Solution

Using the notation of the derivation above, we are given $\alpha = 60°1'6''$, $\beta = 56°3'23''$, and $d = 1000$. Converting to decimal notation, $\alpha \approx 60.018333°$ and $\beta \approx 56.056389°$, so we calculate the approximate height of Mt. Baldy as follows.

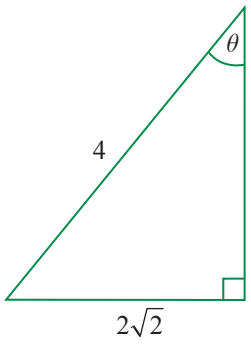
$$\begin{aligned} h &= \frac{1000}{\cot(56.056389°) - \cot(60.018333°)} \\ &\approx \frac{1000}{0.673078 - 0.576924} \\ &\approx 10,400 \text{ feet} \end{aligned}$$

7.2 EXERCISES

PRACTICE

Use the information contained in each figure to determine the values of the six trigonometric functions of θ. Rationalize all denominators in your answers. See Example 1.

1.



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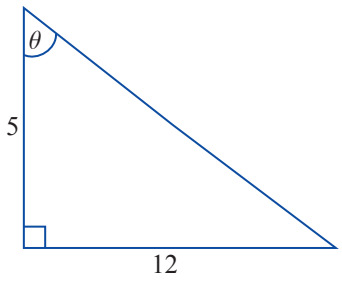

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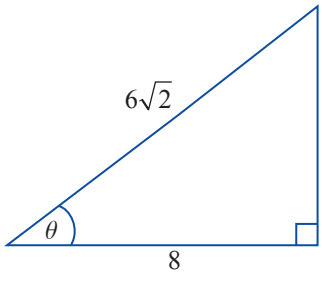

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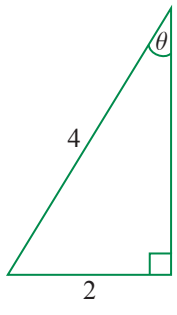

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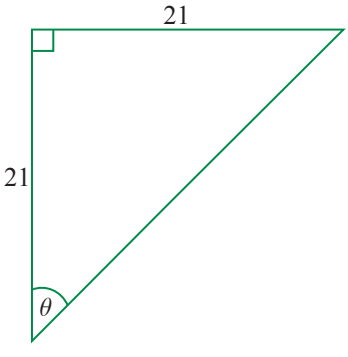

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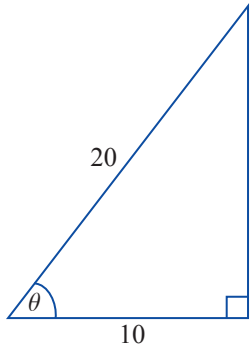

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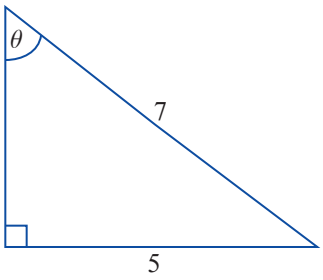

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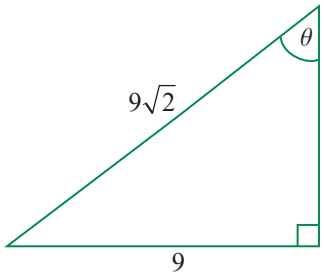

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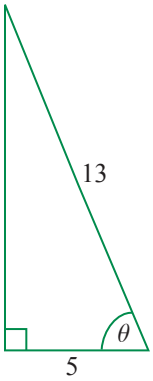

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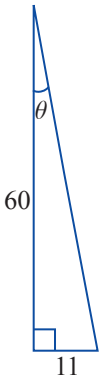

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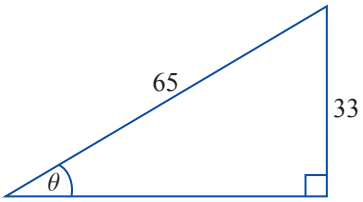

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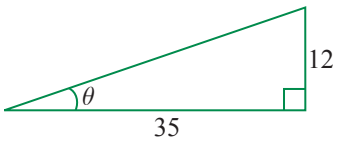

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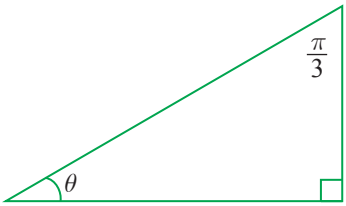

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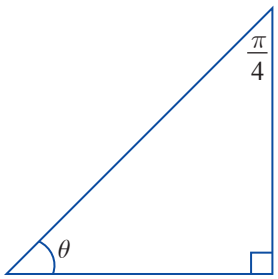


Evaluate the expressions, using a calculator if necessary. Rationalize all denominators in your answers. See Examples 2 and 3.

