for large n (the value for n at which this happens will vary depending on the technology used). This means that, after a few months, store A can count on roughly 60% of the town's customers and store B can count on roughly 40% (the actual identities of the customers will keep changing from month to month, but the relative proportions will have stabilized). We can verify that the situation is stable by applying the transition matrix to an assumed 1000 customers split 60:40.

$$\begin{bmatrix} 0.7 & 0.45 \\ 0.3 & 0.55 \end{bmatrix}\begin{bmatrix} 600 \\ 400 \end{bmatrix} = \begin{bmatrix} 600 \\ 400 \end{bmatrix}$$

You will see another approach to determining the stable long-term state in the exercises.

11.4 EXERCISES

PRACTICE

Given $A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 0 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -5 \\ 3 & 0 \\ -2 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -1 \\ 6 & 10 \\ -3 & 7 \end{bmatrix}$, and $D = \begin{bmatrix} 3 & 2 & 5 \\ -2 & -4 & 1 \end{bmatrix}$,

determine the following, if possible. See Examples 1, 3, and 4.

1. $3A - B$ **2.** $B - 2D$ **3.** $3C$ **4.** $\frac{1}{2}D$

5. $3D + C$ **6.** $A + B + C$ **7.** $2A + 2B$ **8.** $\frac{3}{2}B + \frac{1}{2}C$

9. $C - 3A$ **10.** $3C - A$ **11.** $4A - 3D$ **12.** $2(A - 3B)$

Determine values of the variables that will make the following equations true, if possible. See Examples 1–4.

13. $\begin{bmatrix} 2a & b & 3 \\ -5 & 9 & 7 \end{bmatrix} = \begin{bmatrix} 6 & -1 & 3 \\ -5 & 9 & c-3 \end{bmatrix}$

14. $\begin{bmatrix} x \\ -9 \\ -1+z \end{bmatrix} = \begin{bmatrix} 8 \\ 3y \\ 5 \end{bmatrix}$

15. $\begin{bmatrix} a & 2b & c \end{bmatrix} + 3\begin{bmatrix} a & 2 & -c \end{bmatrix} = \begin{bmatrix} 8 & 2 & 2 \end{bmatrix}$

16. $\begin{bmatrix} w & 5x \\ 2y & z \end{bmatrix} - 5\begin{bmatrix} w & x \\ y & -z \end{bmatrix} = \begin{bmatrix} w+5 & 0 \\ 6 & 1 \end{bmatrix}$

17. $\begin{bmatrix} 3x \\ 2y \end{bmatrix} + \begin{bmatrix} x \\ -y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

18. $\begin{bmatrix} 2a & 3b & c \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$

19. $\begin{bmatrix} x \\ 3x \end{bmatrix} - \begin{bmatrix} y \\ 2y \end{bmatrix} = \begin{bmatrix} 5 \\ 20 \end{bmatrix}$

20. $7\begin{bmatrix} -1 \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 5x \end{bmatrix} + 3\begin{bmatrix} y \\ 1 \end{bmatrix}$

21. $2\begin{bmatrix} x \\ 2y \end{bmatrix} - 3\begin{bmatrix} 5y \\ -3x \end{bmatrix} = \begin{bmatrix} -9 \\ 31 \end{bmatrix}$

22. $2\begin{bmatrix} 3r & s & 2t \end{bmatrix} - \begin{bmatrix} r & s & t \end{bmatrix} = \begin{bmatrix} 15 & 3 & 9 \end{bmatrix}$

23. $2\begin{bmatrix} 2x^2 & x \\ 7x & 4 \end{bmatrix} - \begin{bmatrix} 5x \\ x-2 \end{bmatrix} = \begin{bmatrix} 2x & 0 \\ 6 & x^2 \end{bmatrix}$

24. $\begin{bmatrix} -x \\ 3 \end{bmatrix} - 5\begin{bmatrix} 2 \\ y \end{bmatrix} = \begin{bmatrix} -2y \\ 3x \end{bmatrix}$

25. $3\begin{bmatrix} 2a \\ -a \end{bmatrix} - 3\begin{bmatrix} 3b \\ 2b \end{bmatrix} = \begin{bmatrix} 3 \\ -54 \end{bmatrix}$

26. $2\begin{bmatrix} -s \\ -7 \end{bmatrix} + 2\begin{bmatrix} -2r \\ r \end{bmatrix} = -2\begin{bmatrix} 8 \\ s \end{bmatrix}$

Evaluate the following matrix products, if possible. See Examples 5 and 6.

