NOTE

The same concept can be used to focus sunlight, intensely heating a small area at the focus. This is called a parabolic furnace.

Solution

First, we need to draw a picture of the situation. In order to make the math as easy as possible, we can locate the origin of our coordinate system at the vertex of a parabolic cross section of the mirror, and we can assume the parabola opens upward.

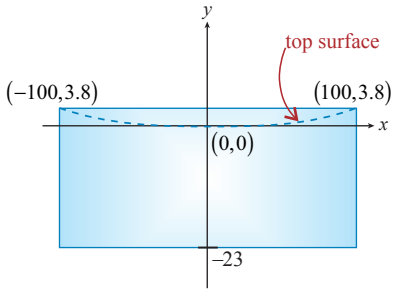


FIGURE 9

Since we placed the vertex at $(0,0)$, we know the equation $x^2 = 4py$ describes the shape of the cross section for some value p. If we can determine p, we can find the focus of the parabola.

To find p, we need the coordinates of another point on the parabola. The difference in thickness of the mirror between the center and the outer rim is 3.8 inches, and the mirror has a diameter of 200 inches, so the two points $(-100,3.8)$ and $(100,3.8)$ must lie on the graph. Plugging a point into the equation $x^2 = 4py$, we can solve for p.

$$\begin{aligned} (100)^2 &= 4p(3.8) \\ 10000 &= 15.2p \\ p &\approx 657.9 \text{ inches} \\ p &\approx 54.8 \text{ feet} \end{aligned}$$

We know that the focus of a parabola is p units from the vertex, so the focus of the Hale Telescope is nearly 55 feet from the mirror.

10.2 EXERCISES

PRACTICE

Graph the following parabolas and determine the focus and directrix of each. See Examples 1 and 2.

1. $(x+1)^2 = 4(y-3)$
2. $y^2 - 4y = 8x + 4$
3. $(y-4)^2 = -2(x-1)$
4. $(y-1)^2 = 8(x+3)$
5. $(x-2)^2 = 4(y+1)$
6. $(y+1)^2 = -12(x+1)$
7. $y^2 = 6x$
8. $x^2 = 2y$

9. $x^2 = 7y$

10. $x^2 = -5y$

11. $y = -12x^2$

12. $x = -4y^2$

13. $x = \frac{1}{6}y^2$

14. $\frac{1}{5}x = -y^2$

15. $y^2 + 16x = 0$

16. $-6x - 2y^2 = 0$

17. $4y + 2x^2 = 4$

18. $2y^2 - 10x = 10$

19. $y^2 + 2y + 12x + 37 = 0$

20. $x^2 - 8y = 6x - 1$

21. $x^2 + 6x + 8y = -17$

22. $x^2 + 2x + 8y = 31$

23. $y^2 + 6y - 2x + 13 = 0$

24. $y^2 - 2y - 4x + 13 = 0$

Match the following equations to their graphs.

25. $(x+2)^2 = 3(y-1)$

26. $(y-1)^2 = 2(x+2)$

27. $y^2 = 4(x+1)$

28. $x^2 = 2(y+1)$

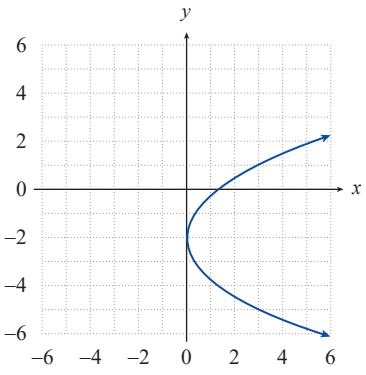
29. $(x-1)^2 = -(y-2)$

30. $(y+2)^2 = 3x$

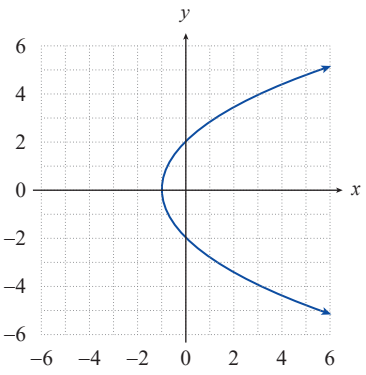
31. $(x-2)^2 = 4y$

32. $y^2 = -2(x+1)$

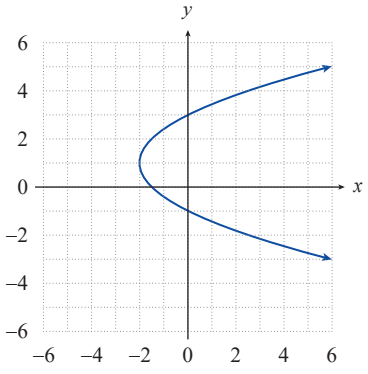
a.



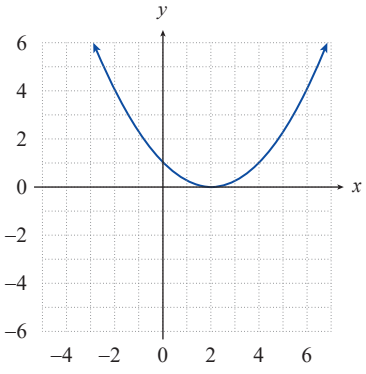
b.



