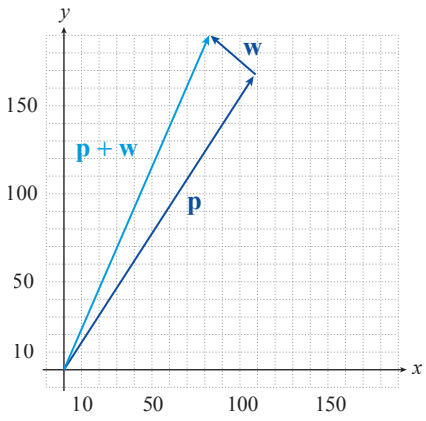


FIGURE 10

$$\mathbf{p} = 200\langle \cos 57°, \sin 57° \rangle \approx \langle 108.9, 167.7 \rangle$$

$$\mathbf{w} = 35\langle \cos 137°, \sin 137° \rangle \approx \langle -25.6, 23.9 \rangle$$

The plane's true velocity is now $\mathbf{p} + \mathbf{w} = \langle 83.3, 191.6 \rangle$. It may also be useful to determine that the speed of the plane is now $\|\mathbf{p} + \mathbf{w}\| = \sqrt{(83.3)^2 + (191.6)^2} \approx 208.9$ miles per hour, and that its bearing is 66.5° North of East (which, using conventional bearing notation, would be written as N 23.5° E). This last angle is derived from the fact that $\tan\theta = \dfrac{191.6}{83.3}$, so $\theta = \tan^{-1}\left(\dfrac{191.6}{83.3}\right) \approx 66.5°$. Figure 10 illustrates the three vectors in this problem.

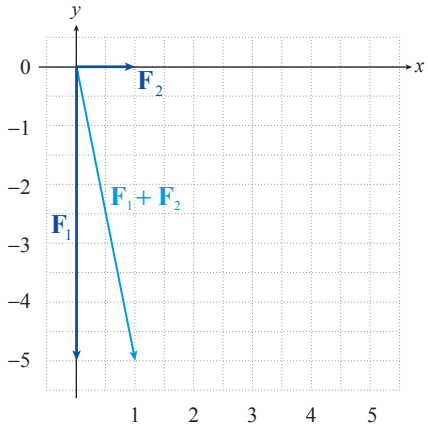


FIGURE 11

Example 7: Applying Vector Operations

A cat is slowly pushing a 5-pound plant across a table, with the intention of knocking it off the edge (determining why cats feel the need to do so is beyond the scope of this text). The cat is pushing with a force of 1 pound. What is the total force being applied to the plant?

Solution

Weight is itself a force—it is the force due to gravity that Earth exerts on an object. Forces exerted on an object are added as vectors, and the result is the total applied force. If we let $\mathbf{F}_1$ denote the weight of the plant and $\mathbf{F}_2$ the force exerted by the cat, the force $\mathbf{F}$ on the plant is

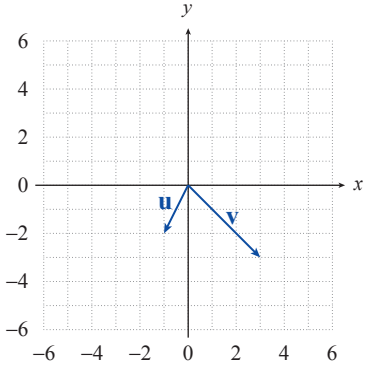
$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \langle 0, -5 \rangle + \langle 1, 0 \rangle = \langle 1, -5 \rangle.$$

The magnitude of $\mathbf{F}$ is $\sqrt{1+25} \approx 5.1$ pounds, and Figure 11 illustrates the situation.

9.6 EXERCISES

PRACTICE

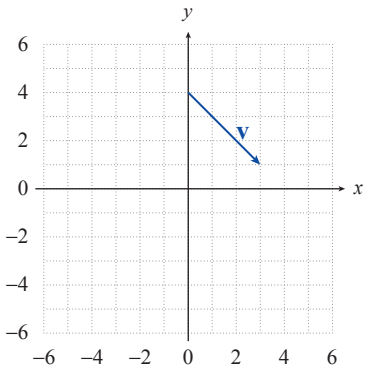
Use the figure to sketch a graph for the specified vector. See Examples 1 and 2.



