What we were not given initially is the length labeled $y$ and the actual angle $\theta$. But from Figure 16, we can now recognize that the triangle is a familiar one and that $\theta' = \frac{\pi}{6}$, so $\theta = \frac{5\pi}{6}$. Finally, it must be the case that $y = 1$, and therefore $\tan\theta = -\frac{1}{\sqrt{3}}$.

### Example 8: Using the Relationships between Trigonometric Functions

Given that $\cot\theta = 0.4$ and $\theta$ lies in the first quadrant, determine $\sin\theta$.

#### Solution

All trigonometric functions are ratios, so it will probably be useful to express cotangent as a fraction. The result will help us construct a right triangle that relates to the given information. To that end, note that $\cot\theta = 0.4 = \frac{4}{10} = \frac{2}{5}$. If we take the numerator and denominator as the lengths of the adjacent and opposite sides of a right triangle, we are led to the diagram in Figure 17.

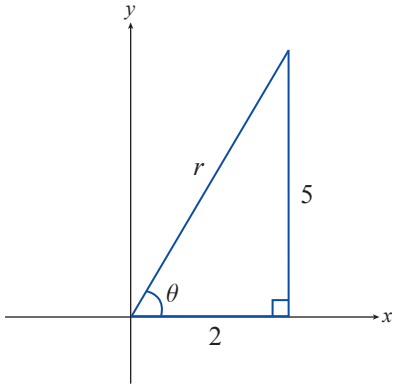


FIGURE 17

Now we can use the Pythagorean Theorem to determine that $r = \sqrt{4+25} = \sqrt{29}$, and so $\sin\theta = \frac{5}{\sqrt{29}}$.

## 7.3 EXERCISES

### PRACTICE

Determine the point $(x, y)$ on the unit circle associated with each real number $s$. See Example 1.

**1.** $s = \frac{\pi}{6}$ **2.** $s = 225°$ **3.** $s = -120°$

**4.** $s = -\frac{\pi}{4}$ **5.** $s = \pi$ **6.** $s = -\frac{8\pi}{3}$

**7.** $s = -750°$ **8.** $s = 855°$ **9.** $s = \frac{31\pi}{6}$

Determine all real numbers $s$ associated with each point $(x, y)$ on the unit circle. See Example 2.

**10.** $(x, y) = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ **11.** $(x, y) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

**12.** $(x, y) = (0, -1)$ **13.** $(x, y) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Determine the values of the six trigonometric functions of each angle $\theta$, using a calculator and rounding your answers to four decimal places if necessary. See Examples 3, 4, and 5.

**14.** $\theta = 45°$ **15.** $\theta = \frac{\pi}{2}$ **16.** $\theta = 60°$

**17.** $\theta = \frac{3\pi}{4}$ **18.** $\theta = \frac{5\pi}{2}$ **19.** $\theta = -520°$

**20.** $\theta = 305°$ **21.** $\theta = -1105°$ **22.** $\theta = 6\pi$

**23.** $\theta = 670°$ **24.** $\theta = \frac{3\pi}{2}$ **25.** $\theta = -215°$

**26.** $\theta = \frac{5\pi}{4}$ **27.** $\theta = 780°$ **28.** $\theta = -445°$

Determine the reference angle associated with the given angle. See Example 4.

**29.** $\theta = 98°$ **30.** $\theta = \frac{9\pi}{2}$ **31.** $\theta = -60°$

**32.** $\theta = \frac{5\pi}{4}$ **33.** $\theta = \frac{5\pi}{2}$ **34.** $\theta = 313°$

**35.** $\theta = \frac{7\pi}{6}$ **36.** $\theta = -168°$ **37.** $\theta = \frac{6\pi}{5}$

**38.** $\theta = 216°$ **39.** $\theta = \frac{3\pi}{2}$ **40.** $\theta = -330°$

**41.** $\theta = \frac{7\pi}{4}$ **42.** $\theta = 718°$ **43.** $\theta = 105°$

Determine the quadrant in which the terminal side of the angle $\theta$ is located.

**44.** $\sin\theta > 0$ and $\tan\theta < 0$ **45.** $\sin\theta < 0$ and $\cot\theta > 0$

**46.** $\tan\theta > 0$ and $\sec\theta > 0$ **47.** $\cos\theta > 0$ and $\cot\theta < 0$

**48.** $\sec\theta < 0$ and $\csc\theta < 0$ **49.** $\cot\theta > 0$ and $\csc\theta > 0$

**50.** $\cot\theta > 0$ and $\cos\theta < 0$ **51.** $\sin\theta > 0$ and $\sec\theta < 0$

In Exercises 52–61, match the given angle $\theta$ with the correct reference angle $\theta'$ among the answer choices **a.**–**c.** Answers will be used more than once.

**a.** $\theta' = 30°$ **b.** $\theta' = 45°$ **c.** $\theta' = 60°$

**52.** $\theta = 300°$ **53.** $\theta = 150°$ **54.** $\theta = -135°$ **55.** $\theta = 210°$

**56.** $\theta = -120°$ **57.** $\theta = 315°$ **58.** $\theta = 510°$ **59.** $\theta = 600°$

**60.** $\theta = 855°$ **61.** $\theta = 480°$

In Exercises 62–76, **a.** rewrite the expression in terms of the given angle's reference angle, and then **b.** evaluate the result, using a calculator and rounding your answers to four decimal places if necessary. See Example 5.

