Example 8: Escape Speed

The speed required for an object to escape from the gravitational pull of a planet is called the **escape speed** of the planet. The escape speed is given by the equation $v_e = \sqrt{\frac{2GM}{r}}$, where v_e is the escape speed, G is the universal gravitation constant, M is the mass of the planet, and r is the radius of the planet. Solve this equation for r.

Solution

We follow the same procedure for solving radical equations.

$v_e = \sqrt{\frac{2GM}{r}}$ — The radical expression is already isolated.

$v_e^2 = \frac{2GM}{r}$ — Square both sides to eliminate the radical.

$r = \frac{2GM}{v_e^2}$ — Solve for r.

1.9 EXERCISES

PRACTICE

Solve the following rational equations. See Examples 1 and 2.

1. $\frac{2x^3+4x^2}{x^2-4x-12} = \frac{-7x-6}{x-6}$

2. $\frac{-x^2}{x-1} - 3 = 0$

3. $\frac{3}{x-2} + \frac{2}{x+1} = 1$

4. $\frac{x}{x-1} + \frac{2}{x-3} = -\frac{2}{x^2-4x+3}$

5. $\frac{1}{t-3} + \frac{1}{t+2} = \frac{1}{t+3}$

6. $\frac{z}{6+z} + \frac{z-1}{6-z} = \frac{z}{6-z}$

7. $\frac{y}{y-1} + \frac{2}{y-3} = \frac{y^2}{y^2-4y+3}$

8. $\frac{2}{2x+1} - \frac{x}{x-4} = \frac{-3x^2+x-4}{2x^2-7x-4}$

9. $\frac{2}{2b+1} + \frac{2b^2-b+4}{2b^2-7b-4} = \frac{b}{b-4}$

10. $\frac{2}{n+3} + \frac{3}{n+2} = \frac{6}{n}$

11. $\frac{1}{x-3} + \frac{1}{x+3} = \frac{2x}{x^2-9}$

12. $\frac{3}{x-1} - \frac{3}{x+2} = \frac{9}{x^2+x-2}$

13. $\frac{1}{|x-3|} = 2$

14. $\frac{3}{|x+1|} = 1$

15. $\frac{1}{|x-3|} + \frac{1}{|x+1|} = 1$

16. $\frac{1}{x-2} + \frac{2}{|x-1|} = 2$

Solve the following radical equations. See Example 6.

17. $\sqrt{4-x}-x=2$

18. $\sqrt{3y+4}+\sqrt{5y+6}=2$

19. $\sqrt{3-3x}-3=\sqrt{3x+2}$

20. $\sqrt{x^2-4x+5}-x+2=0$

21. $\sqrt{x^2-4x+4}+2=3x$

22. $\sqrt{50+7s}-s=8$

23. $\sqrt[3]{3-2x}-\sqrt[3]{x+1}=0$

24. $\sqrt[4]{x^2-x}=\sqrt[4]{x-1}$

25. $\sqrt[4]{2x+3}=-1$

26. $\sqrt{11x+3}+4x=18$

27. $\sqrt{2b-1}+3=\sqrt{10b-6}$

28. $\sqrt{5x+5}=\sqrt{4x-7}+2$

29. $\sqrt{x+10}+1=x-1$

30. $\sqrt{x+1}+10=x-1$

31. $\sqrt{x^2-10}-1=x+1$

32. $\sqrt[3]{5x^2-14x}=-2$

33. $\sqrt[5]{7t^2+2t}=\sqrt[5]{5t^2+4}$

34. $\sqrt[3]{y^3-7y+2}=\sqrt[3]{2-3y}$

35. $\sqrt{14y^2-18y+4}+2=2y$

36. $\sqrt{9x+4}=\sqrt{7x+1}+1$

37. $\sqrt{4z+41}+3=z+2$

Solve the following equations. See Example 7.

