TECHNOLOGY

To find and graph the logarithmic function of best fit for a given set of points using a TI-84 Plus, perform the same steps as described in Section 3.2 for linear regression except select LnReg from the CALC menu.

Example 9: Logarithmic Regression

A manufacturer of aviation instruments has designed a new pressure altimeter, a device that determines altitude as a function of atmospheric pressure. The data in Table 1 was collected in order to calibrate the device, where the pressure p (in pascals) was measured by the new device and its altitude h above sea level (in meters) was measured by another instrument of known accuracy. Use the data and logarithmic regression to find a logarithmic function that models altitude as a function of pressure and plot its graph along with the given points.

Pressure p (in pascals)	40,000	50,000	60,000	70,000	80,000	90,000	100,000
Altitude h (in meters)	7309	5670	4279	3064	1982	1005	113

TABLE 1

Solution

Entering the data into a TI-84 Plus and performing a logarithmic regression results in the function $h(p) = 90{,}678.9 - 7859.23 \ln p$. This is the logarithmic curve of best fit for the given data. The data is displayed in Figure 2, and the graph is shown in Figure 3.

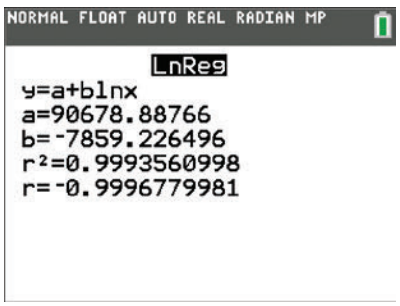

FIGURE 2

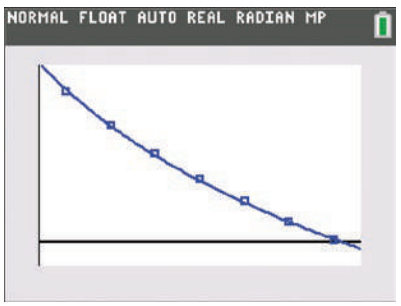

FIGURE 3

6.4 EXERCISES

PRACTICE

Use the properties of logarithms to expand the following expressions as much as possible. Simplify any numerical expressions that can be evaluated without a calculator. See Example 3.

1. $\log_5\left(125x^3\right)$
2. $\ln\left(\dfrac{x^2y}{3}\right)$
3. $\ln\left(\dfrac{e^2p}{q^3}\right)$
4. $\log(100x)$
5. $\log_9\left(9xy^{-3}\right)$
6. $\log_6\left(\sqrt[3]{\dfrac{p^2}{q}}\right)$
7. $\ln\left(\dfrac{\sqrt{x^3}pq^5}{e^7}\right)$
8. $\log_a\sqrt[5]{\dfrac{a^4b}{c^2}}$
9. $\log\left(\log\left(100x^3\right)\right)$

10. $\log_3(9x + 27y)$

11. $\log\left(\frac{10}{\sqrt{x+y}}\right)$

12. $\ln\left(\ln\left(e^{e^x}\right)\right)$

13. $\log_2\left(\frac{y^2+z}{16x^4}\right)$

14. $\log\left(\log\left(100{,}000^{2^x}\right)\right)$

15. $\log_b\left(\sqrt{\frac{x^4y}{z^2}}\right)$

16. $\ln\left(7x^2 - 42x + 63\right)$

17. $\log_b\left(ab^2c^b\right)$

18. $\ln\left(\ln\left(e^{e^x}\right)\right)$

Use the properties of logarithms to condense the following expressions as much as possible, writing each answer as a single term with a coefficient of 1. See Example 4.

