But does $p(x) \to -\infty$ as $x \to \infty$? No, if we multiply out, the leading term of this polynomial would be x^3, which has a positive leading coefficient. To fix this, we multiply the entire polynomial by -1.

$$\begin{aligned} p(x) &= -(x+3)(x-2)(x-5) \\ &= -x^3 + 4x^2 + 11x - 30 \end{aligned}$$

b. Once again, $p(x)$ is a product of linear factors, identified by the required zeros.

$$p(x) = (x+5)(x+2)(x-1)(x-3)$$

Our second condition is that the y-intercept must be $(0,15)$. If we substitute $x = 0$, we see that $p(0) = (5)(2)(-1)(-3) = 30$, so the y-intercept is $(0,30)$. To fix this, we might try subtracting 15 from the polynomial.

$$p(x) \stackrel{?}{=} (x+5)(x+2)(x-1)(x-3) - 15$$

But, we can not do this because it causes -5, -2, 1 and 3 to no longer be zeros! Instead, we multiply $p(x)$ by $\frac{1}{2}$.

$$\begin{aligned} p(x) &= \frac{1}{2}(x+5)(x+2)(x-1)(x-3) \\ &= \frac{1}{2}x^4 + \frac{3}{2}x^3 - \frac{15}{2}x^2 - \frac{19}{2}x + 15 \end{aligned}$$

5.2 EXERCISES

PRACTICE

Use polynomial long division to rewrite each of the following fractions in the form $q(x) + \frac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder. See Examples 1 through 3.

1. $\dfrac{6x^4 - 2x^3 + 8x^2 + 3x + 1}{2x^2 + 2}$

2. $\dfrac{5x^2 + 9x - 6}{x + 2}$

3. $\dfrac{x^3 - 6x^2 + 12x - 10}{x^2 - 4x + 4}$

4. $\dfrac{7x^5 - x^4 + 2x^3 - x^2}{x^2 + 1}$

5. $\dfrac{4x^3 - 6x^2 + x - 7}{x + 2}$

6. $\dfrac{x^3 + 2x^2 - 4x - 8}{x - 3}$

7. $\dfrac{3x^5 + 18x^4 - 7x^3 + 9x^2 + 4x}{3x^2 - 1}$

8. $\dfrac{9x^5 - 10x^4 + 18x^3 - 28x^2 + x + 3}{9x^2 - x - 1}$

9. $\dfrac{2x^5 - 5x^4 + 7x^3 - 10x^2 + 7x - 5}{x^2 - x + 1}$

10. $\dfrac{14x^5 - 2x^4 + 27x^3 - 3x^2 + 9x}{2x^3 + 3x}$

11. $\dfrac{x^4+x^2-20x-8}{x-3}$

12. $\dfrac{2x^5-3x^2+1}{x^2+1}$

13. $\dfrac{9x^3+2x}{3x-5}$

14. $\dfrac{-4x^5+8x^3-2}{2x^3+x}$

15. $\dfrac{2x^2+x-8}{x+3}$

16. $\dfrac{5x^5+x^4-13x^3-2x^2+6x}{x^3-2x}$

17. $\dfrac{2x^3-3ix^2+11x+(1-5i)}{2x-i}$

18. $\dfrac{9x^3-(18+9i)x^2+x+(-2-i)}{x-2-i}$

19. $\dfrac{3x^3+ix^2+9x+3i}{3x+i}$

20. $\dfrac{35x^4+(14-10i)x^3-(7+4i)x^2+2ix}{7x-2i}$

Use synthetic division to determine if the given value for c is a zero of the corresponding polynomial. If not, determine $p(c)$. See Example 4.

21. $p(x)=32x^5-80x^4+80x^3-40x^2+10x+2;\ c=1$

22. $p(x)=32x^5-80x^4+80x^3-40x^2+10x+2;\ c=\dfrac{1}{2}$

23. $p(x)=12x^4-7x^3-32x^2-7x+6;\ c=2$

24. $p(x)=12x^4-7x^3-32x^2-7x+6;\ c=1$

25. $p(x)=12x^4-7x^3-32x^2-7x+6;\ c=\dfrac{1}{3}$

26. $p(x)=2x^2-(3-5i)x+(3-9i);\ c=-2$

27. $p(x)=8x^4-2x+6;\ c=1$

28. $p(x)=x^4-1;\ c=1$

29. $p(x)=x^5+32;\ c=-2$

30. $p(x)=3x^5+9x^4+2x^2+5x-3;\ c=-3$

31. $p(x)=2x^2-(3-5i)x+(3-9i);\ c=-3i$

32. $p(x)=x^2-6x+13;\ c=2$

33. $p(x)=x^2-6x+13;\ c=3-2i$

34. $p(x)=3x^3-13x^2-28x-12;\ c=-2$

35. $p(x)=3x^3-13x^2-28x-12;\ c=6$

36. $p(x)=2x^3-8x^2-23x+63;\ c=2$

37. $p(x)=2x^3-8x^2-23x+63;\ c=5$

38. $p(x)=x^4-3x^3-3x^2+11x-6;\ c=1$

39. $p(x)=x^4-3x^3-3x^2+11x-6;\ c=-2$

40. $p(x)=x^4-3x^3-3x^2+11x-6;\ c=3$

Use synthetic division to rewrite each of the following fractions in the form $q(x)+\frac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder. See Example 5.

41. $\frac{x^3+x^2-18x+9}{x+5}$

42. $\frac{-2x^5+4x^4+3x^3-7x^2+3x-2}{x-2}$

43. $\frac{x^8+x^7-3x^3-3x^2+3}{x+1}$

44. $\frac{x^8-5x^7-3x^3+15x^2-2}{x-5}$

45. $\frac{4x^3-(16+4i)x^2+(14+4i)x+(-6-2i)}{x-3-i}$

46. $\frac{x^6-2x^5+2x^4+4x^2-8x+8}{x-1+i}$

47. $\frac{x^5-3x^4+x^3-5x^2+18}{x-2}$

48. $\frac{x^5-3x^4+x^3-5x^2+18}{x-3}$

49. $\frac{x^4+(i-1)x^3+(1-i)x^2+ix}{x+i}$

50. $\frac{x^6+8x^5+x^3+8x^2-14x-112}{x+8}$

51. $\frac{2x^3-10ix^2+5x+(8-3i)}{x-3i}$

52. $\frac{4x^5-6x^4+10x^3-4x^2-4x}{x-1}$

WRITING & THINKING

Construct a polynomial function with the stated properties. See Example 6.

53. Second-degree, zeros of –4 and 3, and goes to $-\infty$ as $x \to -\infty$.

54. Third-degree, zeros of –2, 1, and 3, and a y-intercept of –12.

55. Second-degree, zeros of $2-3i$ and $2+3i$, and a y-intercept of –13.

56. Third-degree, zeros of $1-i$, $2+i$, and –1, and a leading coefficient of –2.

57. Fourth-degree and a single x-intercept of 3.

58. Second-degree, zeros of $-\frac{3}{4}$ and 2, and a y-intercept of 6.

59. Fourth-degree, zeros of –3, –2, and 1, and a y-intercept of 18.

60. Third-degree, zeros of 1, 2, and 3, and passes through the point $(4, 12)$.

APPLICATIONS

61. A box company makes a variety of boxes, all with volume given by the formula $x^3+10x^2+31x+30$. If the height is given by $x+3$, what is the formula for the surface area of the base?