5.3 EXERCISES

PRACTICE

List all of the potential rational zeros of the following polynomials. Then use polynomial division and the quadratic formula, if necessary, to identify the actual zeros. See Example 1.

1. $f(x) = 3x^3 + 5x^2 - 26x + 8$
2. $g(x) = -2x^3 + 11x^2 + x - 30$
3. $p(x) = x^4 - 5x^3 + 10x^2 - 20x + 24$
4. $h(x) = x^3 - 3x^2 + 9x + 13$
5. $q(x) = x^3 - 10x^2 + 23x - 14$
6. $r(x) = x^4 + x^3 + 23x^2 + 25x - 50$
7. $s(x) = 2x^3 - 9x^2 + 4x + 15$
8. $t(x) = x^3 - 6x^2 + 13x - 20$
9. $j(x) = 3x^4 - 3$
10. $k(x) = x^4 - 10x^2 + 24$
11. $m(x) = x^3 + 11x^2 - x - 11$
12. $g(x) = x^3 - 6x^2 - 5x + 30$

Using the Rational Zero Theorem or your answers to the preceding problems, solve the following polynomial equations.

13. $x^4 + x - 2 = -2x^4 + x + 1$
14. $x^4 + 10 = 10x^2 - 14$
15. $x^3 - 3x^2 + 9x + 13 = 0$
16. $3x^3 + 5x^2 = 26x - 8$
17. $x^4 + 10x^2 - 20x = 5x^3 - 24$
18. $-2x^3 + 11x^2 + x = 30$
19. $2x^3 - 12x^2 + 26x = 40$
20. $2x^3 + 9x^2 + 4x = 15$
21. $x^4 + x^3 + 23x^2 = 50 - 25x$
22. $x^3 + 23x = 10x^2 + 14$
23. $x^3 + 11x^2 = 11 + x$
24. $-6x^2 + x^3 = 5x - 30$

Use Descartes' Rule of Signs to determine the possible numbers of positive and negative real zeros of each of the following polynomials. See Example 2.

25. $f(x) = x^3 + 8x^2 + 17x + 10$
26. $g(x) = x^3 + 2x^2 - 5x - 6$
27. $f(x) = x^3 - 6x^2 + 3x + 10$
28. $g(x) = x^3 + 6x^2 + 11x + 6$
29. $f(x) = x^4 - 5x^3 - 2x^2 + 40x - 48$
30. $g(x) = x^3 + 3x^2 + 3x + 9$
31. $f(x) = x^4 - 25$
32. $g(x) = x^4 - 7x^3 + 5x^2 + 31x - 30$
33. $f(x) = 5x^5 - x^4 + 2x^3 + x - 9$
34. $g(x) = -6x^7 - x^5 - 7x^3 - 2x$
35. $f(x) = -5x^{11} - 14x^9 - 10x^7 - 15x^5$
36. $g(x) = 2x^4 + 7x^3 + 28x^2 + 112x - 64$

Use synthetic division to identify upper and lower bounds of the real zeros of the following polynomials. See Example 3.

37. $f(x) = x^3 + 4x^2 + x - 4$

38. $f(x) = 2x^3 - 3x^2 - 8x - 3$

39. $f(x) = x^3 - 6x^2 + 3x + 10$

40. $g(x) = x^3 + 6x^2 + 11x + 6$

41. $f(x) = x^4 - 5x^3 - 2x^2 + 40x - 48$

42. $g(x) = x^3 + 3x^2 + 3x + 9$

43. $f(x) = x^4 - 25$

44. $g(x) = x^4 - 7x^3 + 5x^2 + 31x - 30$

45. $f(x) = 2x^3 - 7x^2 - 28x - 12$

46. $g(x) = x^5 + x^4 - 9x^3 - x^2 + 20x - 12$

Using your answers to the preceding problems, polynomial division, and the quadratic formula, if necessary, find all of the zeros of the following polynomials.