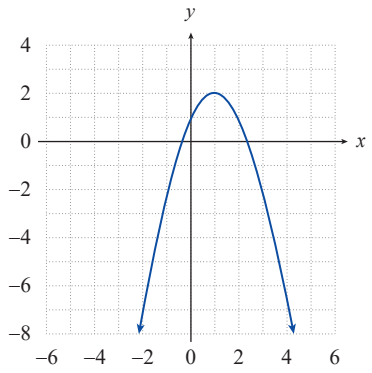
c.



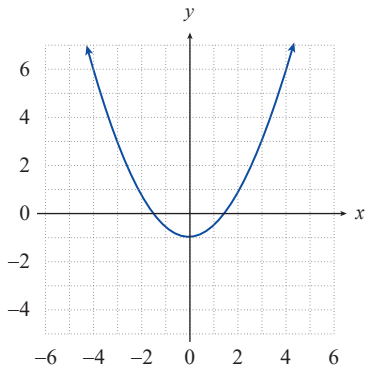
d.



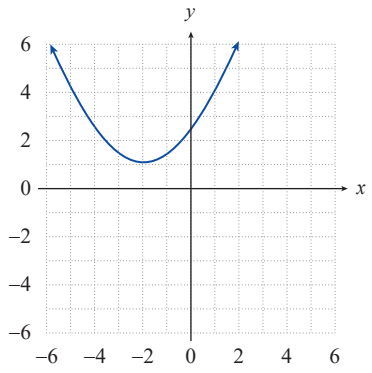
e.



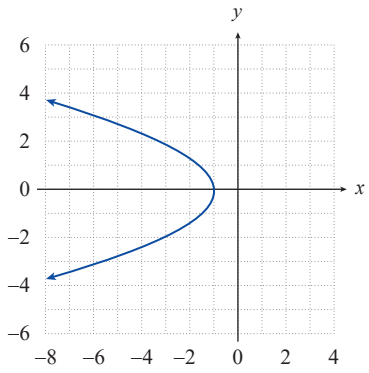
f.



g.



h.



Find the equation, in standard form, for the parabola with the given properties or with the given graph. See Example 3.

33. Focus at $(-2,1)$, directrix is the y-axis.

34. Focus at $(-2,1)$, directrix is the x-axis.

35. Vertex at $(3,-1)$, focus at $(3,1)$.

36. Symmetric with respect to the line $y=1$, directrix is the line $x=2$, and $p=-3$.

37. Vertex at $(3,-2)$, directrix is the line $x=-3$.

38. Vertex at $(7,8)$, directrix is the line $x=\frac{27}{4}$.

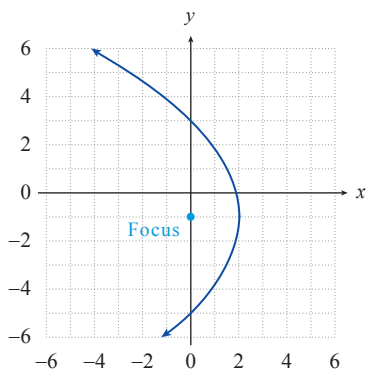
39. Focus at $\left(-3,-\frac{3}{2}\right)$, directrix is the line $y=-\frac{1}{2}$.

40. Vertex at $(3,16)$, focus at $(3,11)$.

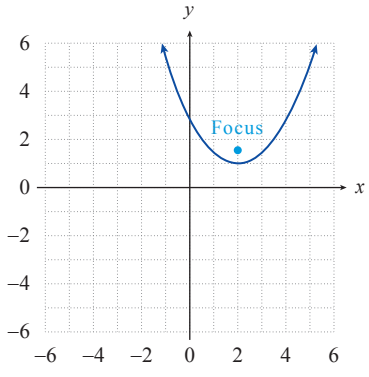
41. Vertex at $(-4,3)$, focus at $\left(-\frac{3}{2},3\right)$.

42. Symmetric with respect to the x-axis, focus at $(-3,0)$, and $p=2$.

43.



44.



APPLICATIONS

45. One design for a solar furnace is based on the paraboloid formed by rotating the parabola $x^2 = 8y$ around its axis of symmetry. The object to be heated in the furnace is then placed at the focus of the paraboloid (assume that x and y are in units of feet). How far from the vertex of the paraboloid is the hottest part of the furnace?

46. A certain brand of satellite dish antenna is a paraboloid with a diameter of 6 feet and a depth of 1 foot. How far from the vertex of the dish should the receiver of the antenna be placed given that the receiver should be located at the focus of the paraboloid?

47. A spotlight is made by placing a strong lightbulb inside a reflective paraboloid formed by rotating the parabola $x^2 = 6y$ around its axis of symmetry (assume that x and y are in units of inches). In order to have the brightest, most concentrated light beam, how far from the vertex should the bulb be placed?

TECHNOLOGY

Use a graphing utility to graph the following equations.

48. $3x^2 - 4y + 24x = -56$

49. $y^2 + 2y = 8x - 41$

50. $y^2 - 6y + 4x = -17$

51. $x^2 - 6x + 12y + 21 = 0$