

**Example 6:** Evaluating Logarithmic Expressions

Evaluate the following logarithmic expressions.

a.  $\ln(\sqrt[3]{e})$

b.  $\log 1000$

c.  $\ln(4.78)$

d.  $\log(10.5)$

**Solution**

a.  $\ln(\sqrt[3]{e}) = \ln\left(e^{\frac{1}{3}}\right) = \frac{1}{3}$

No calculator is necessary for this problem, just an application of an elementary property of logarithms.

b.  $\log 1000 = \log(10^3) = 3$

Again, no calculator is required.

c.  $\ln(4.78) \approx 1.564$

This time, a calculator is needed, and only an approximate answer can be given. Be sure to use the correct logarithm.

d.  $\log(10.5) \approx 1.021$

Again, we must use a calculator, though we can say beforehand that the answer should be only slightly larger than 1, as  $\log 10 = 1$  and 10.5 is only slightly larger than 10.

**6.3 EXERCISES****PRACTICE**

Write the following equations in logarithmic terms.

1.  $625 = 5^4$

2.  $216 = 6^3$

3.  $x^3 = 27$

4.  $b^2 = 3.2$

5.  $4.2^3 = C$

6.  $1.3^2 = V$

7.  $4^x = 31$

8.  $16^{2x} = 215$

9.  $(4x)^{\sqrt{3}} = 13$

10.  $e^x = \pi$

11.  $2^{e^x} = 11$

12.  $4^e = N$

Write the following logarithmic equations as exponential equations.

13.  $\log_3 81 = 4$

14.  $\log_2 \left(\frac{1}{8}\right) = -3$

15.  $\log_b 4 = \frac{1}{2}$

16.  $\log_y 9 = 2$

17.  $\log_2 15 = b$

18.  $\log_5 8 = d$

19.  $\log_5 W = 12$

20.  $\log_7 T = 6$

21.  $\log_\pi (2x) = 4$

22.  $\log_{\sqrt{3}} (2\pi) = x$

23.  $\ln 2 = x$

24.  $\ln(5x) = 3$

Sketch the graphs of the following functions. State their domain and range. See Examples 2 and 3.

25.  $f(x) = \log_3(x - 1)$

26.  $g(x) = \log_5(x + 2) - 1$

27.  $r(x) = \log_{\frac{1}{2}}(x - 3)$

28.  $p(x) = 3 - \log_2(x + 1)$

29.  $q(x) = \log_3(2 - x)$

30.  $s(x) = \log_{\frac{1}{3}}(5 - x)$

31.  $h(x) = \log_7(x - 3) + 3$

32.  $m(x) = \log_{\frac{1}{2}}(1 - x)$

33.  $f(x) = \log_3(6 - x)$

34.  $p(x) = 4 - \log(x + 3)$

35.  $s(x) = -\log_{\frac{1}{3}}(-x)$

36.  $g(x) = \log_5(2x) - 1$

Match the graph of the appropriate equation to the logarithmic function.

37.  $f(x) = \log_2(x - 1)$

38.  $f(x) = \log_2(2 - x)$

39.  $f(x) = \log_2(-x)$

40.  $f(x) = \log_2(x - 3)$

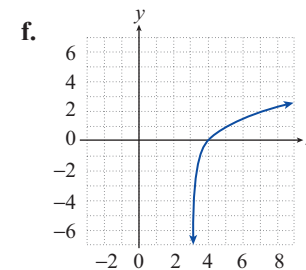
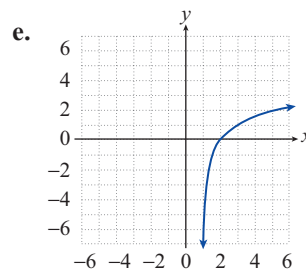
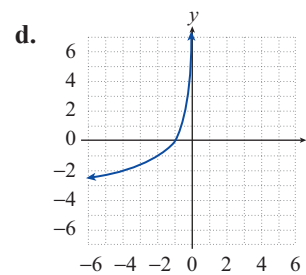
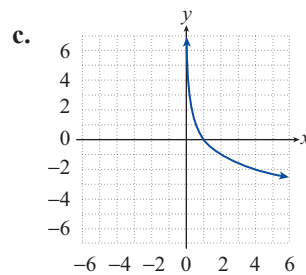
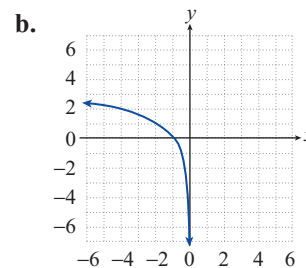
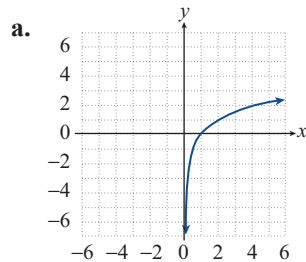
41.  $f(x) = 1 - \log_2 x$

