8.3 EXERCISES

PRACTICE

Use the given information to determine $\cos(2x)$, $\sin(2x)$, and $\tan(2x)$, if possible. See Example 1.

1. $\sin x = \frac{3}{5}$ and $\cos x$ is positive

2. $\tan x = -4$ and $\sin x$ is negative

3. $\cos x = -\frac{2}{\sqrt{6}}$ and $\sin x$ is positive

4. $\sin x = \frac{1}{\sqrt{5}}$ and $\tan x$ is positive

5. $\tan x = \frac{1}{\sqrt{3}}$ and $\cos x$ is negative

6. $\cos x = -3$ and $\tan x$ is negative

Use a power-reducing identity to rewrite the given expression as directed. See Example 3.

7. Rewrite $\sin^3 x$ in terms containing only first powers of sine and cosine.

8. Rewrite $\sin^4 x$ in terms containing only first powers of cosine.

9. Rewrite $\sin^4 x \cos^2 x$ in terms containing only first powers of cosine.

10. Rewrite $\cos^3 x \sin^2 x$ in terms containing only first powers of cosine.

11. Rewrite $\tan^4 x \sin x$ in terms containing only first powers of sine and cosine.

12. Rewrite $\sin^8 x$ in terms containing only first powers of cosine.

Determine the exact value of each of the following expressions. See Examples 4 and 5.

13. $\sin\left(\frac{3\pi}{8}\right)$

14. $\tan(112°30')$

15. $\cos\left(-\frac{\pi}{12}\right)$

16. $\tan\left(\frac{7\pi}{12}\right)$

17. $\sin 75°$

18. $\cos 165°$

Use the product-to-sum identities to rewrite the given expression as a sum or difference. See Example 6.

19. $\sin(3x)\cos(3x)$

20. $\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)$

21. $5\cos 105° \sin 15°$

22. $2\cos 75° \cos 45°$

23. $\sin(x+y)\sin(x-y)$

24. $\sin\left(\frac{5\pi}{6}\right)\cos\left(\frac{\pi}{6}\right)$

25. $\sin\left(\frac{5\pi}{4}\right)\sin\left(\frac{2\pi}{3}\right)$

26. $\cos\beta\cos(3\beta)$

27. $2\cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{6}\right)$

Use the sum-to-product identities to rewrite the given expression as a product. See Example 7.

28. $\sin\left(\frac{\pi}{4}\right)+\sin\left(\frac{3\pi}{4}\right)$

29. $\sin(6x)+\sin(2x)$

30. $\cos 60° + \cos 30°$

31. $\cos(3\beta)-\cos\beta$

32. $\sin\pi-\sin\left(\frac{\pi}{2}\right)$

33. $\sin 135° - \sin 15°$

34. $\cos(6x)-\cos(2x)$

35. $\cos\left(\frac{7\pi}{6}\right)-\cos\left(\frac{\pi}{4}\right)$

36. $\sin(\pi+\theta)+\sin(\pi-\theta)$

WRITING & THINKING

Verify the following trigonometric identities. See Example 2.

37. $\tan(3x)=\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$

38. $\sin(2x)=\frac{2\tan x}{1+\tan^2 x}$

39. $\frac{\sin(4x)-\sin(2x)}{\cos(4x)+\cos(2x)}=\tan x$

40. $\frac{\sin(3x)}{\sin x}=3-4\sin^2 x$

41. $2\sin^2(3x)=1-\cos(6x)$

42. $\sin(3x)=3\sin x\cos^2 x-\sin^3 x$

43. Two of the double-angle identities were proved in this section. Prove the remaining three double-angle identities.

44. The power-reducing identity for sine was proved in this section. Prove the remaining two power-reducing identities.

45. Prove the half-angle identities for sine and cosine by replacing x with $\frac{x}{2}$ in an appropriately chosen identity.

46. One of the formulas in the tangent half-angle identity was proved in this section. Prove the second formula in the tangent half-angle identity.

47. As mentioned in this section, $\cos(nx)$ can be expressed as a polynomial of degree n in $\cos x$; such polynomials are called Chebyshev polynomials. For $\sin(nx)$, the equivalent rewriting is a product of $\sin x$ and a polynomial of degree $n-1$ in $\cos x$. Expand $\sin(nx)$ and $\cos(nx)$ for $n = 2$, 3, and 4 and compare the results.