

9.5 EXERCISES

💡 PRACTICE

Graph and determine the magnitudes of the following complex numbers. See Example 1.

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|-------------|--------------|-------------|
| 1. $3 + 5i$ | 2. $-1 + 3i$ | 3. $2 - 4i$ |
| 4. $-6 - i$ | 5. $4 + 4i$ | 6. $5 + 2i$ |

Sketch z_1 , z_2 , $z_1 + z_2$, and $z_1 z_2$ on the same complex plane.

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|----------------------------------|-----------------------------------|
| 7. $z_1 = 7 + 2i, z_2 = -2 + 3i$ | 8. $z_1 = -1 - 2i, z_2 = 4 + 4i$ |
| 9. $z_1 = 3 + i, z_2 = 5 - i$ | 10. $z_1 = 5 - 2i, z_2 = -1 + 5i$ |

Graph the regions of the complex plane defined by the following. See Example 2.

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| 11. $\{z \mid z < 3\}$ | 12. $\{z \mid 1 \leq z \leq 4\}$ |
| 13. $\{z \mid z \geq 1\}$ | 14. $\{z = a + bi \mid a > 1, b > 2\}$ |
| 15. $\{z = a + bi \mid a \geq b\}$ | 16. $\{z \mid z = 4\}$ |

Write each of the following complex numbers in trigonometric form. See Example 3.

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|----------------------------|----------------------|---------------------|
| 17. $-3 - i$ | 18. $5 - 3i$ | 19. $1 + 2i$ |
| 20. $3 + \sqrt{3}i$ | 21. $4 + 2i$ | 22. $5 - \sqrt{2}i$ |
| 23. $\sqrt{2} - \sqrt{2}i$ | 24. $2\sqrt{3} - 2i$ | 25. $3 + 4i$ |
| 26. $1 + i$ | 27. $4 - 4\sqrt{3}i$ | 28. $2\sqrt{2} - i$ |

Write each of the following complex numbers in standard form. See Example 4.

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| 29. $3\left(\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right)$ | 30. $\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)$ |
| 31. $2\left(\cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)\right)$ | 32. $6(\cos \pi + i\sin \pi)$ |
| 33. $5\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$ | 34. $\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)$ |
| 35. $\frac{3}{2}(\cos 150^\circ + i\sin 150^\circ)$ | 36. $4(\cos 210^\circ + i\sin 210^\circ)$ |
| 37. $5[\cos(78^\circ 20') + i\sin(78^\circ 20')]$ | 38. $3[\cos(121^\circ 40') + i\sin(121^\circ 40')]$ |

Perform the following operations and show the answer in both trigonometric form and standard form. See Examples 5 and 6.

$$39. [4(\cos 60^\circ + i \sin 60^\circ)][4(\cos 330^\circ + i \sin 330^\circ)]$$

$$40. [3(\cos 180^\circ + i \sin 180^\circ)][4(\cos 30^\circ + i \sin 30^\circ)]$$

$$41. \left[\sqrt{2} \left(\cos \left(\frac{5\pi}{4} \right) + i \sin \left(\frac{5\pi}{4} \right) \right) \right] \left[3\sqrt{3} \left(\cos \left(\frac{\pi}{6} \right) + i \sin \left(\frac{\pi}{6} \right) \right) \right]$$

$$42. \left[10 \left(\cos \left(\frac{5\pi}{6} \right) + i \sin \left(\frac{5\pi}{6} \right) \right) \right] \left[6 \left(\cos \left(\frac{2\pi}{3} \right) + i \sin \left(\frac{2\pi}{3} \right) \right) \right]$$

$$43. (-1 + 3i)(\sqrt{3} + i)$$

$$44. 2i(4 + 5i)$$

$$45. \frac{6(\cos 225^\circ + i \sin 225^\circ)}{3(\cos 45^\circ + i \sin 45^\circ)}$$

$$46. \frac{8(\cos 135^\circ + i \sin 135^\circ)}{4(\cos 30^\circ + i \sin 30^\circ)}$$

$$47. \frac{10 \left(\cos \left(\frac{5\pi}{3} \right) + i \sin \left(\frac{5\pi}{3} \right) \right)}{3 \left(\cos \left(\frac{7\pi}{6} \right) + i \sin \left(\frac{7\pi}{6} \right) \right)}$$

$$48. \frac{12 \left(\cos \left(\frac{10\pi}{3} \right) + i \sin \left(\frac{10\pi}{3} \right) \right)}{6(\cos(2\pi) + i \sin(2\pi))}$$

$$49. \frac{-i}{1+i}$$

$$50. \frac{-2-2i}{4+3i}$$

$$51. \frac{2e^{\frac{2\pi}{3}i}}{e^{\frac{\pi}{4}i}}$$

$$52. (e^{210^\circ i})(e^{90^\circ i})$$

Plot both complex numbers and, on the same graph, plot their product. See Example 7.

$$53. \left[4 \left(\cos \left(\frac{5\pi}{6} \right) + i \sin \left(\frac{5\pi}{6} \right) \right) \right] [2(\cos \pi + i \sin \pi)]$$

$$54. \left[5 \left(\cos \left(\frac{3\pi}{2} \right) + i \sin \left(\frac{3\pi}{2} \right) \right) \right] \left[3 \left(\cos \left(\frac{5\pi}{6} \right) + i \sin \left(\frac{5\pi}{6} \right) \right) \right]$$

$$55. (2 - 5i)(\sqrt{2} + 2i)$$

$$56. (-4 - 2i)(-\sqrt{3} + 4i)$$

$$57. \left(2e^{\frac{\pi}{3}i} \right) \left(3e^{\frac{5\pi}{4}i} \right)$$

$$58. \left(5e^{\frac{5\pi}{3}i} \right) (e^{\pi i})$$

Use De Moivre's Theorem to calculate the following. See Example 8.

$$59. (1 - \sqrt{3}i)^5$$

$$60. \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right)^{22}$$

$$61. (5 + 3i)^{17}$$

$$62. (-\sqrt{3} + i)^{13}$$

$$63. \left(\cos \left(\frac{\pi}{4} \right) + i \sin \left(\frac{\pi}{4} \right) \right)^8$$

$$64. [2(\cos 135^\circ + i \sin 135^\circ)]^4$$

Find the indicated roots of the following and graphically represent each set in the complex plane. See Examples 9 and 10.

65. The fourth roots of -1 .

66. The cube roots of $64i$.

67. The square roots of $2\sqrt{3} + 2i$.

68. The fourth roots of $-1 - i$.

69. The fourth roots of 256.

70. The fourth roots of $16\left(\cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)\right)$.

71. The square roots of $4(\cos 120^\circ + i\sin 120^\circ)$.

Solve the following equations. See Examples 9 and 10.

72. $z^3 - i = 0$

73. $z^2 - 4\sqrt{3} - 4i = 0$

74. $z^4 + 81i = 0$

75. $z^5 + 32 = 0$

76. $z^3 + 4\sqrt{2} - i = 0$

77. $z^2 + 25i = 0$