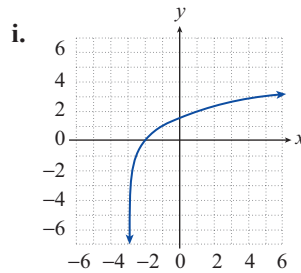
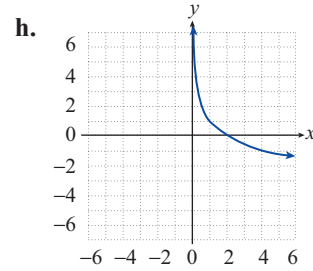
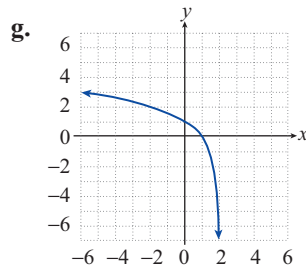
42.  $f(x) = -\log_2 x$

43.  $f(x) = -\log_2(-x)$

44.  $f(x) = \log_2 x$

45.  $f(x) = \log_2(x + 3)$





Evaluate the following logarithmic expressions without the use of a calculator. See Examples 4 and 6.

46.  $\log_7(\sqrt{7})$

47.  $\log_{\frac{1}{2}} 4$

48.  $\log_9\left(\frac{1}{81}\right)$

49.  $\log_3 27$

50.  $\log_{27} 3$

51.  $\log_9\left(\frac{1}{3}\right)$

52.  $\log_{27} 9$

53.  $\log_{\frac{1}{16}}\left(\frac{1}{8}\right)$

54.  $\log_3(\log_{27} 3)$

55.  $\ln e^{2.89}$

56.  $\log(0.0001)$

57.  $\log_a\left(a^{\frac{5}{3}}\right)$

58.  $\ln\left(\frac{1}{e}\right)$

59.  $\log(\log(10^{10}))$

60.  $\log_3 1$

61.  $\ln(\sqrt[5]{e})$

62.  $\log_{\frac{1}{16}} 4$

63.  $\log_8 4^{\log 1000}$

Use the elementary properties of logarithms to solve the following equations. See Example 5.

64.  $\log_{16} x = \frac{3}{4}$

65.  $\log_{16}\left(x^{\frac{1}{2}}\right) = \frac{3}{4}$

66.  $\log_{16} x = -\frac{3}{4}$

67.  $\log_5(5^{\log_3 x}) = 2$

68.  $\log_a(a^{\log_b x}) = 0$

69.  $\log_3(9^{2x}) = -2$

70.  $\log_{\frac{1}{3}}(3^x) = 2$

71.  $\log_7(3x) = -1$

72.  $4^{\log_3 x} = 0$

73.  $\log(x^{10}) = 10$

74.  $\log_x\left(\log_{\frac{1}{2}}\left(\frac{1}{4}\right)\right) = 1$

75.  $6^{\log_x(e^2)} = e$

**Hint:** Note that  $\log_a b = \log_{a^2} b^2$ . This follows from the fact that  $\log_a b = y \Leftrightarrow b = a^y \Leftrightarrow b^2 = a^{2y} = (a^2)^y \Leftrightarrow \log_{a^2} b^2 = y$ .

Solve the following logarithmic equations, using a calculator if necessary to evaluate the logarithms. See Examples 5 and 6. Express your answer either as a fraction or a decimal rounded to two decimal places.

76.  $\log(3x) = 2.1$

77.  $\log(x^2) = -2$

78.  $\ln(x + 1) = 3$

79.  $\ln(2x) = -1$

80.  $\ln(e^x) = 5.6$

81.  $\ln(\ln(x^2)) = 0$

82.  $\log 19 = 3x$

83.  $\log(e^x) = 5.6$

84.  $\log_9(2x - 1) = 2$

85.  $\log(\log(x - 2)) = 1$

86.  $\log(300^{\log x}) = 9$