15. sine and cosecant of $\frac{\pi}{4}$

16. cosine and tangent of $\frac{\pi}{7}$

17. $\sec 60°$

18. $\tan 71°$ and $\cot 71°$

19. $\csc\left(\frac{\pi}{6}\right)$

20. sine of $\frac{3\pi}{7}$

21. secant and tangent of $5°$

22. cosine of $28.37°$

23. cotangent of $\frac{\pi}{3}$

24. $\sin\left(\frac{2\pi}{5}\right)$ and $\cos\left(\frac{2\pi}{5}\right)$

25. $\tan 87.2°$

26. $\csc 54°$

Use a calculator to evaluate each of the following expressions. Round your answers to four decimal places. See Example 3.

27. $\sin 84°$ **28.** $\cos 72°$ **29.** $\tan 46°$ **30.** $\csc 17°$

31. $\sec 88°$ **32.** $\cot 59°$ **33.** $\tan\left(\frac{2\pi}{5}\right)$ **34.** $\cos\left(\frac{\pi}{4}\right)$

35. $\sin\left(\frac{\pi}{8}\right)$ **36.** $\cot\left(\frac{2\pi}{7}\right)$ **37.** $\sec\left(\frac{\pi}{3}\right)$ **38.** $\csc\left(\frac{5\pi}{11}\right)$

Convert each expression from degrees, minutes, seconds (DMS) notation to decimal notation. Round your answers to four decimal places.

39. $38°54'19''$ **40.** $56°12'1''$ **41.** $25°18'90''$

42. $6°8'50''$ **43.** $21°39'56''$ **44.** $88°30'600''$

Determine the value of the given trigonometric expression given the value of another trigonometric expression. Round your answers to four decimal places.

45. Find $\sin\theta$ if $\csc\theta = 8.7$.

46. Find $\cos\theta$ if $\sec\theta = -\frac{7}{4}$.

47. Find $\tan\theta$ if $\cot\theta = \frac{\sqrt{15}}{3}$.

48. Find $\cot\theta$ if $\tan\theta = 2.5$.

49. Find $\sec\theta$ if $\cos\theta = 0.2$.

50. Find $\csc\theta$ if $\sin\theta = -\frac{1}{5}$.

Determine whether the following statements are true or false. Use a calculator when necessary.

51. If $\sin\theta = 0.8$, then $\csc\theta = 1.25$.

52. If $\cos\theta = 0.96$, then $\sec\theta = 1\frac{1}{24}$.

53. If $\tan\theta = 4\frac{4}{9}$, then $\cot\theta = 0.225$.

54. If $\sin\theta = 0.5625$, then $\csc\theta = 2.48$.

55. If $\cos\theta = 0.75$, then $\sec\theta = \frac{8}{3}$.

56. If $\tan\theta = 0.2540$, then $\cot\theta = 3.937$.

APPLICATIONS

Use an appropriate trigonometric function and a calculator if necessary to solve each of the following problems. Round your answers to two decimal places. See Examples 4 and 5.

57. A hang glider wants to determine if a certain vertical cliff is a suitable height for her liftoff. From a distance of 40 yards, she measures the angle from the ground to the cliff's tip as $88°55'24''$. How high is the cliff in feet?

58. A mahi-mahi is hooked on 70 feet of fishing line, 10 feet of which is above the surface of the water. The angle of depression from the water's surface to the line is $40°$. How deep is the fish?

59. A filing cabinet is 3 feet and 4 inches tall from the floor. If a piece of string is stretched from the top of the cabinet to a point on the floor, and the angle between the string and the floor is $11°$, what is the length of the string?

