## Example 9: Solving Equations Using Inverse Trigonometric Functions

Solve the equation $6\sin x - 2 = \sin x$ on the interval $[0, 2\pi)$.

### Solution

$$6\sin x - 2 = \sin x$$
$$5\sin x = 2$$
$$\sin x = 0.4$$

The trigonometric function can be easily isolated.

$$x = \sin^{-1}(0.4) \quad \text{or} \quad x = \pi - \sin^{-1}(0.4)$$

Note that since $\sin x$ is positive, $x$ lies in the first or second quadrant. The solution in the first quadrant is $\sin^{-1}(0.4)$, and the solution in the second quadrant has $\sin^{-1}(0.4)$ as its reference angle.

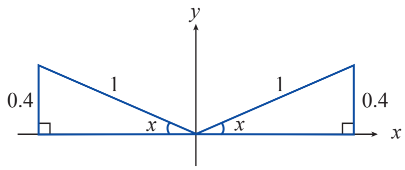


**FIGURE 5**

## 8.4 EXERCISES

### PRACTICE

Use trigonometric identities and algebraic methods, as necessary, to solve the following trigonometric equations. See Examples 1 through 7.

1. $2\sin x + 1 = 0$
2. $4\sin^2 x + 2 = 3$
3. $\sqrt{2} - 2\cos x = 0$
4. $4\cos x = 2$
5. $2\cos x - \sqrt{3} = 0$
6. $\sin(2x) = \sqrt{3}\cos(2x)$
7. $-\dfrac{1}{\sqrt{48}\sin x} = \dfrac{1}{8}$
8. $\sin^2 x - \sin x = 2\sin x - 2$
9. $\sqrt{3}\tan x + 1 = -2$
10. $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$
11. $\sec^2 x - 1 = 0$
12. $\sin^2 x = \sin^2 x + \cos^2 x$
13. $\sec x + \tan x = 1$
14. $\cos x + \sin x \tan x = 2$
15. $\sin^2 x + \cos^2 x + \tan^2 x = 0$
16. $\cos^2 x = 3\sin^2 x$
17. $2\cos^2 x - 3 = 5\cos x$
18. $\sin^3 x = \sin x$
19. $\dfrac{\cot(2x)}{\sqrt{3}} = -1$
20. $\cos x - 1 = \sin x$

Use trigonometric identities, algebraic methods, and inverse trigonometric functions, as necessary, to solve the following trigonometric equations on the interval $[0, 2\pi)$. See Examples 8 and 9.

**21.** $2\sin^2 x + 7\sin x = 4$

**22.** $\tan^2 x = \tan x + 6$

**23.** $2\cos^2 x - 1 = 0$

**24.** $\sec^2 x - 3 = -\tan x$

**25.** $0.05\sin^3 x = 0.1\sin x$

**26.** $12\sin x - 1 = 8\sin x$

**27.** $2\cos^2 x + 11\cos x = -5$

**28.** $10\sin x - 16 = -11 + 18\sin x$

**29.** $\sec^2 x - 2 + 8\tan x = 42$

**30.** $\sin(-x) = 5\sin x - 3$

Verify that the *x*-values given are solutions to the given equation.

**31.** $2\cos x + 1 = 0$

**a.** $x = \frac{2\pi}{3}$ **b.** $x = \frac{4\pi}{3}$

**32.** $3\sec^2 x - 4 = 0$

**a.** $x = \frac{\pi}{6}$ **b.** $x = \frac{5\pi}{6}$

**33.** $2\sin^2 x - \sin x - 1 = 0$

**a.** $x = \frac{\pi}{2}$ **b.** $x = \frac{7\pi}{6}$

**34.** $\tan^2(3x) = 3$

**a.** $x = \frac{\pi}{9}$ **b.** $x = \frac{2\pi}{9}$

**35.** $\csc^4 x - 4\csc^2 x = 0$

**a.** $x = \frac{\pi}{6}$ **b.** $x = \frac{5\pi}{6}$

**36.** $3\cot^2 x - 1 = 0$

**a.** $x = \frac{\pi}{3}$ **b.** $x = \frac{2\pi}{3}$

**37.** $2\cot x + 1 = -1$

**a.** $x = \frac{3\pi}{4}$ **b.** $x = \frac{7\pi}{4}$

**38.** $\csc^2 x = 2\cot x$

**a.** $x = \frac{\pi}{4}$ **b.** $x = \frac{5\pi}{4}$

**39.** $2\sec x + 1 = \sec x + 3$

**a.** $x = \frac{\pi}{3}$ **b.** $x = \frac{5\pi}{3}$

**40.** $2\sin^2(2x) = 1$

**a.** $x = \frac{\pi}{8}$ **b.** $x = \frac{3\pi}{8}$

Determine if the value given is a solution to the trigonometric equation. If the value of *x* is not a solution, give all solutions to the equation.