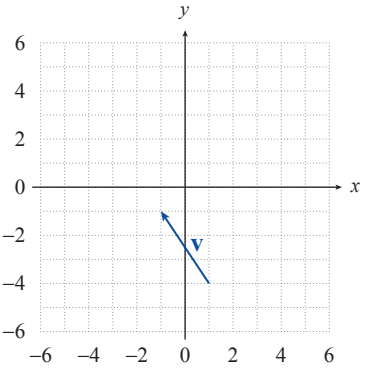
1. $-\mathbf{u}$
2. $2\mathbf{u} + \mathbf{v}$
3. $3\mathbf{v}$
4. $-\dfrac{1}{2}\mathbf{u} - \mathbf{v}$
5. $2\mathbf{u} - 2\mathbf{v}$
6. $\mathbf{u} + 3\mathbf{v}$

Find the component form and the magnitude of vector **v** for each of the following. See Example 3.

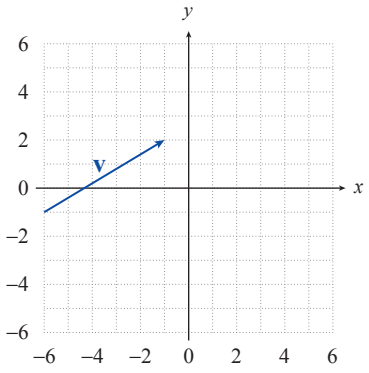
7.



8.



9.



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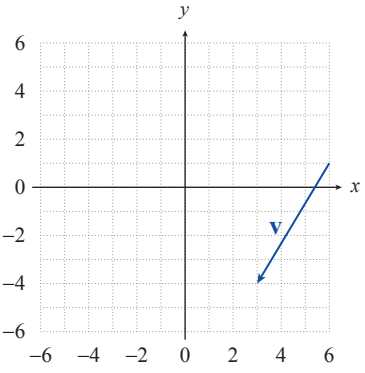


Find the component form and the magnitude of a vector **v** defined by the given points. Assume the first point given is the initial point and the second point given is the terminal point. See Example 3.

11. $(-2,4),(3,3)$

12. $(1,6),(2,3)$

13. $(5,-2),(-2,5)$

14. $(4,0),(-1,7)$

15. $(3,4),(-1,-2)$

16. $(1,-6),(0,0)$

For each of the following, calculate **a.** $2\mathbf{u}+\mathbf{v}$, **b.** $-\mathbf{u}+3\mathbf{v}$, and **c.** $-2\mathbf{v}$. See Example 3.

17. $\mathbf{u}=\langle -2,4\rangle, \mathbf{v}=\langle 2,0\rangle$

18. $\mathbf{u}=\langle 4,1\rangle, \mathbf{v}=\langle 2,5\rangle$

19. $\mathbf{u}=\langle 2,0\rangle, \mathbf{v}=\langle -3,4\rangle$

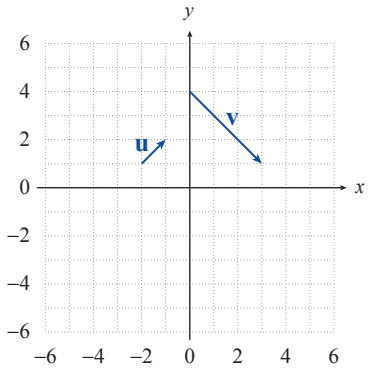
20. $\mathbf{u}=\langle 1,3\rangle, \mathbf{v}=\langle 4,4\rangle$

21. $\mathbf{u}=\langle -1,-4\rangle, \mathbf{v}=\langle -3,-2\rangle$

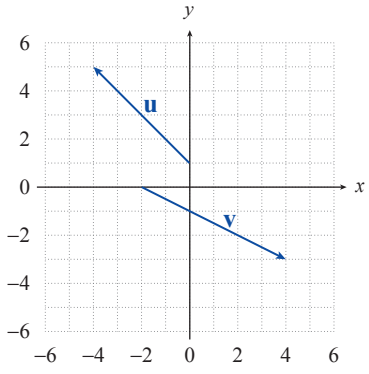
22. $\mathbf{u}=\langle 0,-5\rangle, \mathbf{v}=\langle -1,2\rangle$

For each of the following graphs, determine the component forms of $-\mathbf{u}$, $2\mathbf{u} - \mathbf{v}$, and $\mathbf{u} + \mathbf{v}$ and find the magnitudes of $\mathbf{u}$ and $\mathbf{v}$. See Example 3.