**62.** $\tan 98°$ **63.** $\sin\left(\frac{9\pi}{2}\right)$ **64.** $\cos(-60°)$

**65.** $\tan\left(\frac{5\pi}{4}\right)$ **66.** $\cos\left(\frac{5\pi}{2}\right)$ **67.** $\sin 313°$

**68.** $\cos\left(\frac{7\pi}{6}\right)$ **69.** $\tan(-168°)$ **70.** $\cos\left(\frac{6\pi}{5}\right)$

**71.** $\sin 216°$ **72.** $\tan\left(\frac{3\pi}{2}\right)$ **73.** $\cos(-330°)$

**74.** $\sin\left(\frac{7\pi}{4}\right)$ **75.** $\tan 718°$ **76.** $\sin 105°$

Use the appropriate identity to answer each of the following questions. Choose only one answer per question.

**77.** Which choice is equivalent to $\sin 18°$?

**a.** $\tan 72°$ **b.** $\cos 72°$ **c.** $\csc 72°$ **d.** $\sec 162°$ **e.** $\cos 162°$

**78.** Which choice is equivalent to $\sec\left(\frac{\pi}{6}\right)$?

**a.** $\csc\left(\frac{\pi}{3}\right)$ **b.** $\cos\left(\frac{\pi}{2}\right)$ **c.** $\sin\left(\frac{\pi}{6}\right)$ **d.** $\cos\left(\frac{\pi}{3}\right)$ **e.** $\tan\left(\frac{\pi}{6}\right)$

**79.** Which choice is equivalent to $\tan\left(\frac{\pi}{12}\right)$?

**a.** $\sin\left(\frac{\pi}{2}\right)$ **b.** $\cos\left(\frac{\pi}{12}\right)$ **c.** $\cot\left(\frac{\pi}{2}\right)$ **d.** $\cot\left(\frac{\pi}{12}\right)$ **e.** $\cot\left(\frac{5\pi}{12}\right)$

**80.** Which choice is equivalent to $\cos 87°$?

**a.** $\sin 93°$ **b.** $\cos 93°$ **c.** $\sin 273°$ **d.** $\sec 3°$ **e.** $\sin 3°$

Express each of the following in terms of the appropriate cofunction, and verify the equivalence of the two expressions, using a calculator and rounding your answers to four decimal places if necessary. See Example 6.

**81.** $\cot 135°$ **82.** $\sec\left(\frac{\pi}{2}\right)$ **83.** $\sin(-60°)$

**84.** $\cos\left(-\frac{3\pi}{4}\right)$ **85.** $\csc\left(\frac{5\pi}{6}\right)$ **86.** $\cot 313°$

**87.** $\cos\left(-\frac{3\pi}{6}\right)$ **88.** $\csc(-168°)$ **89.** $\sin\left(-\frac{4\pi}{5}\right)$

**90.** $\sec 216°$ **91.** $\csc\left(\frac{3\pi}{2}\right)$ **92.** $\cos(-15°)$

**93.** $\cot\left(\frac{\pi}{4}\right)$ **94.** $\tan(-105°)$ **95.** $\sec 105°$

Using a calculator, determine $\tan\theta$ and $\cot\theta$ for each of the following exercises. Round your answers to three decimal places.

**96.** $\sin\theta = 0.978$ and $\cos\theta = 0.208$ **97.** $\sin\theta = 0.588$ and $\cos\theta = -0.809$

**98.** $\sin\theta = -0.966$ and $\cos\theta = -0.259$ **99.** $\sin\theta = -0.866$ and $\cos\theta = -0.5$

**100.** $\sin\theta = -0.699$ and $\cos\theta = 0.743$ **101.** $\sin\theta = -0.995$ and $\cos\theta = -0.105$

Use the given information about each angle to evaluate the expressions, if possible. If no angle with the stated properties exists, determine the reason. See Examples 7 and 8.

**102.** Given that $\cos\theta = \frac{\sqrt{12}}{4}$ and $\tan\theta$ is negative, determine $\theta$ and $\tan\theta$.

**103.** Given that $\csc\theta = 1.25$ and the terminal side of $\theta$ lies in the second quadrant, determine $\cot\theta$.

**104.** Given that $\tan\theta = \frac{\sqrt{3}}{3}$ and $\sin\theta$ is positive, determine $\theta$ and $\sin\theta$.

**105.** Given that $\sin\theta = 2$ and $\theta$ is positive, determine $\tan\theta$.

**106.** Given that $\cot\theta = \frac{3}{4}$ and $\sin\theta$ is negative, determine $\sec\theta$.

**107.** Given that $\sin\theta = \frac{\sqrt{3}}{2}$ and $\theta$ lies in the second quadrant, determine $\theta$ and $\tan\theta$.

**108.** Given that $\sec\theta = 0.3$ and the terminal side of $\theta$ lies in the fourth quadrant, determine $\csc\theta$.

The three cofunction identities presented in this section have three companion identities, as follows:

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right), \quad \sec\theta = \csc\left(\frac{\pi}{2} - \theta\right), \quad \text{and} \quad \tan\theta = \cot\left(\frac{\pi}{2} - \theta\right).$$

Express each of the following in terms of the appropriate cofunction. Evaluate both the given expression and the cofunction expression with your calculator to verify that the expressions are equivalent.

**109.** $\sin\left(\frac{7\pi}{4}\right)$ **110.** $\csc\left(\frac{8\pi}{3}\right)$ **111.** $\cot\left(\frac{3\pi}{4}\right)$

**112.** $\cos\left(-\frac{5\pi}{3}\right)$ **113.** $\tan 15°$ **114.** $\sec(-315°)$

## WRITING & THINKING

**115.** Prove the three cofunction identities given in the directions for Exercises 109–114. (**Hint:** For the first identity, begin with the observation that $\sin\theta = \sin\left(\frac{\pi}{2} - \left(\frac{\pi}{2} - \theta\right)\right)$ and then apply one of the three original cofunction identities.)