- The observation that the graph of $p$ crosses the $y$-axis at the easily computed point $\left(0, p(0)\right)$.
- The technique of polynomial long division, useful in dividing one polynomial by another of the same or smaller degree.
- The technique of synthetic division, a shortcut that applies when dividing a polynomial by a polynomial of the form $x - c$. Recall that the remainder of this division is the value $p(c)$.
- The Rational Zero Theorem, which provides a list of potential rational zeros for polynomials with integer coefficients.
- Descartes' Rule of Signs, which provides guidance on the number of positive and negative real zeros that a real-coefficient polynomial might have.
- The Upper and Lower Bounds rule, which indicates an interval in which to search for all the zeros of a real-coefficient polynomial.
- The Intermediate Value Theorem, which can be used to "home in" on a real zero of a given polynomial.

As you solve various polynomial problems, try to keep the big picture in mind. Often, it is useful to literally keep a picture, namely the graph of the polynomial, in mind even if the problem does not specifically involve graphing.

## 5.4 EXERCISES

Throughout these exercises, a graphing utility may be helpful in identifying zeros and in checking your graphing, if permitted by your instructor.

### PRACTICE

Sketch the graph of each factored polynomial. See Example 1.

**1.** $f(x) = (x+1)^4 (x-2)^3 (x-1)$

**2.** $g(x) = -x^3 (x-1)(x+2)^2$

**3.** $f(x) = -x(x+2)(x-1)^2$

**4.** $g(x) = (x+2)(x-1)^3$

**5.** $f(x) = (x-1)^4 (x-2)(x-3)$

**6.** $g(x) = (x+1)^2 (x-2)^3$

**7.** $f(x) = (x-4)(x+2)^2 (x-3)^3$

**8.** $g(x) = (x+3)(x-1)^5$

Use all available methods to factor each of the following polynomials completely, and then sketch the graph of each one. See Example 1.

**9.** $f(x) = x^5 + 4x^4 + x^3 - 10x^2 - 4x + 8$

**10.** $p(x) = 2x^3 - x^2 - 8x - 5$

**11.** $s(x) = -x^4 + 2x^3 + 8x^2 - 10x - 15$

**12.** $f(x) = -x^3 + 6x^2 - 12x + 8$

**13.** $H(x) = x^4 - x^3 - 5x^2 + 3x + 6$

**14.** $h(x) = x^5 - 11x^4 + 46x^3 - 90x^2 + 81x - 27$

**15.** $f(x) = 2x^3 + 11x^2 + 20x + 12$

**16.** $g(x) = x^4 + 3x^3 - 5x^2 - 21x - 14$

Use all available methods to solve each polynomial equation.

**17.** $x^5 + 4x^4 + x^3 = 10x^2 + 4x - 8$

**18.** $x^4 + 15 = 2x^3 + 8x^2 - 10x$

**19.** $x^4 + x^3 + 3x^2 + 5x - 10 = 0$

**20.** $x^3 - 9x^2 = 30 - 28x$

**21.** $x^5 + x^4 - x^3 + 7x^2 - 20x + 12 = 0$

**22.** $2x^4 - 5x^3 - 2x^2 + 15x = 0$

**23.** $x^5 + 15x^3 + 16 = x^4 + 15x^2 + 16x$

**24.** $x^3 - 5 = 5x^2 - 9x$

Use all available methods (in particular, the Conjugate Roots Theorem, if applicable) to factor each of the following polynomials completely, making use of the given zero if one is given. See Example 2.

