

Example 5: Roots and Complex Numbers

Simplify the following expressions.

a. $(2 - \sqrt{-3})^2$

b. $\frac{\sqrt{4}}{\sqrt{-4}}$

Solution

$$\begin{aligned} \text{a. } (2 - \sqrt{-3})^2 &= (2 - \sqrt{-3})(2 - \sqrt{-3}) \\ &= 4 - 4\sqrt{-3} + \sqrt{-3}\sqrt{-3} \\ &= 4 - 4i\sqrt{3} + (i\sqrt{3})^2 \\ &= 4 - 4i\sqrt{3} - 3 \\ &= 1 - 4i\sqrt{3} \end{aligned}$$

Each $\sqrt{-3}$ is converted to $i\sqrt{3}$ before multiplying.

$$\begin{aligned} \text{b. } \frac{\sqrt{4}}{\sqrt{-4}} &= \frac{2}{2i} \\ &= \frac{1}{i} \\ &= -i \end{aligned}$$

We simplify each radical before dividing.

We already simplified $\frac{1}{i}$ in Example 4c, so we quickly obtain the correct answer of $-i$.**1.5 EXERCISES****PRACTICE**

Evaluate the following square root expressions. See Example 1.

1. $\sqrt{-25}$

2. $\sqrt{-12}$

3. $-\sqrt{-27}$

4. $-\sqrt{-100}$

5. $\sqrt{-32x}$, $x > 0$

6. $\sqrt{-x^2}$

7. $\sqrt{-29}$

8. $(-i)^2\sqrt{-64}$

Simplify the following complex expressions. See Examples 2, 3, and 4.

9. $(4 - 2i) - (3 + i)$

10. $(4 - i)(2 + i)$

11. $(3 - i)^2$

12. i^7

13. $(7i - 2) + (3i^2 - i)$

14. $(3 + i)(3 - i)$

15. $(5 - 3i)^2$

16. $(5 + i)(2 - 9i)$

17. i^{13}

18. $(9 - 4i)(9 + 4i)$

19. $11i^{314}$

20. i^{132}

21. $(7 - 3i)^2$

22. $(4 - 3i)(7 + i)$

23. $(3i)^2$

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|-------------------------|----------------------|---|
| 24. $(1+i)+i$ | 25. $i(5-i)$ | 26. $i^{-11}\left(\frac{6}{i^3}\right)$ |
| 27. $(10i^2-9i)+(9+5i)$ | 28. $(-5i)^3$ | 29. $i^7\left(\frac{49}{7i^2}\right)$ |
| 30. $\frac{1+2i}{1-2i}$ | 31. $\frac{10}{3-i}$ | 32. $\frac{i}{2+i}$ |
| 33. $\frac{1}{i^9}$ | 34. $(2+5i)^{-1}$ | 35. i^{-25} |
| 36. $\frac{1}{i^{27}}$ | 37. $\frac{52}{5+i}$ | 38. $(2-3i)^{-1}$ |
| 39. $\frac{4i}{5+7i}$ | 40. i^{-4} | 41. $\frac{5+i}{4+i}$ |

Simplify the following expressions. See Example 5.

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|--------------------------------|------------------------------|--------------------------------------|
| 42. $(3+\sqrt{-2})^2$ | 43. $(1+\sqrt{-6})^2$ | 44. $\frac{\sqrt{18}}{\sqrt{-2}}$ |
| 45. $(\sqrt{-32})(-\sqrt{-2})$ | 46. $(\sqrt{-9})(\sqrt{-2})$ | 47. $\frac{\sqrt{-98}}{3i\sqrt{-2}}$ |
| 48. $(\sqrt{-8})(\sqrt{-2})$ | 49. $(5+\sqrt{-3})^2$ | 50. $\frac{\sqrt{-72}}{5i\sqrt{-2}}$ |

APPLICATIONS

51. Electrical engineers often use j , rather than i , to represent imaginary numbers. This is to prevent confusion with their use of i , which often represents current. Under this convention, assume the impedance of a particular part of a series circuit is $4-3j$ ohms and the impedance of another part of the circuit is $2+6j$ ohms. Find the total impedance of the circuit. (Impedances in series are simply added.)
52. Consider the formula $V = IZ$, where V is voltage (in volts), I is current (in amps), and Z is impedance (in ohms). If you know the current of a circuit is $5-4j$ amps and the impedance is $8+2j$ ohms, find the voltage.
53. If you know the voltage of a circuit is $35+5j$ volts and the current is $3+j$ amps, find the impedance.

WRITING & THINKING

54. Explain why it may be useful to be able to use imaginary numbers in real-world math.

 TECHNOLOGY

Use a graphing utility to simplify the following complex expressions.

55. $\frac{3-2i}{1+i}$

56. $(3-2i)^4$

57. $\frac{2500}{(3+i)^4}$

58. $(2-5i)(3+7i)(1-4i)$

59. $(1+i)^5(3-i)^2$

60. $\frac{3+7i}{(2-5i)(1+3i)}$

61. $\frac{6+3i}{2-4i}$

62. $(5-3i)^5$

63. $\frac{400}{(6+2i)^3}$

64. $(6-3i)(8+i)(7-4i)$

65. $(5-3i)^4(7+2i)^3$

66. $\frac{4+3i}{(7-2i)(5+4i)}$