60. A tree being cut down makes a 70° angle with the ground when the tip of the tree is directly above a spot that is 40 feet from the base of the tree. Find the height of the tree.

61. Stephen is standing 15 yards from a stream, but instead of walking directly towards the stream, he decides to take a more scenic (though straight-line) path to the stream. If the angle between the scenic route and the stream is 18°, how far did Stephen walk?

62. The builder of a parking garage wants to build a ramp at an angle of 16° that covers a horizontal span of 40 feet. What is the vertical rise of the ramp?

63. A kitesurfer's lines are 20 m long and make an angle of 37° with the ocean while heading away from the beach under current wind conditions. How high above the water is the kite flying?

64. An anthropologist studying a tribe of indigenous people wants to know the dimensions of their stone-hewn temple. After walking 15 m from the structure, she measures the angle to its top to be 53°. What is the height of the temple?

65. A radio tower has a 64-foot shadow cast by the sun. If the angle from the tip of the shadow to the top of the tower is 78.5°, what is the height of the radio tower?

66. A ladder is propped up to a barn at an 80° angle. If the ladder is 22 feet long, what is the approximate height where the top of the ladder touches the barn?

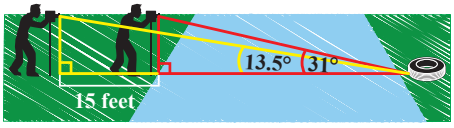
67. The ramp of a moving truck touches the ground 12 feet away from the end of the truck. If the ramp makes an angle of 30° relative to the ground, what is the length of the ramp?

68. The angle of elevation of a flying kite is $61°7'21''$. If the other end of the 40-foot string attached to the kite is tied to the ground, what is the approximate height of the kite?

69. A length of rope is attached from the top of a dock to the rope tie device located on the underside of the boat at the water's surface. The rope is 33 feet in length and has an angle of elevation relative to the surface of the water of 12°. How high above the water does the dock sit?

In Exercises 70–74, use the formula from Example 6.

70. A surveyor wants to find the width of a river without crossing it. He sights an abandoned tire on the opposite bank (the banks are straight and parallel) and measures the angle from where he stands relative to the shore to be 31°. After walking precisely 15 feet away from the tire, he measures the same angle to be 13.5°. How wide is the river?



71. A drawbridge operator in a control room observes a sailboat approaching and finds the angle of depression to the boat to be 9°. Twenty minutes later, the angle to the same boat is 19°. If the sailboat has traveled 68.2 m, how high above water is the control room?

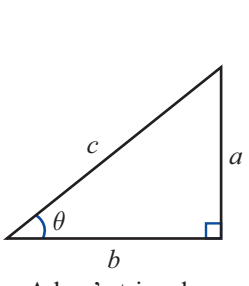
72. A birdwatcher discovers a hawk's nest in a tree some distance away. She wants to determine its height, so she measures the angle from the level ground to the nest at 40°. After approaching 25 feet closer to the tree, she finds the same angle to be 52.5°. How high does the nest sit, in feet?

73. A surveyor standing some distance from a plateau measures the angle of elevation from the ground to the top of the plateau to be $46°57'12''$. The surveyor then walks forward 800 feet and measures the angle of elevation to be $55°37'70''$. What is the height of the plateau?

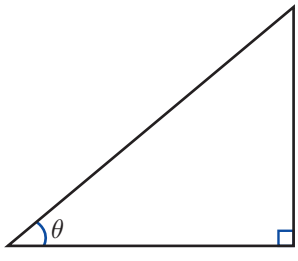
74. A surveyor standing some distance from a hill measures the angle of elevation from the ground to the top of the hill to be $83°45'97''$. The surveyor then steps back 300 feet and measures the angle of elevation to be $75°44'16''$. What is the height of the hill?

WRITING & THINKING

75. To physically demonstrate how the trigonometric functions are defined in terms of ratios of sides of a triangle, Adam draws the triangle shown. Belinda draws another triangle, beginning with the same angle but then proceeding to make her triangle significantly larger, also shown. Nevertheless, when Adam measures the lengths of his triangle's sides and calculates the values of all six trigonometric functions for the angle, and Belinda does the same for her triangle, they obtain the same results. What explains this outcome?



Adam's triangle



Belinda's triangle