. See Example 4.

39. $\mathbf{u} = \langle 2, -3 \rangle$, $\mathbf{v} = \langle 1, 6 \rangle$

40. $\mathbf{u} = \langle -12, 30 \rangle$, $\mathbf{v} = \left\langle \frac{1}{2}, -\frac{5}{4} \right\rangle$

41. $\mathbf{u} = 2\mathbf{i} - 2\mathbf{j}$, $\mathbf{v} = -\mathbf{i} - \mathbf{j}$

42. $\mathbf{u} = \mathbf{i}$, $\mathbf{v} = -2\mathbf{i} + 2\mathbf{j}$

Find the projection of $\mathbf{u}$ onto $\mathbf{v}$, and then write $\mathbf{u}$ as a sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$. See Example 5.

43. $\mathbf{u} = \langle 1, 3 \rangle$, $\mathbf{v} = \langle 4, 2 \rangle$

44. $\mathbf{u} = \langle 2, 2 \rangle$, $\mathbf{v} = \langle 1, -7 \rangle$

45. $\mathbf{u} = \langle 3, -5 \rangle$, $\mathbf{v} = \langle 6, 2 \rangle$

46. $\mathbf{u} = \langle 0, 3 \rangle$, $\mathbf{v} = \langle 2, 6 \rangle$

47. $\mathbf{u} = \langle -3, -3 \rangle$, $\mathbf{v} = \langle -4, -1 \rangle$

48. $\mathbf{u} = \langle 4, 2 \rangle$, $\mathbf{v} = \langle 1, 5 \rangle$

Find the work done on a particle moving from J to K if the magnitude and direction of the force are given by $\mathbf{F}$. See Example 7.

49. $J = (1, 4)$, $K = (5, 6)$, $\mathbf{F} = \langle 2, 3 \rangle$

50. $J = (-3, 2)$, $K = (0, 5)$, $\mathbf{F} = \langle 4, 2 \rangle$

51. $J = (3, 0)$, $K = (-4, -2)$, $\mathbf{F} = -\mathbf{i} + 2\mathbf{j}$

52. $J = (3, -3)$, $K = (5, 1)$, $\mathbf{F} = 6\mathbf{i} - 3\mathbf{j}$

APPLICATIONS

53. A truck with a gross weight of 25,000 pounds is parked on an 8° slope. What force is required to prevent the truck from rolling down the hill?

54. A child sits in his go-cart at the start position of a race atop a hill. If the hill has a slope of 3°, and the child and go-cart have a total weight of 250 pounds, what force is required to keep them stationary at the start position?

55. A woman on skis holds herself stationary, with the use of her ski poles, on a slope that is 45° from the horizontal. If the woman and her skis have a total weight of 155 pounds, what is the force required to prevent her from sliding down the slope?

56. A loaded furniture dolly is being pulled up a 15° ramp by a mover. When he pauses to rest, he has to exert 103.53 pounds of force just to keep the dolly stationary. How much does the loaded dolly weigh?

57. A child pulls a sled over the snow, exerting a force of 25 pounds on the attached rope. The rope is 35° from the horizontal. If the child pulls the sled a distance of 80 feet, what work has been done?

58. The world's strongest man pulls a log 200 feet, and the tension in the cable connecting the man and log is 3000 pounds. What is the work being done if the cable is being held 15° from the horizontal?

59. A recreational vehicle pulls a passenger car behind it, exerting 1250 pounds on the attachment point. The angle of attachment is 30° from the horizontal. If the RV pulls the car a distance of 2 miles, what work has been done?