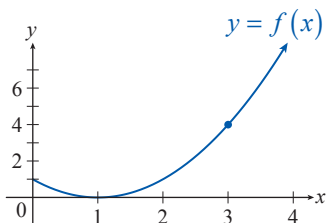


13.2 EXERCISES

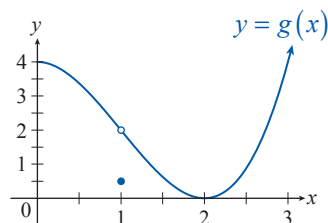
PRACTICE

Use the graph of the function to find the indicated limit (if it exists).

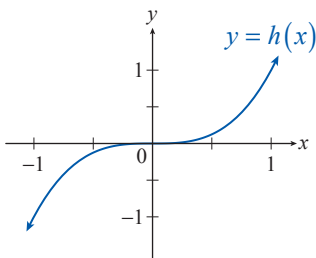
1. $\lim_{x \rightarrow 3} f(x)$



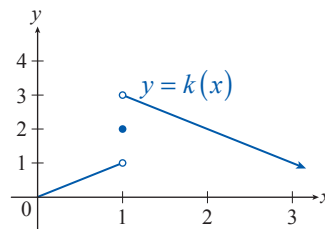
2. $\lim_{x \rightarrow 1} g(x)$



3. $\lim_{x \rightarrow 0} h(x)$



4. $\lim_{x \rightarrow 1} k(x)$



Create a table of values to estimate the value of the indicated limit without graphing the function. Choose the last x -value so that it is no more than 0.001 units from the given c -value.

5. $\lim_{x \rightarrow \sqrt{2}} \frac{x^2 - 2}{x - \sqrt{2}}$

6. $\lim_{x \rightarrow 3} \frac{x^3 - 9x^2 + 27x - 27}{x - 3}$

7. $\lim_{x \rightarrow 1} \frac{x^{10} - 1}{x - 1}$

8. $\lim_{x \rightarrow 0} \frac{4 \sin x}{3x}$

9. $\lim_{x \rightarrow \pi} \frac{2 \cos x - 1}{1 - \sin x}$

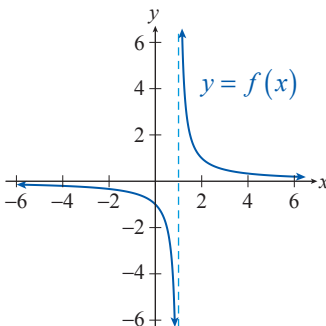
10. $\lim_{x \rightarrow 7} \frac{x^2 - 49}{x - 7}$

11. $\lim_{x \rightarrow 7^+} \frac{x^2 + 49}{x - 7}$

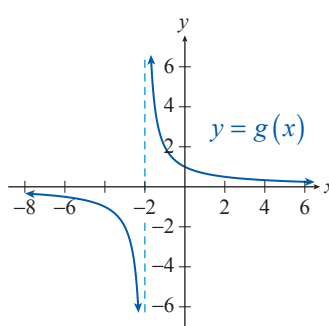
12. $\lim_{x \rightarrow 0^+} \frac{\sqrt{4+x}}{x}$

Use one-sided limit notation to describe the behavior of the function near its vertical asymptote(s).

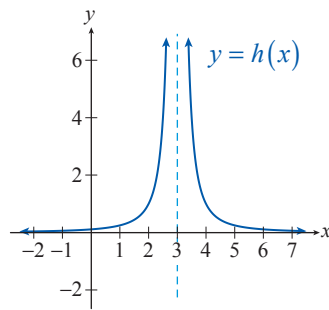
13.