**25.** $f(x) = x^4 - 9x^3 + 27x^2 - 15x - 52$; $3 - 2i$ is a zero.

**26.** $g(x) = x^3 - (1-i)x^2 - (8-i)x + (12-6i)$; $2 - i$ is a zero.

**27.** $f(x) = x^3 - (2+3i)x^2 - (1-3i)x + (2+6i)$; 2 is a zero.

**28.** $p(x) = x^4 - 2x^3 + 14x^2 - 8x + 40$; $2i$ is a zero.

**29.** $n(x) = x^4 - 4x^3 + 6x^2 + 28x - 91$; $2 + 3i$ is a zero.

**30.** $G(x) = x^4 - 14x^3 + 98x^2 - 686x + 2401$; $7i$ is a zero.

**31.** $f(x) = x^4 - 3x^3 + 5x^2 - x - 10$

**32.** $g(x) = x^6 - 8x^5 + 25x^4 - 40x^3 + 40x^2 - 32x + 16$

**33.** $r(x) = x^4 + 7x^3 - 41x^2 + 33x$

**34.** $d(x) = x^5 - x^4 - 18x^3 + 18x^2 + 81x - 81$

**35.** $P(x) = x^3 - 6x^2 + 28x - 40$

**36.** $g(x) = x^6 - x^4 - 16x^2 + 16$

## WRITING & THINKING

Construct polynomial functions with the stated properties. See Example 3.

**37.** Third-degree, only real coefficients, $-1$ and $5+i$ are two of the zeros, $y$-intercept is $-52$.

**38.** Fourth-degree, only real coefficients, $\sqrt{7}$ and $i\sqrt{5}$ are two of the zeros, $y$-intercept is $-35$.

**39.** Fifth-degree, 1 is a zero of multiplicity 3, $-2$ is the only other zero, leading coefficient is 2.

**40.** Fifth-degree, only real coefficients, 0 is the only real zero, $1+i$ is a zero of multiplicity 1, leading coefficient is 1.

**41.** Fourth-degree, only real coefficients, $x$-intercepts are 0 and 6, $-2i$ is a zero, leading coefficient is 3.

**42.** Fifth-degree, $-2$ is a zero of multiplicity 2, another integer is a zero of multiplicity 3, $y$-intercept is 108, leading coefficient is 1.

**43.** Third-degree, only real coefficients, $-4$ and $3+i$ are two of the zeros, $y$-intercept is $-40$.

**44.** Fifth-degree, 1 is a zero of multiplicity 4, $-2$ is the only other zero, leading coefficient is 4.

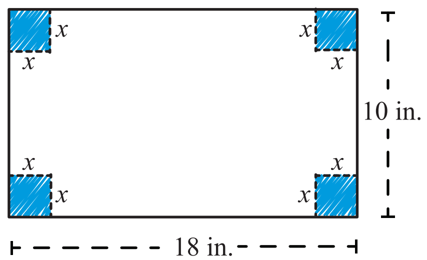
**45.** Third-degree, only real coefficients, $-4$ and $4+i$ are two of the zeros, $y$-intercept is $-68$.

**46.** Assume $f(x)$ is an $n^{\text{th}}$-degree polynomial with real coefficients. Explain why the following statement is true: If $n$ is even, the number of turning points is odd and if $n$ is odd, the number of turning points is even.

## APPLICATIONS

**47.** An open-top box is to be constructed from a 10 inch by 18 inch sheet of tin by cutting out squares from each corner as shown and then folding up the sides. Let $V(x)$ denote the volume of the resulting box.

**a.** Write $V(x)$ as a product of linear factors.
**b.** For which values of $x$ is $V(x)=0$?
**c.** Which answers from part b. are physically possible?



**48.** An open-top box is to be constructed from a 10 inch by 15 inch sheet of tin by cutting out squares from each corner and then folding up the sides. Let $V(x)$ denote the volume of the resulting box.

**a.** Write $V(x)$ as a product of linear factors.
**b.** For which values of $x$ is $V(x)=0$?
**c.** Which of your answers from part b. are physically possible?

**49.** An open-top box is to be constructed from a 9 inch by 17 inch sheet of tin by cutting out squares from each corner and then folding up the sides. Let $V(x)$ denote the volume of the resulting box.

**a.** Write $V(x)$ as a product of linear factors.
**b.** For which values of $x$ is $V(x)=0$?
**c.** Which of your answers from part b. are physically possible?