47. $f(x) = x^3 + 4x^2 - x - 4$

48. $f(x) = 2x^3 - 3x^2 - 8x - 3$

49. $f(x) = x^3 - 6x^2 + 3x + 10$

50. $g(x) = x^3 + 6x^2 + 11x + 6$

51. $f(x) = x^4 - 5x^3 - 2x^2 + 40x - 48$

52. $g(x) = x^3 + 3x^2 + 3x + 9$

53. $f(x) = x^4 - 25$

54. $g(x) = x^4 - 7x^3 + 5x^2 + 31x - 30$

55. $f(x) = 2x^3 - 7x^2 - 28x - 12$

56. $g(x) = x^5 + x^4 - 9x^3 - x^2 + 20x - 12$

Use the Intermediate Value Theorem to show that each of the following polynomials has a real zero between the indicated values. See Example 5.

57. $f(x) = 5x^3 - 4x^2 - 31x - 6;\ -3$ and -1

58. $f(x) = x^4 - 9x^2 - 14;\ 1$ and 4

59. $f(x) = x^4 + 2x^3 - 10x^2 - 14x + 21;\ 2$ and 3

60. $f(x) = -x^3 + 2x^2 + 13x - 26;\ -4$ and -3

Show that each of the following equations must have a solution between the indicated real numbers.

61. $14x + 10x^2 = x^4 + 2x^3 + 21;\ 2$ and 3

62. $x^3 - 2x^2 = 13(x - 2);\ -4$ and -3

Using any of the methods discussed in this section as guides, find all of the real zeros of the following functions.

63. $f(x) = 3x^3 - 18x^2 + 9x + 30$

64. $f(x) = -4x^3 - 19x^2 + 29x - 6$

65. $f(x) = 3x^5 + 7x^4 + 12x^3 + 28x^2 - 15x - 35$

66. $f(x) = 2x^4 + 5x^3 - 9x^2 - 15x + 9$

67. $f(x) = -15x^4 + 44x^3 + 15x^2 - 72x - 28$

68. $f(x) = 2x^4 + 13x^3 - 23x^2 - 32x + 20$

69. $f(x) = 3x^4 + 7x^3 - 25x^2 - 63x - 18$

70. $f(x) = x^5 + 7x^4 + 5x^3 - 43x^2 - 42x + 72$

71. $f(x) = 2x^5 - 3x^4 - 47x^3 + 103x^2 + 45x - 100$

72. $f(x) = x^6 - 125x^4 + 4804x^2 - 57,600$

Using any of the methods discussed in this section as guides, solve the following equations.

73. $x^3 + 6x^2 + 11x = -6$

74. $x^3 - 7x = 6(x^2 - 10)$

75. $x^3 + 9x^2 = 2x + 18$

76. $6x^3 + 14 = 41x^2 + 9x$

77. $4x^3 = 18x^2 + 106x + 48$

78. $3x^3 + 15x^2 - 6x = 72$

79. $8x^4 + 24 + 8x = 2x^3 + 38x^2$

80. $x^4 + 7x^2 = 3x^3 + 21x$

81. $6x^6 - 10x^5 - 9x^4 + 27x^3 = 20x^2 + 18x - 30$

82. $4x^5 - 5x^4 + 20x^2 = 6x^3 + 25x + 30$

WRITING & THINKING

83. Create a proof of the Rational Zero Theorem by following the suggested steps.

a. Assuming $\frac{p}{q}$ is a zero of the polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, show that the equation $a_n\left(\frac{p}{q}\right)^n + a_{n-1}\left(\frac{p}{q}\right)^{n-1} + \cdots + a_1\left(\frac{p}{q}\right) + a_0 = 0$ can be written in the form $a_n p^n + a_{n-1}p^{n-1}q + \cdots + a_1 pq^{n-1} = -a_0 q^n$.

b. It can be assumed that $\frac{p}{q}$ is written in lowest terms (that is, the greatest common divisor of p and q is 1). By examining the left-hand side of the last equation above, show that p must be a divisor of the right-hand side, and hence a factor of a_0.

c. By rearranging the equation so that all terms with a factor of q are on one side, use a similar argument to show that q must be a factor of a_n.