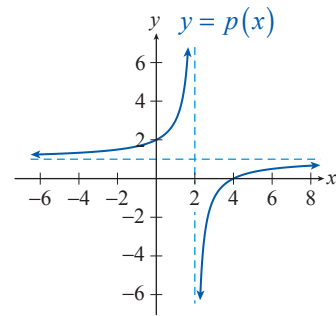
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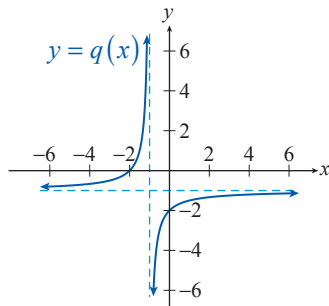
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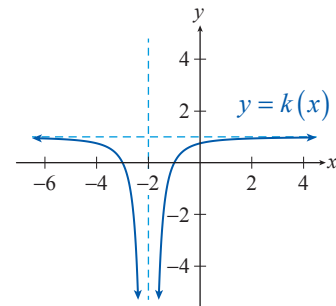
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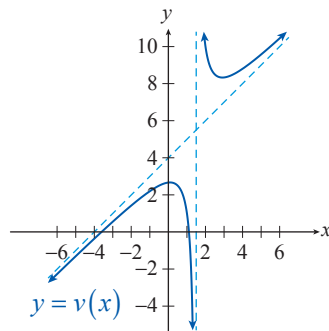
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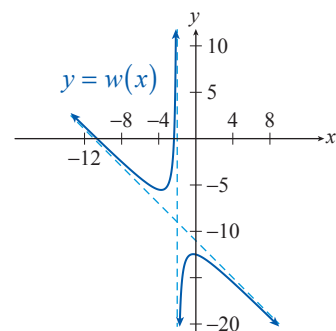
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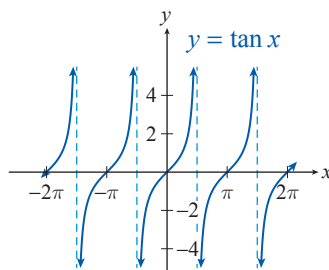
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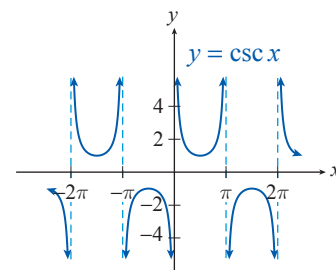
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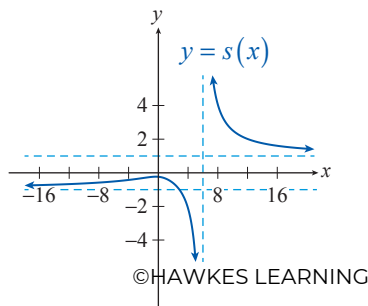
21.



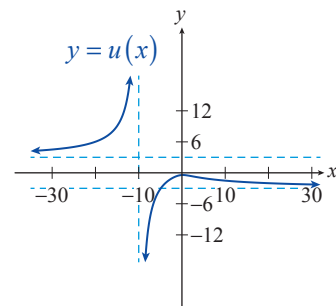
22.



23.



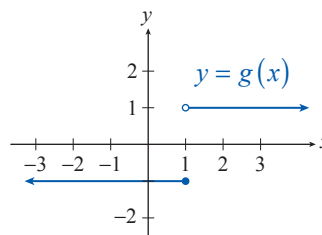
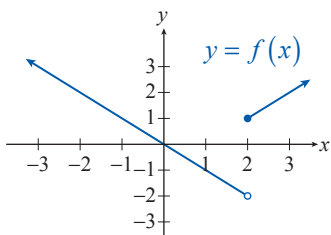
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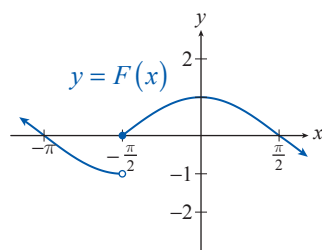
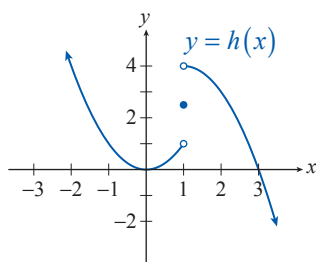
25–36. Consider the functions given in Exercises 13–24. Find their limits at ∞ and $-\infty$ (if they exist). When applicable, use the horizontal asymptote(s) as a guide.

Use the graph to find the indicated one-sided limits, if they exist.

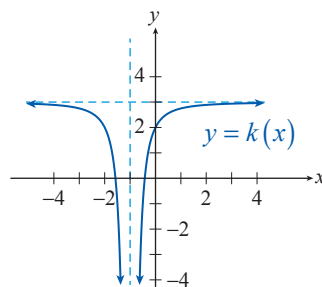
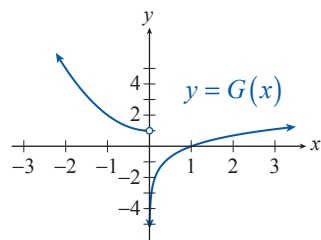
37. a. $\lim_{x \rightarrow 2^-} f(x)$ b. $\lim_{x \rightarrow 2^+} f(x)$ 38. a. $\lim_{x \rightarrow 1^-} g(x)$ b. $\lim_{x \rightarrow 1^+} g(x)$



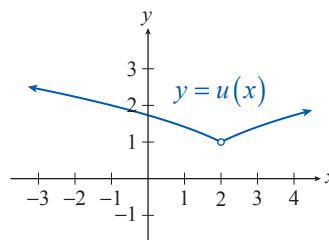
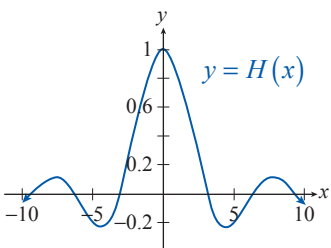
39. a. $\lim_{x \rightarrow 1^-} h(x)$ b. $\lim_{x \rightarrow 1^+} h(x)$ 40. a. $\lim_{x \rightarrow (\frac{\pi}{2})^-} F(x)$ b. $\lim_{x \rightarrow (\frac{\pi}{2})^+} F(x)$