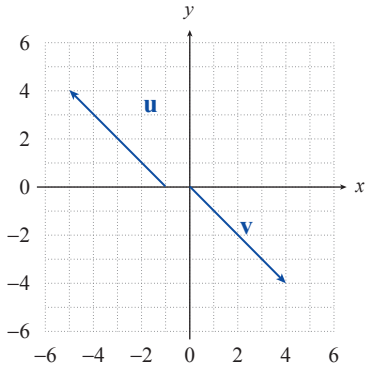
23.



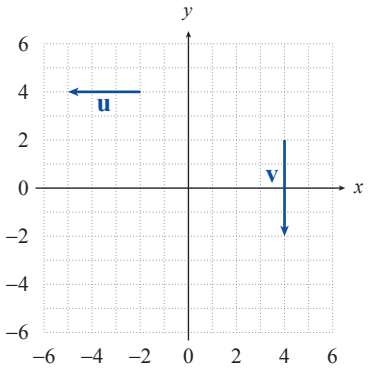
24.



25.



26.



Given the vector $\mathbf{u}$, find **a.** a unit vector pointing in the same direction as $\mathbf{u}$, and **b.** the linear combination of $\mathbf{i}$ and $\mathbf{j}$ that is equivalent to $\mathbf{u}$. See Example 4.

27. $\mathbf{u} = \langle 6, -3 \rangle$

28. $\mathbf{u} = \langle 1, 4 \rangle$

29. $\mathbf{u} = \langle -5, -1 \rangle$

30. $\mathbf{u} = \langle -4, 3 \rangle$

31. $\mathbf{u} = \langle 2, 3 \rangle$

32. $\mathbf{u} = \langle 5, 2 \rangle$

Find the magnitude and direction angle of the vector $\mathbf{v}$.

33. $\mathbf{v} = 5(\cos 30° \mathbf{i} + \sin 30° \mathbf{j})$

34. $\mathbf{v} = 7(\cos 45° \mathbf{i} + \sin 45° \mathbf{j})$

35. $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$

36. $\mathbf{v} = -2\mathbf{i} - 2\mathbf{j}$

Find the component form of $\mathbf{v}$ given its magnitude and the angle it makes with the positive x-axis. See Example 5.

37. $\|\mathbf{v}\| = 6,\ \theta = 30°$

38. $\|\mathbf{v}\| = \dfrac{5}{2},\ \theta = 0°$

39. $\|\mathbf{v}\| = 18,\ \theta = 135°$

40. $\|\mathbf{v}\| = 3\sqrt{3},\ \theta = 90°$

41. $\|\mathbf{v}\| = 1,\ \theta = 120°$

42. $\|\mathbf{v}\| = 4\sqrt{2},\ \theta = 45°$

43. $\|\mathbf{v}\| = 4,\ \mathbf{v}$ in the direction of $2\mathbf{i} + 3\mathbf{j}$

44. $\|\mathbf{v}\| = 7,\ \mathbf{v}$ in the direction of $\mathbf{i} + 4\mathbf{j}$

APPLICATIONS

45. A paper airplane is launched into the air at a speed of 4 ft/s and at an angle of 30° from the horizontal. Express this velocity in vector form.

46. A golf ball is driven into the air at a speed of 75 miles per hour and at an angle of 50° from the horizontal. Express this velocity in vector form.

47. A sailboat is traveling at a speed of 45 miles per hour with a bearing of N 59° W, when it encounters a front with winds blowing at 15 miles per hour with a bearing of S 3° E. What is the resultant true velocity of the sailboat?

48. An underwater missile is traveling at a speed of 350 miles per hour and bearing of S 17° W, when it meets a current traveling at 44 miles per hour in the direction of N 61° W. What is the resultant true velocity of the underwater missile?

49. Prometheus is slowly pushing a 1235-pound boulder across a flat plain with a force of 150 pounds. What is the total force **F** being applied to the boulder, and what is the magnitude of **F**?

50. A boy is pushing a toy truck across the floor. If the toy weighs 3 pounds and the boy is exerting half a pound of pressure on the toy, what is the total force **F** being applied to the toy truck, and what is the magnitude of **F**?