19. $\log x - \log y$

20. $\log_5 x - 2\log_5 y$

21. $\log_5\left(x^2 - 25\right) - \log_5(x-5)$

22. $\ln\left(x^2y\right) - \ln y - \ln x$

23. $\frac{1}{3}\log_2 x + \log_2(x+3)$

24. $\frac{1}{5}\left(\log_7\left(x^2\right) - \log_7(pq)\right)$

25. $\ln 3 + \ln p - 2\ln q$

26. $2\left(\log_5\left(\sqrt{x}\right) - \log_5 y\right)$

27. $\log(x-10) - \log x$

28. $2\log a^2b - \log\left(\frac{1}{b}\right) + \log\left(\frac{1}{a}\right)$

29. $3\left(\ln\left(\sqrt[3]{z^2}\right) - \ln(xy)\right)$

30. $\log_2(4x) - \log_2 x$

31. $\log_5 20 - \log_5 5$

32. $\log 30 - \log 2 - \log 5$

33. $\ln 15 + \ln 3$

34. $\ln 8 - \ln 4 + \ln 3$

35. $0.5\log_3 16 - \log_3 4$

36. $3\log_7 2 - 2\log_7 4$

37. $0.25\ln 81 + \ln 4$

38. $2(\log 4 - \log 1 + \log 2)$

39. $\log 11 + 0.5\log 9 - \log 3$

40. $3\log_4\left(x^2\right) + \log_4\left(x^6\right)$

41. $\log_8\left(2x^2 - 2y\right) - 0.25\log_8 16$

42. $\log_{3x} x^2 + \log_{3x} 18 - \log_{3x} 6$

Use the properties of logarithms to write each of the following as a single term that does not contain a logarithm.

43. $5^{2\log_5 x}$

44. $10^{\log y^2 - 3\log x}$

45. $e^{2-\ln x + \ln p}$

46. $e^{5\left(\ln\sqrt[5]{3} + \ln x\right)}$

47. $10^{\log x^3 - 4\log y}$

48. $a^{\log_a b + 4\log_a \sqrt{a}}$

49. $10^{2\log x}$

50. $10^{4\log x - 2\log x}$

51. $\log_4 16 \cdot \log_x x^2$

52. $e^{\ln x + 2 + \ln x^2}$

53. $4^{\log_4(3x) + 0.5\log_4\left(16x^2\right)}$

54. $4^{2\log_2 6 - \log_2 9}$

Evaluate the following logarithmic expressions. See Example 5.

55. $\log_4 17$

56. $2\log_{\frac{1}{3}} 5$

57. $\log_9 8$

58. $\log_2 0.01$

59. $\log_{12} 10.5$

60. $\log(\ln 2)$

61. $\log_6 3^4$

62. $\log_7 14.3$

63. $\log_{\frac{1}{2}} \pi^{-2}$

64. $\log_{\frac{1}{5}} 626$ **65.** $\ln(\log 123)$ **66.** $\log_{17} 0.041$

67. $\log 16$ **68.** $\log_3 9$ **69.** $\log_5 20$

70. $\log_8 26$ **71.** $\log_4 0.25$ **72.** $\log_{1.8} 9$

73. $\log_{2.5} 34$ **74.** $\log_{0.5} 10$ **75.** $\log_4 2.9$

76. $\log_{0.4} 14$ **77.** $\log_{0.2} 17$ **78.** $\log_{0.16} 2.8$

Without using a calculator, evaluate the following expressions.

79. $\log_4 16$ **80.** $\log_5 25^3$ **81.** $\ln e^4 + \ln e^3$

82. $\log_4 \dfrac{1}{64}$ **83.** $\ln e^{1.5} - \log_4 2$ **84.** $\log_2 8^{(2\log_2 4 - \log_2 4)}$

Find the value of x in each of the following equations. Express your answer as exact as possible, or as a decimal rounded to two decimal places.

85. $\log_x 1024 = 4$ **86.** $\log_6 729 = x$ **87.** $\log_2 529 = x$

88. $\log_4 625 = x$ **89.** $\log_x 729 = 9$ **90.** $\log_4 x = 8$

91. $\log_{12} x = 1$ **92.** $\log_x 16{,}807 = 7$ **93.** $\log_4 x = 10$

APPLICATIONS

94. A certain brand of tomato juice has a $[H_3O^+]$ concentration of 6.31×10^{-5} moles/liter. What is the pH of this brand?

