

Example 11: Solving Equations by Factoring

Solve the equation $x^{-2} - 7x^{-1} + 12 = 0$ by factoring.

Solution

While the given equation is not quadratic, it certainly bears a strong resemblance to one that is. In fact, if we make the substitution $y = x^{-1}$, we obtain the quadratic equation

$$y^2 - 7y + 12 = 0,$$

which we can solve by factoring the trinomial as $(y-3)(y-4)$. This gives us $y = 3$ or $y = 4$. Translating this back to the variable x , we have $x^{-1} = 3$ or $x^{-1} = 4$, so $x = \frac{1}{3}$ or $x = \frac{1}{4}$.

1.8 EXERCISES**PRACTICE**

Solve the following quadratic equations by factoring. See Example 1.

1. $2x^2 - x = 3$

2. $x^2 - 14x + 49 = 0$

3. $y(2y+9) = -9$

4. $3x^2 + 33 = 2x^2 + 14x$

5. $5x^2 + 2x + 3 = 4x^2 + 6x - 1$

6. $(x-7)^2 = 16$

7. $4x^2 - 9 = 0$

Solve the following quadratic equations by taking square roots. See Example 2.

8. $(x-3)^2 = 9$

9. $(a-2)^2 = -5$

10. $(y-18)^2 - 1 = 0$

11. $9 = (3s+2)^2$

12. $(2x-1)^2 = 8$

13. $x^2 - 4x + 4 = 49$

Solve the following quadratic equations by completing the square. See Example 3.

14. $x^2 + 8x + 7 = -8$

15. $2x^2 + 6x - 10 = 10$

16. $2x^2 + 7x - 15 = 0$

17. $4x^2 - 4x - 63 = 0$

18. $4x^2 - 56x + 195 = 0$

19. $y^2 + 22y + 96 = 0$

Solve the following quadratic equations using the quadratic formula. See Example 4.

20. $3x^2 - 4 = -x$

21. $2.6z^2 - 0.9z + 2 = 0$

22. $a(a+2) = -1$

23. $3x^2 - 2x = 0$

24. $6x^2 + 5x - 4 = 3x - 2$

25. $4x^2 - 14x - 27 = 3$

Calculate the discriminant and use it to determine the number and type of solutions of the following quadratic equations. See Example 5.

26. $2x^2 - x + 5 = 0$

27. $x^2 + x - 5 = 0$

28. $-3x^2 - 2x + 2 = 0$

29. $2x^2 - 4x + 2 = 0$

Solve the following quadratic equations using any appropriate method. See Example 6.

30. $y^2 + 9y = -40.50$

31. $(z-11)^2 = 9$

32. $256t^2 - 324 = 0$

33. $(9y-6)^2 = 121y^2$

34. $2x^2 + 8x - 3 = 6x$

35. $4z^2 + 14z = 10z - 3$

36. $x^2 - 6x = 27$

37. $y^2 - 2y + 1 = -289$

38. $-3(b+5)^2 = -768$

39. $7x^2 - 42x = 0$

40. $5x^2 - 5x - 10 = 0$

41. $4w^2 + 10w + 5 = 3w^2 + 18w - 10$

42. $|x^2 - 3x| = 2$ (**Hint:** Replace $|x^2 - 3x|$ first with $x^2 - 3x$ and solve the resulting equation, then replace it with $-(x^2 - 3x)$ and solve the resulting equation.)

43. $|x^2 - x| = 2$

44. $|x^2 - 8| = 1$

Solve the following quadratic-like equations. See Example 8.

45. $(x-1)^2 + (x-1) - 12 = 0$

46. $(y-5)^2 - 11(y-5) + 24 = 0$

47. $(x^2+1)^2 + (x^2+1) - 12 = 0$

48. $(x^2-13)^2 + (x^2-13) - 12 = 0$

49. $(x^2 - 2x + 1)^2 + (x^2 - 2x + 1) - 12 = 0$

50. $2y^{\frac{2}{3}} + y^{\frac{1}{3}} - 1 = 0$

51. $2x^{\frac{2}{3}} - 7x^{\frac{1}{3}} + 3 = 0$

52. $(x^2 - 6x)^2 + 4(x^2 - 6x) - 5 = 0$

53. $(y^2 - 5)^2 + 5(y^2 - 5) - 36 = 0$

54. $(x^2 + 7)^2 + 8(x^2 + 7) + 12 = 0$

55. $(t^2 - t)^2 - 8(t^2 - t) + 12 = 0$

56. $2x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 2 = 0$

57. $3x^{\frac{2}{3}} - x^{\frac{1}{3}} - 2 = 0$

58. $5y^{\frac{2}{3}} + 33y^{\frac{1}{3}} + 18 = 0$

Solve the following polynomial equations by factoring. See Example 9.

59. $a^3 - 3a^2 = a - 3$

60. $2x^3 + x^2 + 2x + 1 = 0$

61. $x^4 + 5x^2 - 36 = 0$

62. $y^3 + 8 = 0$

63. $5s^3 + 6s^2 - 20s = 24$

64. $8a^3 - 27 = 0$

65. $16a^4 = 81$

66. $6x^3 + 8x^2 = 14x$

67. $14x^3 + 27x^2 - 20x = 0$

68. $5z^3 + 28z^2 = 49z$

69. $27x^3 + 64 = 0$

70. $x^3 - 4x^2 + x = 4$

Solve the following equations by factoring. See Examples 10 and 11.

