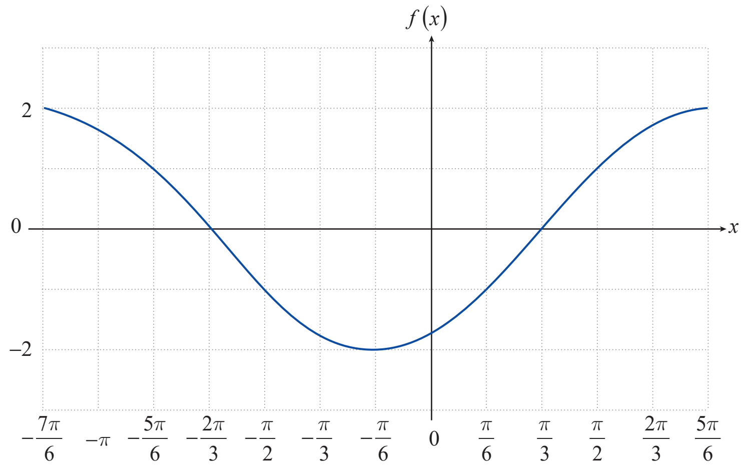

FIGURE 7: $f(x)=2\sin\left(x-\frac{\pi}{3}\right)$

8.2 EXERCISES

PRACTICE

Use the sum and difference identities to determine the exact value of each of the following expressions. See Examples 1, 2, and 3.

1. $\cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

2. $\sin\left(\frac{\pi}{6}+\frac{3\pi}{4}\right)$

3. $\tan\left(\frac{4\pi}{3}+\frac{5\pi}{4}\right)$

4. $\sin\left(\frac{2\pi}{3}+\frac{\pi}{4}\right)$

5. $\tan\left(\frac{\pi}{3}-\frac{3\pi}{4}\right)$

6. $\tan\left(\frac{4\pi}{3}-\frac{5\pi}{4}\right)$

7. $\cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$

8. $\sin\left(\frac{5\pi}{4}-\frac{\pi}{3}\right)$

9. $\sin\left(\frac{7\pi}{4}+\frac{2\pi}{3}\right)$

10. $\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

11. $\tan 75°$

12. $\tan 15°$

13. $\sin 165°$

14. $\cos(-15°)$

15. $\tan 255°$

16. $\cos 195°$

17. $\cos 165°$

18. $\sin\left(\frac{\pi}{12}\right)$

19. $\tan\left(\frac{5\pi}{12}\right)$

20. $\cos\left(\frac{7\pi}{12}\right)$

21. $\cos\left(\frac{25\pi}{12}\right)$

22. $\sin\left(\frac{13\pi}{12}\right)$

23. $\sin\left(\frac{11\pi}{12}\right)$

24. $\tan\left(\frac{7\pi}{12}\right)$

25. $\sin\left(\frac{5\pi}{12}\right)$

26. $\tan\left(\frac{\pi}{12}\right)$

27. Suppose that $\sin\alpha = \frac{4}{5}$ and $\sin\beta = \frac{5}{13}$ and both α and β are in quadrant I. Find $\cos(\alpha - \beta)$.

28. Suppose that $\sin\alpha = -\frac{15}{17}$ and $\cos\beta = -\frac{3}{5}$ and the terminal sides of both α and β are in quadrant III. Find $\sin(\alpha - \beta)$.

29. Suppose that $\cos\alpha = -\frac{15}{17}$ and $\cos\beta = -\frac{3}{5}$, the terminal side of α is in quadrant II, and the terminal side of β is in quadrant III. Find $\sin(\alpha + \beta)$.

30. Suppose that $\cos\alpha = -\frac{24}{25}$ and $\sin\beta = \frac{5}{13}$, the terminal side of α is in quadrant III, and β is in quadrant I. Find $\cos(\alpha + \beta)$.

31. Suppose that $\cos\alpha = \frac{2}{5}$ and $\cos\beta = \frac{1}{5}$ and both α and β are in quadrant I. Find $\sin(\beta - \alpha)$.

32. Suppose that $\cos\alpha = -\frac{2}{3}$ and $\sin\beta = -\frac{2\sqrt{2}}{3}$, the terminal side of α is in quadrant III, and the terminal side of β is in quadrant IV. Find $\tan(\alpha + \beta)$.

Use the sum and difference identities to rewrite each of the following expressions as a trigonometric function of one angle, and then evaluate the result. See Example 4.

33. $\sin 15° \cos 30° + \cos 15° \sin 30°$

34. $\cos\left(\frac{5\pi}{12}\right)\cos\left(\frac{2\pi}{3}\right) + \sin\left(\frac{5\pi}{12}\right)\sin\left(\frac{2\pi}{3}\right)$

35. $\frac{\tan 100° + \tan 35°}{1 - \tan 100° \tan 35°}$

36. $\sin 125° \cos 35° - \cos 125° \sin 35°$

37. $\frac{\tan\left(\frac{5\pi}{16}\right) - \tan\left(\frac{\pi}{16}\right)}{1 + \tan\left(\frac{5\pi}{16}\right)\tan\left(\frac{\pi}{16}\right)}$

38. $\cos 15° \cos 15° - \sin 15° \sin 15°$

39. $\sin 70° \cos 80° + \cos 70° \sin 80°$

40. $\cos\left(\frac{\pi}{5}\right)\cos\left(\frac{3\pi}{10}\right) - \sin\left(\frac{\pi}{5}\right)\sin\left(\frac{3\pi}{10}\right)$

41. $\cos 182° \cos 47° + \sin 182° \sin 47°$

42. $\frac{\tan\left(\frac{5\pi}{12}\right) + \tan\left(\frac{3\pi}{4}\right)}{1 - \tan\left(\frac{5\pi}{12}\right)\tan\left(\frac{3\pi}{4}\right)}$

43. $\frac{\tan 70° - \tan 10°}{1 + \tan 70° \tan 10°}$

44. $\sin\left(\frac{5\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)-\cos\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{12}\right)$

Evaluate each of the following expressions. See Example 6.