38. $(x+3)^{\frac{1}{4}}+2=0$

39. $(2x-5)^{\frac{1}{4}}=(x-1)^{\frac{1}{4}}$

40. $(2x-1)^{\frac{2}{3}}=x^{\frac{1}{3}}$

41. $(3y^2+9y-5)^{\frac{1}{2}}=y+3$

42. $(3x-5)^{\frac{1}{5}}=(x+1)^{\frac{1}{5}}$

43. $w^{\frac{3}{5}}+8=0$

44. $z^{\frac{4}{3}}-\frac{16}{81}=0$

45. $x^{\frac{2}{3}}-\frac{25}{49}=0$

46. $(x^2+21)^{\frac{-3}{2}}=\frac{1}{125}$

47. $(x-2)^{\frac{2}{3}}=(14-x)^{\frac{1}{3}}$

48. $(x^2+7)^{\frac{-3}{2}}=\frac{1}{64}$

49. $(y-2)^{\frac{2}{3}}=(13y-66)^{\frac{1}{3}}$

Solve each of the following formulas for the indicated variable. See Example 8.

50. The formula $T=2\pi\sqrt{\frac{l}{g}}$ gives the period T of a pendulum of length l. Solve this formula for l.

51. The formula $c=\sqrt{a^2+b^2}$ gives the length of the hypotenuse c of a right triangle. Solve this formula for a.

52. Einstein's Theory of Relativity states that $E=mc^2$. Solve this equation for c.

53. The formula $\omega=\sqrt{\frac{k}{m}}$ gives the angular frequency ω of a mass m suspended from a spring of spring constant k. Solve this formula for m.

54. The formula $V = \frac{4}{3}\pi r^3$ gives the volume of a sphere with radius r. Solve the equation for r.

55. The formula $F = \frac{mv^2}{r}$ gives the force on an object in circular motion. Solve the equation for v.

56. The formula for lateral acceleration, used in automotives, is $a = \frac{1.227r}{t^2}$. Solve this equation for t.

57. According to one guideline regarding body mass index (BMI), a healthy mass for an adult male can be found using the formula $m = 23h^2$, where m is expressed in kilograms and h in meters. Solve this equation for h.

58. Kepler's Third Law is $T^2 = \frac{4\pi^2 r^3}{GM}$. It relates the period T of a planet to the radius r of its orbit and the sun's mass M. Solve this formula for r.

59. The equation $r = \frac{2gm}{c^2}$ is the Schwarzschild Radius Formula used to find the radius of a black hole in space. Solve the equation for c.

60. The total mechanical energy of an object with mass m at height h in a closed system can be written as $ME = \frac{1}{2}mv^2 + mgh$. Solve for v, the velocity of the object, in terms of the given quantities.

61. Recall, the formula for the Pythagorean Theorem states that $a^2 + b^2 = c^2$. Solve this formula for b.

62. In a circuit with an AC power source, the total impedance Z depends on the resistance R, the capacitance C, the inductance L, and the frequency of the current ω according to $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$. Solve this equation for the inductance L.

63. The formula used to find the orbital period for circular Keplerian orbits is $P = \frac{2\pi}{\sqrt{\frac{u}{a^3}}}$. Solve this equation for a.

APPLICATIONS

64. If Joanne were to paint her living room alone, it would take 5 hours. Her sister Lisa could do the job in 7 hours. How long would it take them working together?

65. The hot water tap can fill a given sink in 4 minutes. If the cold water tap is turned on as well, the sink fills in 1 minute. How long would it take for the cold water tap to fill the sink alone?

66. The hull of Jack's yacht needs to be cleaned. He can clean it by himself in 5 hours, but he asks his friend Thomas to help him. If it takes 3 hours for the two men to clean the hull of the boat, how long would it have taken Thomas alone?

67. Two hoses, one of which has a flow rate three times the other, can together fill a tank in 3 hours. How long does it take each of the hoses individually to fill the tank?

68. Officials begin to release water from a full man-made lake at a rate that would empty the lake in 12 weeks, but a river that can fill the lake in 30 weeks is replenishing the lake at the same time. How long does it take to empty the lake?

69. In order to flush deposits from a radiator, a drain that can empty the entire radiator in 45 minutes is left open at the same time it is being filled at a rate that would fill it in 30 minutes. How long does it take for the radiator to fill?

70. Jimmy and Janice are picking strawberries. Janice can fill a bucket in a half hour, but Jimmy continues to eat the strawberries that Janice has picked at a rate of one bucket per 1.5 hours. How long does it take Janice to fill her bucket?

71. A farmer can plow a given field in 2 hours less time than it takes his son. If they acquire two tractors and work together, they can plow the field in 5 hours. How long does it take the father alone? Round your answer to one decimal place.