71. $3x^{\frac{11}{3}} + 2x^{\frac{8}{3}} - 5x^{\frac{5}{3}} = 0$

72. $(y-6)^{\frac{-5}{2}} + 7(y-6)^{\frac{-3}{2}} = 0$

73. $y^{-2} - 2y^{-1} + 1 = 0$

74. $2x^{\frac{13}{5}} - 5x^{\frac{8}{5}} + 2x^{\frac{3}{5}} = 0$

75. $(2x-5)^{\frac{1}{3}} - 3(2x-5)^{\frac{-2}{3}} = 0$

76. $x^{-4} - 13x^{-2} + 36 = 0$

77. $y^{\frac{7}{2}} - 5y^{\frac{5}{2}} + 6y^{\frac{3}{2}} = 0$

78. $(t+4)^{\frac{2}{3}} + 2(t+4)^{\frac{8}{3}} = 0$

79. $y^{-2} - 2y^{-1} - 35 = 0$

80. $x^{\frac{11}{2}} - 6x^{\frac{9}{2}} + 9x^{\frac{7}{2}} = 0$

81. $5y^{\frac{11}{3}} + 3y^{\frac{8}{3}} - 2y^{\frac{5}{3}} = 0$

82. $(3x-3)^{\frac{-1}{3}} - 5(3x-3)^{\frac{-4}{3}} = 0$

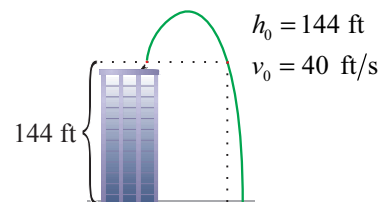
83. $x^{-2} + 8x^{-1} + 15 = 0$

84. $(y+3)^{\frac{2}{5}} + 4(y+3)^{\frac{7}{5}} = 0$

APPLICATIONS

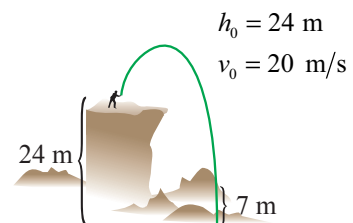
85. How long would it take for a ball dropped from the top of a 144-foot building to hit the ground?

86. Suppose that instead of being dropped, as in Exercise 85, a ball is thrown upward with a velocity of 40 feet per second from a height of 144 feet. Assuming it misses the building on the way back down, how long after being thrown will it hit the ground?



87. A slingshot is used to shoot a BB at a velocity of 96 feet per second straight up from ground level. When will the BB reach its maximum height of 144 feet?

88. A rock is thrown off a cliff with a velocity of 20 meters per second. It is thrown upward from a height of 24 meters and misses the cliff on the way back down. When will the rock be 7 meters from ground level? (Round your answer to one decimal place.)



89. Luke, an experienced bungee jumper, leaps from a tall bridge and falls toward the river below. The bridge is 170 feet above the water and Luke's bungee cord is 110 feet long unstretched. When will Luke's cord begin to stretch? (Round your answer to one decimal place.)

 **WRITING & THINKING**

90. Compare the answers to Exercises 85 and 87 and explain why they are the same.

Use the connection between solutions of polynomial equations and polynomial factoring to answer the following questions. See the discussion after Example 3.

91. Factor the quadratic $x^2 - 6x + 13$.
92. Factor the quadratic $9x^2 - 6x - 4$.
93. Factor the quadratic $4x^2 + 12x + 1$.
94. Factor the quadratic $25x^2 - 10x + 2$.
95. Determine b and c so that the equation $x^2 + bx + c = 0$ has the solution set $\{-3, 8\}$.
96. Find b , c , and d so the equation $x^3 + bx^2 + cx + d = 0$ has solutions of -3 , -1 , and 5 .
97. Find b , c , and d so the equation $x^3 + bx^2 + cx + d = 0$ has solutions of -2 , 0 , and 6 .
98. Find b and c so the equation $x^3 + bx^2 + cx = 0$ has solutions of 0 , 1 , and -7 .
99. Find a , c , and d so the equation $ax^3 + 4x^2 + cx + d = 0$ has solutions of -4 , 6 , and -6 .
100. Find a , b , and d so the equation $ax^3 + bx^2 + 3x + d = 0$ has solutions of -3 , $-\frac{1}{2}$, and 0 .
101. Find a , b , and c so the equation $ax^3 + bx^2 + cx + 6 = 0$ has solutions of $-\frac{3}{5}$, $\frac{2}{3}$, and 1 .

 **TECHNOLOGY**

Use a graphing utility to solve the following polynomial equations.

102. $5x^2 - 3x = 17$
103. $5x^2 - 3x = -17$
104. $(a + 4)(4a - 3) = 5$
105. $10\pi r + \pi r^2 = 107$
106. $4.8x^2 + 3.5x - 9.2 = 0$
107. $(3x - 1)(3 - x) = 2x + 5$