

Example 12: Factoring Expressions with Noninteger Rational Exponents

Factor each of the following algebraic expressions.

a. $3x^{-\frac{2}{3}} - 6x^{\frac{1}{3}} + 3x^{\frac{4}{3}}$

b. $(x-1)^{\frac{1}{2}} - (x-1)^{-\frac{1}{2}}$

Solution

$$\begin{aligned} \text{a. } 3x^{-\frac{2}{3}} - 6x^{\frac{1}{3}} + 3x^{\frac{4}{3}} &= 3x^{-\frac{2}{3}}(1 - 2x + x^2) \\ &= 3x^{-\frac{2}{3}}(x^2 - 2x + 1) \\ &= 3x^{-\frac{2}{3}}(x-1)(x-1) \\ &= 3x^{-\frac{2}{3}}(x-1)^2 \end{aligned}$$

Under the guidelines above, we factor out $3x^{-\frac{2}{3}}$. Note that we use the properties of exponents to obtain the terms in the second factor.

We notice that the second factor is a second-degree trinomial, which is itself factorable. In fact, it is an example of a perfect square trinomial.

$$\begin{aligned} \text{b. } (x-1)^{\frac{1}{2}} - (x-1)^{-\frac{1}{2}} &= (x-1)^{-\frac{1}{2}}[(x-1) - 1] \\ &= (x-1)^{-\frac{1}{2}}(x-2) \end{aligned}$$

In this example we factor out $(x-1)^{-\frac{1}{2}}$ again using the properties of exponents to obtain the terms in the second factor.

1.3 EXERCISES**PRACTICE**

Classify each of the following expressions as either a polynomial or not a polynomial. For those that are polynomials, identify the degree of the polynomial and the number of terms (use the words monomial, binomial, and trinomial if applicable). See Example 1.

1. $3x^{\frac{3}{2}} - 2x$

2. $17x^2y^5 + 2z^3 - 4$

3. $5x^{10} + 3x^3 - 2y^3z^8 + 9$

4. πx^3

5. 8

6. 0

7. $7^3xy^2 + 4y^4$

8. abc^2d^3

9. $4x^2 + 7xy + 5y^2$

10. $3n^4m^{-3} + n^2m$

11. $\frac{y^2z}{4} + 2yz^4$

12. $6x^4y + 3x^2y^2 + xy^5$

Write each of the following polynomials in descending order, and identify **a.** the degree of the polynomial, and **b.** the leading coefficient.

13. $-4x^{10} - x^{13} + 9 + 7x^{11}$

14. $9x^8 - 9x^{10}$

15. $4s^3 - 10s^5 + 2s^6$

16. $4 - 2x^5 + x^2$ 17. $9y^6 - 2 + y - 3y^5$ 18. $4n + 6n^2 - 3$
 19. $8z^2 + \pi z^5 - 2z + 1$ 20. $-6y^5 - 3y^7 + 12y^6$

Add or subtract the polynomials, as indicated. See Example 2.

21. $(-4x^3y + 2xz - 3y) - (2xz + 3y + x^2z)$ 22. $(4x^3 - 9x^2 + 1) + (-2x^3 - 8)$
 23. $(x^2y - xy - 6y) + (xy^2 + xy + 6x)$ 24. $(5x^2 - 6x + 2) - (4 - 6x - 3x^2)$
 25. $(a^2b + 2ab + ab^2) - (ab^2 + 5ab + a^2b)$ 26. $(x^4 + 2x^3 - x + 5) - (x^3 - x - x^4)$
 27. $(xy - 4y + xy^2) + (3y - x^2y - xy)$ 28. $(-8x^4 + 13 - 9x^2) - (8 - 2x^4)$

Multiply the polynomials, as indicated. See Examples 3 and 4.

29. $(3a^2b + 2a - 3b)(ab^2 + 7ab)$ 30. $(x^2 - 2y)(x^2 + y)$
 31. $(3a + 4b)(a - 2b)$ 32. $(x + xy + y)(x - y)$
 33. $(6x - 3y)(x + 6y)$ 34. $(5y + x)(4y - 2x)$
 35. $(7y^2 + x)(y^2 - 5x)$ 36. $(y^2 + x)(3y^2 - 7x)$
 37. $(6xy^2 - 3x + 4y)(x^2y + 6xy)$ 38. $(2xy^2 + 4y - 6x)(x^2y - 5xy)$

Use a special product formula to perform the indicated operations. See Example 5.

39. $(3a + b)^2$ 40. $(x - 5y)^2$
 41. $(2x - 3y)(2x + 3y)$ 42. $(x - 3y)^2$
 43. $(-x - 2y)^2$ 44. $(\sqrt{2x} - \sqrt{3y})(\sqrt{2x} + \sqrt{3y})$
 45. $\left(\frac{1}{x} - y\right)\left(\frac{1}{x} + y\right)$ 46. $[(x - y) - z][(x - y) + z]$

Factor each polynomial by factoring out the greatest common factor. See Example 6.