45. $\tan\left(\cos^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{12}{13}\right)\right)$

46. $\cos\left(\arctan 1+\arccos\left(\frac{1}{2}\right)\right)$

47. $\sin\left(\arctan\sqrt{3}+\arctan\left(\frac{4}{3}\right)\right)$

48. $\tan\left(\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)-\cos^{-1}\left(\frac{5}{13}\right)\right)$

Express each of the following as an algebraic function of *x*. See Example 7.

49. $\sin\left(\sin^{-1}(2x)+\cos^{-1}(2x)\right)$

50. $\sin\left(\arctan(2x)-\arccos(2x)\right)$

51. $\cos\left(\arctan(2x)-\arcsin x\right)$

52. $\cos\left(\cos^{-1}x-\sin^{-1}x\right)$

53. $\cos\left(\arccos x+\arcsin(2x)\right)$

54. $\sin\left(\sin^{-1}x-\cos^{-1}x\right)$

Express each of the following functions in terms of a single sine function, and graph the result. See Example 9.

55. $f(x)=\sin x+\cos x$

56. $g(x)=\sin x+\sqrt{3}\cos x$

57. $h(\beta)=\sin(2\beta)-\cos(2\beta)$

58. $f(\theta)=-\sqrt{3}\sin\theta+\cos\theta$

59. $g(u)=5\sin(5u)+12\cos(5u)$

60. $h(v)=8\cos\left(\frac{v}{2}\right)+6\sin\left(\frac{v}{2}\right)$

WRITING & THINKING

Use the sum and difference identities to verify the following identities. See Example 5.

61. $\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$ (**Hint:** Use sine and cosine.)

62. $\cos^2 u-\sin^2 v=\cos(u+v)\cos(u-v)$

63. $\cos\left(\frac{3\pi}{2}-\alpha\right)=-\sin\alpha$

64. $\sin(\beta-\theta)+\sin(\beta+\theta)=2\sin\beta\cos\theta$

65. $\tan\left(\alpha-\frac{5\pi}{4}\right)=\frac{\tan\alpha-1}{1+\tan\alpha}$

66. $\sec\left(\frac{\pi}{2}-u\right)=\csc u$

67. $\tan(\pi+2\pi)=0$

68. $\sin\left(\frac{5\pi}{6}+\theta\right)=\frac{1}{2}\left(\cos\theta-\sqrt{3}\sin\theta\right)$

69. $\sin(u+v)\sin(u-v)=\sin^2 u-\sin^2 u$

70. $\cos\left(\frac{7\pi}{4}-\beta\right)=\frac{\sqrt{2}}{2}(\cos\beta-\sin\beta)$

71. Use a cofunction identity to prove the sum and difference identities for sine. (**Hint:** Note that $\sin(u+v)=\cos\left(\frac{\pi}{2}-(u+v)\right)=\cos\left(\left(\frac{\pi}{2}-u\right)-v\right)$ and apply the difference identity for cosine.)

72. Given sum identities for sine and cosine, prove the sum identity for tangent.

73. Prove or disprove that $\sin(u+v)+\sin(u-v)=2\sin u\cos v$.

74. Prove or disprove that $\dfrac{\cos(u+v)}{\cos u\ \cos v}=\tan u+\tan v$.

75. Prove or disprove that $\dfrac{\cos(u-v)}{\cos(u+v)}=2\tan u\ \tan v$.

76. Prove or disprove that $\dfrac{\sin(u+v)}{\sin(u-v)}=\dfrac{\tan u+\tan v}{\tan u-\tan v}$.

77. Use the sine and cosine difference formulas to prove $\tan(u-v)=\dfrac{\tan u-\tan v}{1+\tan u\ \tan v}$.

TECHNOLOGY

Using a graphing utility, determine whether the following identities are true or false. (**Hint:** Graph both expressions on each side of the equality separately and determine if the graphs coincide.)

78. $\sin\left(\dfrac{\pi}{2}-\theta\right)=\cos\theta$

79. $\cos\left(\theta-\dfrac{3\pi}{2}\right)=-\sin\theta$

80. $\cot(\pi+\theta)=-\tan\theta$

81. $\sec\left(\dfrac{\pi}{2}-\theta\right)=\csc\theta$

82. $\sin\left(\dfrac{\pi}{6}+\theta\right)=\dfrac{1}{2}\left(\cos\theta+\sqrt{3}\sin\theta\right)$

83. $\dfrac{1+\tan\theta}{1-\tan\theta}=-\tan\theta$