**41.** $2\cos x = -1; \quad x = \frac{4\pi}{3} + 2n\pi$

**42.** $\tan(3x)(\tan x - 1) = 0; \quad x = \frac{\pi}{4} + n\pi$

**43.** $3\sec^2 x = 4; \quad x = \frac{\pi}{6} + n\pi$

**44.** $\sin^2 x - 3\cos^2 x = 0; \quad x = \frac{\pi}{3} + 2n\pi$

**45.** $\sqrt{3}\csc x = 2; \quad x = \frac{2\pi}{3} + 2n\pi$

**46.** $2\sin^2 x - 1 = 0; \quad x = \frac{\pi}{4} + n\pi$

**47.** $\tan x = -\sqrt{3}; \quad x = \frac{\pi}{6} + n\pi$

**48.** $\tan^2(3x) - 3 = 0; \quad x = \frac{2\pi}{9} + \frac{n\pi}{3}$

**49.** $3\cot^2 x = 1; \quad x = \frac{\pi}{3} + n\pi$

**50.** $\cos(2x)(2\cos x + 1) = 0; \quad x = \frac{5\pi}{6} + n\pi$

Use trigonometric identities and algebra, as necessary, to rewrite the following equations to be quadratic-like, and then solve each on the interval $[0,2\pi)$.

**51.** $2\sin^2 x - \sin x - 1 = 0$

**52.** $2\sin^2 x + 3\cos x - 3 = 0$

**53.** $\sin x - \cos x - 1 = 0$

**54.** $\tan x + \sqrt{3} = \sec x$

**55.** $\cos(2x) - \cos x = 0$

**56.** $2\cos^2 x - \sqrt{3}\cos x = 0$

**57.** $\csc^2 x - 2\cot x = 0$

Solve the following equations on the interval $[0°,360°)$. Give the exact answers when appropriate; otherwise, round your answers to one decimal place.

**58.** $\sin^2 x \cos x - \cos x = 0$

**59.** $\cos^2 x = \sin^2 x$

**60.** $\tan x = \cot x$

**61.** $2 \sin x = \csc x + 1$

**62.** $\sec^2 x - 2 \tan x = 4$

**63.** $\sin^2 x = 2 \sin x - 3$

**64.** $2 \cos^2 x - 1 = -2 \cos x$

**65.** $2\sin x \cot x + \sqrt{3}\cot x - 2\sqrt{3}\sin x - 3 = 0$

Solve the algebraic and trigonometric equations given. Restrict the solutions of the trigonometric equations to the interval $[0,2\pi)$. Give the exact answers for $s$; round your answers for $t$ to four decimal places.

**66.** $6s^2 - 13s + 6 = 0$;   $6 \cos^2 t - 13 \cos t + 6 = 0$

**67.** $s^2 + s - 12 = 0$;   $\sin^2 t + \sin t - 12 = 0$

**68.** $2s^2 + 7s - 15 = 0$;   $2 \tan^2 t + 7 \tan t - 15 = 0$

**69.** $4s^2 - 4s - 1 = 0$;   $4 \cos^2 t - 4 \cos t - 1 = 0$

## APPLICATIONS

**70.** If an arrow is shot by an archer with an initial velocity $v_0$ at an angle of $\theta$ in reference to the horizontal, then its range, the horizontal distance it travels, is given by $r = \frac{1}{32}v_0^2\sin(2\theta)$. If the initial velocity is $v_0 = 100$ feet per second and the arrow hits a target 300 feet from where the archer is standing, what is the value, in degrees, of the angle $\theta$? Round your answer to one decimal place.

**71.** A baseball leaves a bat at an angle of $\theta$ in reference to the horizontal. The initial velocity is $v_0 = 95$ feet per second. The ball is caught 160 feet from where it is hit. What is the value, in degrees, of the angle $\theta$ if the range, the horizontal distance traveled by the ball, is given by $r = \frac{1}{32}v_0^2\sin(2\theta)$? Round your answer to one decimal place.

## TECHNOLOGY

Use a graphing utility to approximate the solutions of the given equation on the interval $[0,2\pi)$. Round your answers to four decimal places.

**72.** $x\tan x-3=0$

**73.** $2\sin x+\cos x=0$

**74.** $2\cos^2 x-\sin x=0$

**75.** $\cot^2 x-\sec^2 x=0$

**76.** $2\sin x-\csc^2 x=0$

**77.** $2\sin x=1-2\cos x$

**78.** $\log x=-\sin x$

**79.** $\sin\left(\frac{x}{2}\right)=2\cos(2x)$

Use a graphing utility to solve the following equations on the interval $[0°,360°)$. Remember to change the mode to degrees.

**80.** $2\sin(2x)=\sqrt{3}$

**81.** $\sin(3x)-\frac{1}{2}=0$

**82.** $2\sin(4x)-1=0$

## WRITING & THINKING

**83.** While working Exercises 66–69, what did you observe as the maximum number of real solutions the algebraic equations can have?

**84.** While working Exercises 66–69, what did you observe as the maximum number of real solutions the trigonometric equations can have on the interval $[0,2\pi)$?

**85.** While working Exercises 80–82, what did you observe about the solutions to the equations of the form $y=\sin(ax)$ on the interval $[0°,360°)$?