95. One type of detergent, when added to neutral water with a pH of 7, results in a solution with a $[H_3O^+]$ concentration that is 5.62×10^{-4} times weaker than that of the water. What is the pH of the solution?

96. What is the concentration of $[H_3O^+]$ in lemon juice with a pH of 2.1?

97. An earthquake in Chile in 2019 measured 6.7 on the Richter scale. What was the intensity, relative to a 0-level earthquake, of this event?

98. How much stronger was the 2001 Gujarat earthquake (6.9 on the Richter scale) than the 2019 earthquake described in Exercise 97?

99. A construction worker operating a jackhammer would experience noise with an intensity of 20 watts/meter^2 if it weren't for ear protection. Given that $I_0 = 10^{-12}$ watts/meter^2, what is the decibel level for such noise?

100. A microphone picks up the sound of a thunderclap and measures its decibel level as 105. Given that $I_0 = 10^{-12}$ watts/meter^2, with what sound intensity did the thunderclap reach the microphone?

101. Matt, a lifeguard, has to make sure that the pH of the swimming pool stays between 7.2 and 7.6. If the pH is out of this range, he has to add chemicals that alter the pH level of the pool. If Matt measures the $\left[H_3O^+\right]$ concentration in the swimming pool to be 2.40×10^{-8} moles/liter, what is the pH? Does he need to change the pH by adding chemicals to the water?

102. The intensity of a cat's soft purring is measured to be 2.19×10^{-11}. Given that $I_0=10^{-12}$ watts/meter^2, what is the decibel level of this noise?

103. Newton's Law of Cooling states that the rate at which an object cools is proportional to the difference between the temperature of the object and the surrounding temperature. If C denotes the surrounding temperature and T_0 denotes the temperature at time $t = 0$, the temperature of an object at time t is given by $T(t)=C+(T_0-C)e^{-kt}$, where k is a constant that depends on the particular object under discussion.

a. You are having friends over for tea and want to know how long after boiling the water it will be drinkable. If the temperature of your kitchen stays around 74 °F and you found online that the constant k for tea is approximately 0.049, how many minutes after boiling the water will the tea be drinkable (you prefer your tea no warmer than 140 °F)? Recall that water boils at 212 °F.

b. As you intern for your local crime scene investigation department, you are asked to determine at what time a victim died. If you are told k is approximately 0.1947 for a human body and the body's temperature was 72 °F at 1:00 a.m., and the body has been in a storage building at a constant 60 °F, approximately what time did the victim die? Recall the average temperature for a human body is 98.6 °F. Note in this situation, t is measured in hours.

c. When helping your father cook a turkey, you were told to remove the turkey when the thickest part had reached 180 °F. If you remove the turkey and place it on the table in a room that is 72 °F, and it cools to 155 °F in 20 minutes, what will the temperature of the turkey be at lunch time (an hour and 15 minutes after the turkey is removed from the oven)? Should you warm the turkey before eating?

TECHNOLOGY

Use a TI-84 Plus to find and graph a logarithmic function of best fit along with the given data. See Example 9.

104. The menu developers for a chain of coffee shops have conducted experiments to see how long customers take to make a drink choice, based on the number of drinks on the menu. Normalizing so that the average time needed to make a choice given just two drinks is 1 time unit, the average times needed to make a choice given n drinks on the menu are shown in the table below.

Number n of choices	2	3	4	5	6	7
Average time t to make a choice	1	1.4	1.7	2	2.2	2.3

105. The manufacturer of the new pressure altimeter in Example 9 expects the relative error in measured pressure to be larger for lower pressure values, so for pressures in the range of 25,000 pascals to 50,000 pascals altitude recordings were taken at 5000 pascal increments, as shown in the table below.

Pressure p (in pascals)	25,000	30,000	35,000	40,000	45,000	50,000
Altitude h (in meters)	10,541	9321	8257	7309	6452	5670