4.3 EXERCISES

PRACTICE

In each of the following exercises, use the information given to determine **a.** $(f+g)(-1)$, **b.** $(f-g)(-1)$, **c.** $(fg)(-1)$, and **d.** $\left(\frac{f}{g}\right)(-1)$. See Examples 1, 2, and 3.

1. $f(-1)=-3$ and $g(-1)=5$

2. $f(-1)=0$ and $g(-1)=-1$

3. $f(x)=x^2-3$ and $g(x)=x$

4. $f(x)=\sqrt[3]{x}$ and $g(x)=x-1$

5. $f(-1)=15$ and $g(-1)=-3$

6. $f(x)=\frac{x+5}{2}$ and $g(x)=6x$

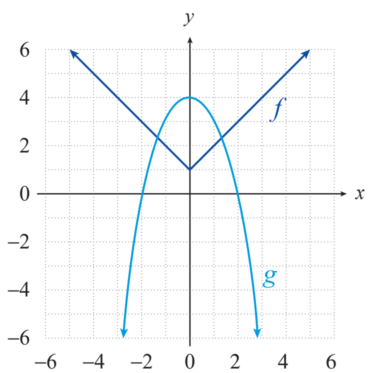
7. $f(x)=x^4+1$ and $g(x)=x^{11}+2$

8. $f(x)=\frac{6-x}{2}$ and $g(x)=\sqrt{\frac{x}{-4}}$

9. $f=\{(5,2),(0,-1),(-1,3),(-2,4)\}$ and $g=\{(-1,3),(0,5)\}$

10. $f=\{(3,15),(2,-1),(-1,1)\}$ and $g(x)=-2$

11.



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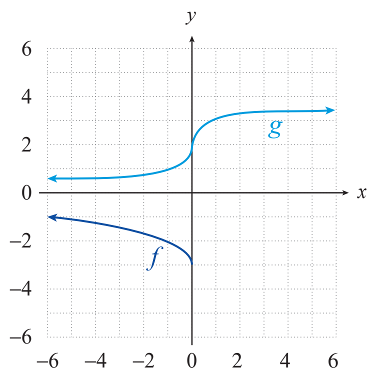

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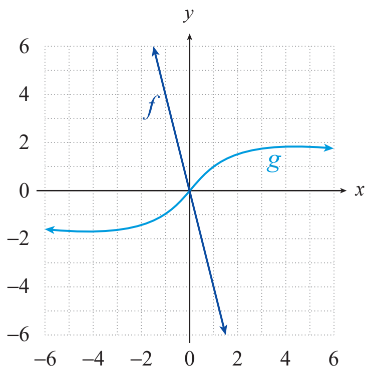

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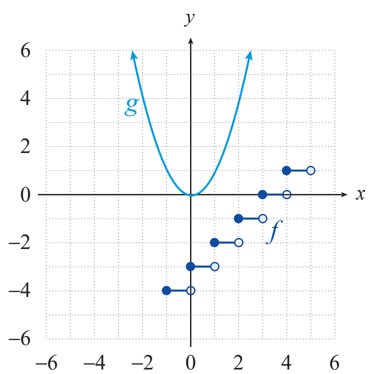


In each of the following exercises, find **a.** the formula and domain for $(f+g)$ and **b.** the formula and domain for $\frac{f}{g}$. See Examples 2 and 3.

15. $f(x)=|x|$ and $g(x)=\sqrt{x}$

16. $f(x)=x^2-1$ and $g(x)=\sqrt[3]{x}$

17. $f(x)=x-1$ and $g(x)=x^2-1$

18. $f(x)=x^{\frac{3}{2}}$ and $g(x)=x-3$

19. $f(x)=3x$ and $g(x)=x^3-8$

20. $f(x)=x^3+4$ and $g(x)=\sqrt{x-2}$

21. $f(x)=-2x^2$ and $g(x)=|x+4|$

22. $f(x)=6x-1$ and $g(x)=x^{\frac{2}{3}}$

In each of the following exercises, use the information given to determine $(f \circ g)(3)$. See Examples 4 and 5.