27. $\begin{bmatrix} 3 & -2 & 1 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 0 & 3 \\ 9 & 4 \end{bmatrix}$

28. $\begin{bmatrix} 0 & -8 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 3 & 7 \end{bmatrix}$

29. $\begin{bmatrix} 3 & 7 \end{bmatrix}\begin{bmatrix} 0 & -8 \\ 5 & 6 \end{bmatrix}$

30. $\begin{bmatrix} 5 & 0 & -3 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ -6 \end{bmatrix}$

31. $\begin{bmatrix} 3 & 9 & -4 \\ 0 & 0 & 2 \\ 5 & -2 & 7 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$

32. $\begin{bmatrix} 4 \\ 2 \\ -6 \end{bmatrix}\begin{bmatrix} 5 & 0 & -3 \end{bmatrix}$

33. $\begin{bmatrix} -3 & -6 & -3 \end{bmatrix}\begin{bmatrix} 6 & 9 \\ 6 & -8 \\ -8 & 8 \end{bmatrix}$

34. $\begin{bmatrix} 4 & -5 \\ 7 & -9 \end{bmatrix}\begin{bmatrix} -8 & 3 \end{bmatrix}$

35. $\begin{bmatrix} -3 \\ -5 \\ -6 \end{bmatrix}\begin{bmatrix} -5 & 1 & 8 \end{bmatrix}$

Given $A = \begin{bmatrix} -3 & 1 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 8 & -5 \end{bmatrix}$, $C = \begin{bmatrix} 4 \\ 7 \\ -2 \end{bmatrix}$, and $D = \begin{bmatrix} -5 & 4 \\ -1 & -1 \end{bmatrix}$, determine the following, if possible. See Examples 5 and 6.

36. AB

37. BA

38. $BA + B$

39. A^2

40. C^2

41. CB

42. D^2

43. $CD + C$

44. DA

45. AD

46. DB

47. $(BD)A$

APPLICATIONS

48. Suppose that each month 20% of store B's customers switch to store A, and 10% of store A's customers switch back to store B. At the start of January, store A has 300 customers and store B has 700. How many customers can each store expect at the start of February? At the start of March?

49. Given the percentages stated in the last problem, what long-term proportion of the town's customers can each store expect? (A graphing utility may be used to compute high powers of the transition matrix, or you can use the method described in the following exercise.)

WRITING & THINKING

50. Suppose P is a 2×2 transition matrix, and we want to determine the effect of applying high powers of P to the matrix

$$\begin{bmatrix} x \\ y \end{bmatrix},$$

where $x + y$ is a fixed constant, say c. (In our competing store situation, $x + y = 1000$.) If the long-term behavior approaches a steady state, as in our two-store example, then there is some value for x and some value for y such that $x + y = c$ and

$$P\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}.$$

In other words, once the steady state has been reached, applying the matrix P to it has no effect on the state.

We can use this fact to actually solve for x and y as follows. Given the matrix

$$P = \begin{bmatrix} 0.7 & 0.45 \\ 0.3 & 0.55 \end{bmatrix},$$

write the equation

$$P\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

in system form. You should find that the two equations that result are actually identical. But if we now also use the fact that $x + y = 1000$, we can solve for the variables and find that $x = 600$ and $y = 400$. Verify that this is indeed the case.

51. Your friend Jared is having trouble with matrices, so you offer to study with him. Check his solution of the following problem. If the solution is incorrect, explain the error that has been made.

$$2\begin{bmatrix} -8 & -9 & -1 \\ -8 & 1 & 5 \end{bmatrix} - 2\begin{bmatrix} -2 & 3 \\ 5 & -7 \\ -8 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & -18 & -2 \\ -16 & 2 & 10 \end{bmatrix} + \begin{bmatrix} 4 & -6 \\ -10 & 14 \\ 16 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -16+4-18-10-2+16 & -16-6-18+14-2+2 \\ -16+4+2-10+10+2 & -16-6+2+14+10+2 \end{bmatrix}$$

$$= \begin{bmatrix} -26 & -26 \\ -8 & 6 \end{bmatrix}$$

TECHNOLOGY

Given $A = \begin{bmatrix} 3.8 & -1.2 & 4.6 \end{bmatrix}, B = \begin{bmatrix} -8.2 & -4.9 \\ 7.4 & -1.3 \\ 3.5 & -2.1 \end{bmatrix}, C = \begin{bmatrix} 6.3 \\ 5.7 \end{bmatrix}$, and $D = \begin{bmatrix} 2.8 & -7.1 \\ -5.4 & 6.6 \end{bmatrix}$,

use a graphing utility to determine the following, if possible.

52. BD

53. CA

54. D^2

55. AB

56. DC

57. BC