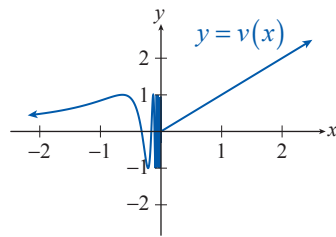
41. a. $\lim_{x \rightarrow 0^-} G(x)$ b. $\lim_{x \rightarrow 0^+} G(x)$ 42. a. $\lim_{x \rightarrow -1^-} k(x)$ b. $\lim_{x \rightarrow -1^+} k(x)$



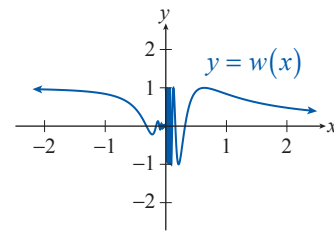
43. a. $\lim_{x \rightarrow 0^-} H(x)$ b. $\lim_{x \rightarrow 0^+} H(x)$ 44. a. $\lim_{x \rightarrow 2^-} u(x)$ b. $\lim_{x \rightarrow 2^+} u(x)$



45. a. $\lim_{x \rightarrow 0^-} v(x)$ b. $\lim_{x \rightarrow 0^+} v(x)$



46. a. $\lim_{x \rightarrow 0^-} w(x)$ b. $\lim_{x \rightarrow 0^+} w(x)$



Use limit notation to describe the unbounded behavior of the given function as x approaches ∞ and/or $-\infty$.

47. $f(x) = x^3$

48. $g(x) = x^2 + 2.1x - 1$

49. $h(x) = -x^4 + 0.2x^3$

50. $k(x) = -0.35x^5 + x + 1.35$

51. $F(x) = \sqrt{x+2}$

52. $G(x) = \sqrt[3]{x+1} - 2.3$

53. $H(x) = |x+2|$

54. $K(x) = -|x+2| - 1$

55. $u(x) = |x-1| + |x+2|$

56. $v(x) = e^{x+2}$

57. $s(x) = -10^{-x} + 1$

58. $t(x) = \ln x - 1$

 **WRITING & THINKING**

Determine whether the given statement is true or false. If false explain or provide a counterexample.

59. If $\lim_{x \rightarrow c} f(x)$ does not exist, then $f(x)$ is undefined at $x = c$.

60. If $f(x)$ is undefined at $x = c$, then $\lim_{x \rightarrow c} f(x)$ does not exist.

61. If $f(x)$ is defined on $(0, \infty)$ and $y = 0$ is a horizontal asymptote for $f(x)$, then there exists a number M such that if $x > M$ then $f(x) < \frac{1}{10^6}$.

62. If $f(x)$ has a vertical asymptote at $x = c$, then either $\lim_{x \rightarrow c} f(x) = \infty$ or $\lim_{x \rightarrow c} f(x) = -\infty$.

63. If $\lim_{x \rightarrow c} f(x)$ does not exist, then $\lim_{x \rightarrow c^-} f(x) \neq \lim_{x \rightarrow c^+} f(x)$ or at least one of $\lim_{x \rightarrow c^-} f(x)$ or $\lim_{x \rightarrow c^+} f(x)$ does not exist.

 TECHNOLOGY

Use a graphing utility to decide whether the given limit exists by evaluating the function at several x -values approaching the indicated c -value. Then graph the function to confirm your findings. Do you obtain misleading graphs when choosing small viewing windows?

64.
$$\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x - 2}$$

65.
$$\lim_{x \rightarrow 3} \frac{x - 2}{x^2 - 5x + 6}$$

66.
$$\lim_{x \rightarrow -1.5} \frac{x^2 - 2.25}{x + 1.5}$$

67.
$$\lim_{x \rightarrow 1} \frac{x - 1}{\sqrt{x} - 1}$$

68.
$$\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3}$$

69.
$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{2x}$$

70.
$$\lim_{x \rightarrow 0^+} \cos\left(\frac{1}{x}\right)$$

71.
$$\lim_{x \rightarrow 0^+} \left[x \cos\left(\frac{1}{x}\right) \right]$$

72. Evaluate the function $f(x) = \left(1 + \frac{1}{x}\right)^x$ for several consecutive positive integers, and try to observe a tendency. Then use a graphing utility to graph $f(x)$ in a large viewing window and try to guess $\lim_{x \rightarrow \infty} f(x)$. Have you seen that number before?