23. $f(-5)=2$ and $g(3)=-5$

24. $f(\pi)=\pi^2$ and $g(3)=\pi$

25. $f(x)=x^2-3$ and $g(x)=\sqrt{x}$

26. $f(x)=\sqrt{x^2-9}$ and $g(x)=1-2x$

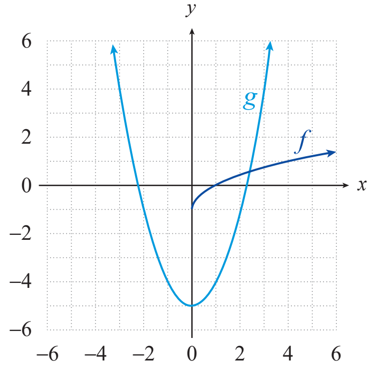
27. $f(x)=2+\sqrt{x}$ and $g(x)=x^3+x^2$

28. $f(x)=x^{\frac{3}{2}}-3$ and $g(x)=\left|\frac{4x}{3}\right|$

29. $f(x)=\sqrt{x+6}$ and $g(x)=\sqrt{4x-3}$

30. $f(x)=\sqrt{\frac{3x}{14}}$ and $g(x)=x^4-x^3-x^2-x$

31.



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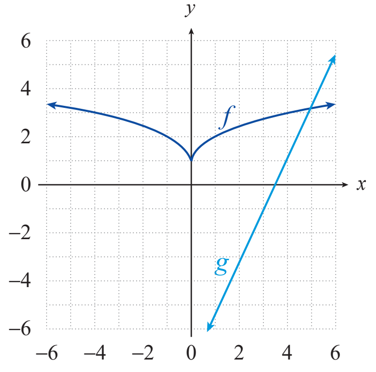

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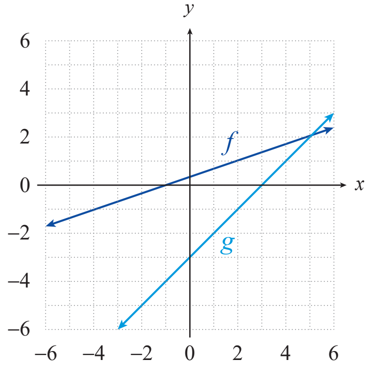


In each of the following exercises, find **a.** the formula and domain for $f \circ g$ and **b.** the formula and domain for $g \circ f$. See Example 6.

34. $f(x)=\frac{1}{x}$ and $g(x)=x-1$

35. $f(x)=\frac{4x-2}{3}$ and $g(x)=\frac{1}{x}$

36. $f(x)=1-x$ and $g(x)=\sqrt{x}$

37. $f(x)=|x-3|$ and $g(x)=x^3+1$

38. $f(x)=x^2+2x$ and $g(x)=x-3$

39. $f(x)=\sqrt{x-1}$ and $g(x)=\frac{x+1}{2}$

40. $f(x)=x^3+4x^2$ and $g(x)=|x|-1$

41. $f(x)=-3x+2$ and $g(x)=x^2+2$

42. $f(x)=x+2$ and $g(x)=\frac{x^2+3}{2}$

43. $f(x)=\sqrt{x-1}$ and $g(x)=x^2$

Write each of the following functions as a composition of two functions. Answers will vary. See Example 7.

44. $f(x)=\sqrt[3]{3x^2-1}$

45. $f(x)=\frac{2}{5x-1}$

46. $f(x)=|x-2|+3$

47. $f(x)=x+\sqrt{x+2}-5$

48. $f(x)=|x^3-5x|+7$

49. $f(x)=\frac{\sqrt{x-3}}{x^2-6x+9}$

50. $f(x)=\sqrt{2x^3-3}-4$

51. $f(x)=|x^2+3x|-3$

52. $f(x)=\frac{3}{4x-2}$

In each of the following exercises, use the information given to find $g(x)$.