47. $4m^2n + 16m^3 + 7m$ 48. $3a^2b + 3a^3b - 9a^2b^2$
 49. $5(a - b^2) + (a - b^2)$ 50. $3x^3y - 9x^4y + 12x^3y^2$
 51. $2x^6 - 14x^3 + 8x$ 52. $27x^7y + 9x^6y - 9x^4yz$
 53. $(x^3 - y)^2 - (x^3 - y)$ 54. $6xy^3 + 9y^3 - 12xy^4$
 55. $12y^6 - 8y^2 - 16y^5$ 56. $(2x + y^2)^4 - (2x + y^2)^6$

Factor each polynomial by grouping. See Example 7.

57. $a^3 + ab - a^2b - b^2$ 58. $ax - 2bx - 2ay + 4by$
 59. $z + z^2 + z^3 + z^4$ 60. $x^2 + 3xy + 3y + x$
 61. $nx^2 - 2y - 2x^2 + ny$ 62. $2ac - 3bd + bc - 6ad$
 63. $ax - 5bx + 5ay - 25by$ 64. $3ac - 5bd + bc - 15ad$

Use the special factoring patterns to factor the following binomials. See Example 8.

65. $4x^2 - 121$

66. $64z^3 + 216$

67. $49a^2 - 144b^2$

68. $x^3 - 27y^3$

69. $25x^4y^2 - 9$

70. $27a^9 + 8b^{12}$

71. $x^3 - 1000y^3$

72. $64x^6 - 125y^3z^9$

73. $m^6 + 125n^9$

74. $49a^6 - 9b^2c^4$

75. $27x^6 - 8y^{12}z^3$

76. $(3x-6)^2 - (y-2x)^2$

77. $16z^2y^4 - 9x^8$

78. $512x^6 + 729y^3$

79. $343y^9 - 27x^3z^6$

80. $(2x+y^2)^2 - (y^2-3)^2$

Factor the following trinomials. See Examples 9, 10, and 11.

81. $x^2 + 2x - 15$

82. $x^2 + 6x + 9$

83. $x^2 - 2x + 1$

84. $x^2 - 5x + 6$

85. $x^2 - 4x + 4$

86. $x^2 + 5x + 4$

87. $y^2 + 14y + 49$

88. $x^2 - 3x - 18$

89. $x^2 + 13x + 22$

90. $y^2 + y - 42$

91. $y^2 - 9y + 8$

92. $6x^2 + 5x - 6$

93. $5a^2 - 37a - 24$

94. $25y^2 + 10y + 1$

95. $5x^2 + 27x - 18$

96. $6y^2 - 13y - 8$

97. $16y^2 - 25y + 9$

98. $10m^2 + 29m + 10$

99. $8a^2 - 2a - 3$

100. $20y^2 + 21y - 5$

101. $12y^2 - 19y + 5$

102. $10y^2 - 11y - 6$

Factor the following algebraic expressions. See Example 12.

103. $(2x-1)^{\frac{3}{2}} + (2x-1)^{\frac{1}{2}}$

104. $2x^{-2} + 3x^{-1}$

105. $7a^{-1} - 2a^{-3}b$

106. $(3z+2)^{\frac{5}{3}} - (3z+2)^{\frac{2}{3}}$

107. $10y^{-2} - 2y^{-5}x$

108. $4y^{-3} + 12y^{-4}$

109. $(5x+7)^{\frac{7}{3}} - (5x+7)^{\frac{4}{3}}$

110. $(8x+6)^{\frac{7}{2}} - (8x+6)^{-\frac{1}{2}}$

111. $7y^{-1} + 5y^{-4}$

112. $5x^{-4} - 4x^{-5}y$

 WRITING & THINKING

- 113.** Pneumothorax is a disease in which air or gas collects between the lung and the chest wall, causing the lung to collapse. When this disease is evident, the following formula is used to determine the degree of collapse of the lungs, represented as a percent:

$$\text{Degree} = 100 \left(1 - \frac{L^3}{H^3} \right)$$

In this formula, L is the diameter of one lung and H is the diameter of one hemithorax (or half the chest cavity). Is this formula a polynomial? If so, find its degree and the number of terms. If not, explain.

- 114.** You are trying to find a formula for the area of a certain trapezoid. You know the height of the trapezoid is x^2 , the bottom base is $2x^2 + 4$, and the top base is $6x + 2$. Insert these values into the formula for the area of a trapezoid. Is the result a polynomial? If so, find the degree of the polynomial, the leading coefficient, and the number of terms in the polynomial. If not, explain.
- 115. a.** Given a rectangular picture frame with sides of $2x + 1$ and $x^3 + 4$, find the area of the picture frame. Is the result a polynomial? If so, find the degree of the polynomial, the leading coefficient, and the number of terms in the polynomial. If not, explain.
- b.** Now find the perimeter of the picture frame. Is this a polynomial? If so, find the degree of the polynomial, the leading coefficient, and the number of terms in the polynomial. If not, explain.