53. $f(x)=|x+3|$ and $(f+g)(x)=|x+3|+\sqrt{x+5}$

54. $f(x)=x$ and $(f\circ g)(x)=\frac{x+12}{-3}$

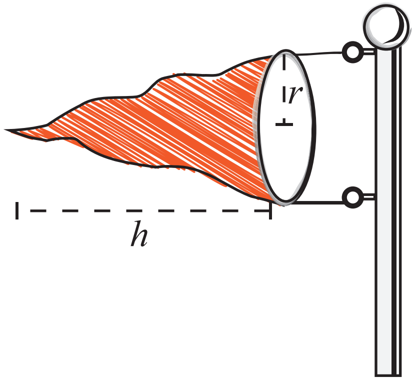
55. $f(x)=x^2-3$ and $(f-g)(x)=x^3+x^2+4$

56. $f(x)=x^2$ and $(g\circ f)(x)=\sqrt{-x^2+5}+4$

APPLICATIONS

57. The volume of a right circular cylinder is given by the formula $V=\pi r^2h$. If the height h is three times the radius r, show the volume V as a function of r.

58. The surface area S of a wind sock is given by the formula $S=\pi r\sqrt{r^2+h^2}$, where r is the radius of the base of the wind sock and h is the height of the wind sock. As the wind sock is being knitted by an automated knitter, the height h increases with time t according to the formula $h(t)=\frac{1}{4}t^2$. Find the surface area S of the wind sock as a function of time t and radius r.



59. The volume V of the wind sock described in the previous exercise is given by the formula $V=\frac{1}{3}\pi r^2 h$ where r is the radius of the wind sock and h is the height of the wind sock. If the height h increases with time t according to the formula $h(t)=\frac{1}{4}t^2$, find the volume V of the wind sock as a function of time t and radius r.

60. A widget factory produces n widgets in t hours of a single day. The number of widgets the factory produces is given by the formula $n(t)=10{,}000t-25t^2$, $0 \le t \le 9$. The cost c in dollars of producing n widgets is given by the formula $c(n)=2040+1.74n$. Find the cost c as a function of time t.

WRITING & THINKING

61. Given two odd functions f and g, show that $f \circ g$ is also odd. Verify this fact with the particular functions $f(x)=\sqrt[3]{x}$ and $g(x)=\frac{-x^3}{3x^2-9}$. Recall that a function is odd if $f(-x)=-f(x)$ for all x in the domain of f.

62. Given two even functions f and g, show that the product is also even. Verify this fact with the particular functions $f(x)=2x^4-x^2$ and $g(x)=\frac{1}{x^2}$. Recall that a function is even $f(-x)=f(x)$ for all x in the domain of f.

As mentioned in Topic 4, a given complex number c is said to be in the Mandelbrot set if, for the function $f(z)=z^2+c$, the sequence of iterates $f(0), f^2(0), f^3(0), \ldots$ stays close to the origin (which is the complex number $0+0i$). It can be shown that if any single iterate falls more than 2 units in distance (magnitude) from the origin, then the remaining iterates will grow larger and larger in magnitude. In practice, computer programs that generate the Mandelbrot set calculate the iterates up to a predecided point in the sequence, such as $f^{50}(0)$, and if no iterate up to this point exceeds 2 in magnitude, the number c is admitted to the set. The magnitude of a complex number $a+bi$ is the distance between the point (a,b) and the origin, so the formula for the magnitude of $a+bi$ is $\sqrt{a^2+b^2}$.

Use the above criterion to determine, without a calculator or computer, if the following complex numbers are in the Mandelbrot set or not.

63. $c=0$ **64.** $c=1$ **65.** $c=i$ **66.** $c=-1$

67. $c=1+i$ **68.** $c=-i$ **69.** $c=1-i$ **70.** $c=-1-i$

71. $c=2$ **72.** $c=-2$